FIVE LIVES OF A *GEOMETRIA SUBTERRANEA* (1708-1785). AUTHORSHIP AND KNOWLEDGE CIRCULATION IN PRACTICAL MATHEMATICS

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ABSTRACT. — In 1708, the subterranean geometer August Beyer (1677-1753) wrote a manuscript entitled *Geometria subterranea*, detailing the instruments and operations of underground surveying, of which several handwritten copies still exist. A modified version of this practical geometry was published by its author in 1749 and a second edition was printed in 1785, well after Beyer's death, by a mathematics professor of the Freiberg mining academy, J.F. Lempe (1757-1801). Analysing successive versions of this text shows the evolution of the discipline in the 18th century.

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In addition, several questions about subterranean geometry have a general interest for the history of practical mathematics. The concept of authorship, in both senses of paternity of a text and of moral authority, proves to be ambigous, and in last resort unfit to understand the evolution and circulation of this kind of useful knowledge. Moreover, the growing institutionalization of engineering training in the 18th century could be thought to imply a swift progress in the mathematization of actual practices. The example of Beyer's *Geometria subterranea* shows that the short-term influence of technical schools was sometimes mixed, while highlighting other circulation realms for practical geometry.

RÉSUMÉ (Cinq vies d'une *Geometria subterranea* (1708-1785). Autorité et circulation des connaissances en mathématiques pratiques.)

En 1708, le géomètre souterrain Auguste Beyer (1677-1753) rédige un manuscrit intitulé *Geometria subterranea* dans lequel il décrit les instruments et opérations de l'arpentage minier. Plusieurs copies manuscrites de ce texte ont été conservées. Une version modifiée de ce texte fut publiée par son auteur en 1749, tandis qu'une seconde édition réalisée par J.F. Lempe (1757-1801), professeur de mathématiques à l'Académie des mines de Freiberg, parut en 1785, c'est-à-dire bien après la mort de Beyer. Les versions successives de ce texte témoignent de l'évolution de la discipline au cours du XVIII^e siècle.

Plusieurs questions relatives à cette discipline ont en outre un intérêt plus général pour l'histoire des mathématiques pratiques. Le concept d'autorité, au double sens de paternité d'un texte et d'influence morale, se revèle ambigu et *in fine* inadapté pour comprendre l'évolution et la circulation de ce type de savoirs utiles. On pourrait de plus penser que l'institutionnalisation croissante de la formation des ingénieurs au xVIII^e siècle implique une mathématisation croissante des pratiques effectives. L'exemple de la *Geometria subterranea* d'Auguste Beyer montre que le succès des écoles techniques fût, dans un premier temps, relativement mitigé, tout en mettant en évidence des dynamiques alternatives de circulation des savoirs géométriques pratiques.

1. INTRODUCTION

This paper describes the evolution of subterranean geometry in the 18th century, a discipline then belonging to the mathematical sciences. We focus on one particular work, a *Geometria subterranea* written around 1708 by the mining official August Beyer (1677-1753). At least five versions of this text have been preserved, three of them manuscripts from the first half of the century, as well as two printed editions respectively published in 1749 and 1785. This dense material allows for a minute analysis of its content and a close comparison of successive rewritings of the original text. We can thus track the evolution of underground surveying over the course of a century.

The fact that a manuscript written in 1708 would still be in use three generations later might be interpreted as a relative stagnation. On the contrary, this article shows that despite an apparent stability, numerous evolutions concerned both the mathematical content and the social structure of this discipline. When Beyer wrote the first version of his manuscript, this topic was neither taught in schools nor at universities while manuscripts were transmitted from master to pupils in a guild-like fashion [Sennewald 2002]. When the last version of this work appeared in 1785, subterranean geometry was a well-identified discipline and a public object of science—at least in the German-speaking world. Mining academies had been created, and a dozen textbooks were available while new instruments or methods were regularly discussed in technical journals.¹

A similar trend could be observed in many other fields of practical mathematics: the 18th century saw a significant increase of the ambitions, if not of the achievements, of mechanical and mathematical sciences. ² New institutions were created for military and civil engineering, from the *École royale des ponts et chaussées* (1747) and the *École royale du génie de Mézières* (1748) in France to the *Bergakademien* in Saxony (1765) and several other German states [Taton 1964 ; Guagnini 2004].

In that context, we will also use Beyer's text as a case study to ask more general questions about the elaboration and diffusion of knowledge in practical mathematical sciences. Practical geometry, and its subdiscipline subterranean geometry, were especially important and widespread in mining regions in order to direct extracting operations, draw maps and settle property limits underground. Following its numerous metamorphoses over the course of the 18th century helps understand its evolution and reveals a general pattern, with analytical methods increasingly replacing the graphical and piecemeal approaches of the previous century.

The importance of the institutionalization of engineering schools in continental Europe in spurring the mathematization of various technical activities has often been underlined.³ Without denying the long-term

¹ For a brief overview of the state of subterranean geometry at the end of the 18th century and especially the creation of the mining academy of Freiberg, see [Morel 2013, p. 164-190].

² About mathematization in the 18th century, see [Lowood 1990] for a summary of the debate and several interesting case studies. About the limits and slow progress of this process see [Vérin 1993, p. 243-333, p. 357], [Belhoste et al. 1990].

³ Institutional history and the history of teaching mathematical and mechanical sciences have produced important works such as [Taton 1964] or [Belhoste 1998]. See [Schubring 2003] for an overwiew about the institutional history of mathematics.

influence of these institutions, their immediate impact on contemporary technicians has to be reassessed and put in perspective. As early as 1782, the professor of mathematics at the mining academy of Freiberg claimed that academic teaching had within a few years deeply improved the practice of subterranean geometry:

And so subterranean geometry stayed, for those who had to perform it, in the usual craft usage, until this most valuable institution, the mining academy that was built here in 1765, gave to everyone who had the capacity and desire of thinking, through the learning of mathematics and other auxiliary sciences, the opportunity not only [to master] the principles of subterranean geometry but also its complete scope, and could convince himself not only of the basics of subterranean geometry, but also of its whole range, and in how many kinds of cases it may be applied usefully to mining, and in what kind of tighter connection it stands with mathematics. ⁴

This assertion, full of emphasis and rhetorical elaboration, sums up almost perfectly the challenges that historians of practical mathematics face about the 18th century. New institutions systematically blamed the artisan character and lack of theory they considered to be inherent to previous methods. They rejected the use of manuscripts and advocated an open circulation of knowledge. They also pretended to have instantly improved actual practices, as if heavy scientific books presenting intricate methods could both convince practitioners and solve every concrete problem at once, without any downside. To balance these obviously one-sided reports, statements from practitioners about the early history of these institutions are generally scarce and equally biased.

To give a more nuanced view of the development of practical mathematics, we need to understand how the practitioners themselves were working, reflecting on and improving their methods in the early eighteenth century. The question of authorship