Louis Nirenberg

Né le 28 Février 1925 au Canada (Hamilton)
Nationalité américaine.

Carrière professionnelle

Louis Nirenberg a été formé au départ à l'Université Mc Gill (université anglophone de Montréal) avant de venir préparer une thèse à l'université de New York où il soutient son PHD en 1949 sous la direction de J. Stoker.

Il rejoint ensuite la faculté de l'université de New York et devient membre du Courant Institut dès sa création. Il y restera sans interruption toute sa vie académique jusqu'à sa retraite en 1999.

Il est professeur émérite à l'université de New York, membre de la National Academy of Sciences (Etats-Unis) et membre associé étranger de l'Académie des Sciences en France depuis 1989.

Prix et distinctions

Nirenberg a obtenu de nombreux prix au cours de sa carrière

- Prix Bôcher de l'AMS (1959)
- Prix Crafoord (1982), décerné par l'académie suédoise dans les domaines où il n'y a pas de prix Nobel
- Médaille nationale de la science (1995) (plus haute distinctinction attribuée pour des contributions à la science aux Etats-Unis)

Encadrement de recherche

Louis Nirenberg a eu 46 étudiants en thèse et plus de 245 descendants (= étudiant d'étudiant …).

Activités de recherche

Difficile de donner une brève description d'une œuvre immense dans la théorie des équations aux dérivées partielles et de l'analyse non-linéaire, qui commence juste après la deuxième guerre mondiale et ne s'est pas encore arrêtée. Etudiant à Mc Gill en physique, il se tourne vers les mathématiques à son arrivée à New York. Sa thèse

préfigure son orientation future. Il y résout en effet un problème de Weyl en géométrie différentielle. Sa preuve s'appuie sur une réduction à la preuve de l'existence d'une solution d'une équation aux dérivées partielles elliptique s'appuyant sur la preuve de nouvelles estimations a priori.
On trouve ici en germe de nombreux aspects de son œuvre. Avec S. Agmon et A. Douglis, Nirenberg
a démontré en toute généralité la régularité des solutions faibles des systèmes elliptiques ; la méthode des quotients différentiels initiée à cette fin est enseignée aux étudiants de master. C'est ensuite l'étude avec Joseph Kohn de problèmes de régularité pour le problème du dbar-Neumann qui conduit à la création et au développement de la théorie des opérateurs pseudo-différentiels, point de départ de l'analyse microlocale qui deviendra une discipline en elle-même à partir des années 70., avec en point de mire la célèbre conjecture de Nirenberg-Trèves sur la résolubilité locale dont la solution finale par N. Dencker ne date que de quatre ans.
Pour revenir à une autre thématique présente dans sa thèse, Nirenberg est l'auteur de nombreuses inégalités jouant un rôle fondamental en analyse linéaire et non linéaire : estimations de Cagliardo-Nirenberg, John-Nirenberg et Caffarelli-Kohn-Nirenberg.

Louis Nirenberg est aussi un as du principe-maximum qu'il a su utiliser pour de nombreux types d'équations elliptiques ou paraboliques héritées de problèmes provenant de la géométrie ou de la physique. Il a dans ce domaine beaucoup interagi avec l'Ecole Française et en particulier avec H. Brezis (sur la positivité des solutions) et H. Berestycki (en continuation du célèbre théorème de symétrie de Gidas-Ni-Nirenberg).

Pour les problèmes fortement non-linéaires Nirenberg a été également un pionnier dans le développement de la théorie des points critiques (on peut citer par exemple sa suite de travaux sur l'équation de Monge-Ampère avec Caffarelli et Spruck) .

Le nom de Nirenberg est aussi inévitablement associé à l'analyse complexe autour de l'étude des structures presque complexes et de la reconnaissance des opérateurs de Cauchy-Riemann Deux contributions (une avec Newlander et l'autre avec Kodaira et Spencer) donnent des réponses à des questions fondamentales de la théorie.

La topologie est également une théorie dans laquelle Nirenberg a su puiser (et développer) des techniques pouvant ensuite jouer un rôle crucial dans l'analyse des équations aux dérivéees partielles. Un de ses aspects est la théorie du degré . Citons ici un résultat très récent obtenu en collaboration avec H. Brézis sur l'existence d'une théorie du degré pour des applications dans la classe VMO (Vanishing Mean Oscillation) qui ne sont même pas continues.

Ainsi par ses travaux , souvent obtenus en collaboration, Louis Nirenberg a marqué la théorie des équations aux dérivées partielles. Au delà de ses travaux et de ses nombreuses directions de recherche, il a été et reste par ces nombreux surveys, conférences et suggestions amicales un inspirateur irremplaçable pour les jeunes et les nombreux spécialistes du monde entier.

En conclusion, ce prix récompense un des plus grands acteurs des mathématiques en analyse, dont l'œuvre couvre toute la deuxième moitié du vingtième siècle et dont l'influence reste considérable en ce début du 21 ème siècle. Tous ceux qui l'ont rencontré, et ils sont nombreux en France, pays qu'il visite fréquemment, peuvent aussi témoigner de son grand humanisme.

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ISSN: 1072-6691. URL: http://ejde.math.txstate.edu or http://ejde.math.unt.edu ftp ejde.math.txstate.edu (login: ftp) LOUIS NIRENBERG AND KLAUS SCHMITT: THE JOY OF DIFFERENTIAL EQUATIONS JEAN MAWHIN
Abstract. A tribute to Louis Nirenberg for his 80th birthday anniversary and to Klaus Schmitt for his 65th birthday anniversary.

2. Louis Nirenberg

À tout seigneur tout honneur, as we say in French, let me start by the inhuman task of describing for you the achievements of Louis Nirenberg, the results of about fifty-five years of cruising in the mathematical sky at the highest altitude. Louis Nirenberg was born February 28 1925. Can you do anything else than mathematics when your birth place is Hamilton. Louis refused taking any advantage of this, and decided to study physics at McGill. But nobody can escape to his predestination. The young physicist started to work, on the atomic bomb, at the National Research Council in Montr´eal, and had for colleagues Richard Courant's older son Ernst and his Canadian wife. He asked her to consult her father-in-law about the best place in New York to study theoretical physics. Courant's answer was an interview at New York University, followed by an assistantship in mathematics. This is the reason why Canada does not have today the world leadership in nuclear weapons, and why its military threatening of United States only exists in the imagination of Michael Moore.

Then comes a first keyword in Louis's career: fidelity. He entered the Mathematics Department of New York University as a graduate student in 1945 and he never left it since for another institution. There should be big celebrations in Courant this year. In addition, more than one quarter of his papers have appeared in the Communications in Pure and Applied Mathematics, with a record of four in 1953, his first year of publication. They include his PhD thesis, defended in 1949, with Jim Stoker as adviser and Kurt Friedrichs as model. The thesis contains the complete positive solution of Weyl's problem in differential geometry : given a Riemannian metric on the unit sphere, with positive Gauss curvature, can you embed this 2-sphere isometrically into 3-space as a convex surface ? So we must retain geometry as another keyword for Nirenberg's contributions, an area that he revisited regularly during his life with results on surfaces with curvature of fixed sign, on the rigidity of closed surfaces, on the intrinsic norms on complex manifolds (with Chern and Levine) and on the characterization of convex bodies. Many historians claim that the whole career of a scientist can be read in his
first publications like in a cristal ball : Louis Nirenberg's proof of Weyl's problem consists in a reduction to a partial differential equation, solved, by a continuity method, through the obtention of new a priori estimates. This equation is elliptic, and ellipticity is definitely another keyword of Nirenberg's mathematical work.

More than one third of his articles contain the work 'elliptic' just in their title, and a much larger fraction deals with elliptic equations or systems. There is hardly one aspect of those equations he has not considered.
The regularity problem for example. Royal paths exist to prove the existence and uniqueness of weak solutions of elliptic problems, but a much more difficult question is to know how regular is a weak solution. Today, no graduate student ignores Nirenberg's method of differences for proving interior and boundary regularity, or the Agmon-Douglis-Nirenberg's extension to general elliptic systems!
For Navier-Stokes equation, essential questions about regularity are still open seventy years after Leray pioneering work (and the nugget worth one million dollars !). Among the best results today is Caffarelli-Kohn-Nirenberg's estimate of the Hausdorf measure of the set of singularities. It is the study, with Joseph Kohn, of regularity problems in several complex variables connected with boundary regularitycfor the dbar-Neumann problem, which led Kohn and Nirenberg to create andcdeveloped the fruitful concept of pseudo-differential operator, generalizing and unifyingcthe singular integral and partial differential operators. Their importance incanalysis needs not to be emphasized. Incidentally, Friedrichs found the name for the new baby.
It is hard to be a democrat when you are an analyst. Your success depends much more on creating and using inequalities than equalities. In this respect, Louis is one of the tycoons of this mathematical wild liberalism. We all know, teach and use the Gagliardo-Nirenberg, the John-Nirenberg and the Caffarelli-Kohn-Nirenberg inequalities, which extend and unify so many fundamental ones. Louis has expressed himself, at several occasions, his love of inequalities : Friedrichs was a great lover of inequalities and that affected me very much. The point of view was that the inequalities are more interesting than the equalities.

I love inequalities. If somebody shows me a new inequality, I
say: Oh, that's beautiful, let me think about it, and I may have
some ideas connected to it.
Those inequalities play a fundamental role in partial differential equations for the obtention of a priori estimates for the possible solutions or through some operator inequalities. Descartes is famous, in his philosophy of existence, for his sentence
Je pense, donc je suis – I am thinking, hence I do exist.
The philosophy of the solutions of partial differential equations could be summarized by something like
I am limited, hence I do exist.
You can now wonder how such an strong advocate of inequalities was able to build so many tight professional and personal relations with French people. You all know the motto of France: *Liberté, égalité, fraternité – Liberty, equality, fraternity !*
His cult of inequalities did not prevent Louis to be elected a foreign member of the French Academy of Science and promoted Doctor honoris causa of the University of Paris ! Much more, Louis has won over the heart of Nanette, a charming French lady! I bed that his fights for liberty and his cult of fraternity have won over his unlimited taste for inequality. When you observe that ninety percents of Louis's papers are written in collaboration, you can add fraternity to the keywords describing his personality. Any afterdinner speech being uncomplete without one bad play upon words, Liberty gives me the opportunity to mention the free boundary value problems and Louis's fundamental contributions, with David Kinderlehrer, to the regularity of their solutions.
Positivity is another feature of Louis's character : almost each of us has received from him, one day or another, a positive encouragement or a stimulating remark. His forty-five PhD students (several ones are here) would all testify in this direction. In elliptic and parabolic differential equations, positivity is an essential manifestation of the maximum principle, and Louis's virtuosity in using this beautiful instrument is

unequalled. Do I need to recall the Gidas-Ni-Nirenberg symmetry theorem and its variations, in particular in joint work with Henri Berestycki and others. Louis Nirenberg is the Paganini of the maximum principle, and the fact is caught for ever in an AMS movie, a deserved reward to a movies lover. Positivity can also occurs in variational problems, for example in Brezis-Nirenbergs discussion
of positive solutions of nonlinear elliptic equations of the Yamabe type, which have inspired so many other mathematicians by opening the way to problems with lack of compactness. This is only one of the many contributions of Nirenberg in critical point theory, advocated and developed in inspiring survey papers. Critical point theory is one of the keys to attack nonlinearity. A pioneer in nonlinear elliptic equations (the topics of his first published paper), Louis has returned, at various stages of his career, to fully nonlinear elliptic equations to make striking breakthroughs, like the ones in a series of papers with Caffarelli and Spruck on Monge-Amp`ere and related equations. His invited lecture at the Internationa Congress of Mathematicians of 1962 in Stockholm, contains two sentences that I always offer as a guide to my students dealing with nonlinear problems. The first one is:

Most results for nonlinear problems are still obtained via linear ones, i.e. despite the fact that the problems are nonlinear no because of it.

The second one comments a result of Moser:
The nonlinear character of the equations is used in an essential way, indeed he obtains results because of the nonlinearity not despite it. Both aspects have been masterfully explored by Louis Nirenberg, but there is some Dr Jekyll and Mr Hyde aspect we cannot hide: this master of nonlinearity never hesitated to betray the club in making striking contributions to linear problems, forcing us to retain linearity as another keyword. Louis probably could argue that some of his fundamental contributions to linear elliptic equations, in particular his generalized Schauder and Sobolev estimates, were motivated by solving new nonlinear problems. But how can he justify the necessary and sufficient conditions for local solvability of general linear partial differential equations obtained with François Treves ?
Another keyword of Louis's mathematical Wonderland is holomorphy. One important question is the study of almost complex structures : how to recognize the Cauchy-Riemann operators when given in some arbitrary coordinate system. For higher dimensions, necessary integrability conditions are needed. Newlander and Nirenberg have shown them to be sufficient as well. Another question is the existence of deformations of complex structures, families of diffeomorphic complex manifolds, differentiable in the parameter. The answer is given by a theorem of Kodaira-Nirenberg-Spencer. Needless to say that some exciting partial differential equations are hidden in the proof.
Topology must be retained as an other keyword of Louis's mathematical activity. In recent work with Haim Brezis, he created a big scandal among topologists when developing – motivated by some nonlinear models of physics – a degree theory for mappings which need not be continuous ! They live instead in the VMO space of functions with vanishing mean oscillation, a close relative of the BMO space of functions with bounded mean oscillation, invented, for other purposes, by Fritz John and Louis Nirenberg, and widely used in many parts of analysis. The BMO space is an enlargement of the space of essentially bounded measurable functions, and the VMO space a subspace of BMO, the closure of the space of continuous functions. But Louis's links with topology are somewhat older. Already in 1970, he introduced the use of stable homotopy in generalizing Landesman-Lazer conditions for bounded nonlinear perturbations of elliptic operators with positive index.
In terms of classical degree, instead of questioning the continuity of the map, he was questioning the equal dimensions of the underlying spaces. This paper has had a

smaller lineage as most other ones, because, as Louis frankly observed : So far, not natural example has come up in which it has been used.
When I am depressed, I like to remember that I gave one, neither natural nor elliptic, just a simple system of two second order ordinary differential equations with three nonlinear boundary conditions. Other important contributions of Brezis-Nirenberg were inspired by Landesman-Lazer's paper.
Incidentally, I am not sure everybody is aware of the role played by Louis in advertising, within the PDE world, Landesman-Lazer's result. In French language, we like to oppose the savoir faire, the ability, to the faire savoir, the advertising. I think that the marvelous saga of the Landesman-Lazer problem is a wonderful combination of Landesman-Lazer "savoir faire" with Louis Nirenberg's "faire savoir". And this is far from an unique example of the generosity (another keyword) and enthusiasm of Louis for presenting, in his marvellous lectures and in survey papers, the results of other mathematicians. This is often an opportunity for a silver nugget to be changed in a golden one. In such a lecture, Louis always starts by apologizing that he is not really an expert in the area he is presenting. Each evening, before sleeping, I ask God to make me a non-expert like Louis Nirenberg. To punish me for this arrogance, God has changed me into an afterdinner speaker, transforming this personal punishment into a collective one.
The world would not turn round if this combination of exceptional personality and outstanding achievements had not been recognized and crowned by distinctions and awards. I mention only a few, to avoid starting another lecture and becoming a tormentor for Louis's simplicity. The Bocher, Crafoord and Steele prizes, the National Medal of Science, fellowship from all important academies in the world and honorary degrees from many prestigious universities. They show the universal appreciation of a man whose human qualities are at the level of his mathematical accomplishments.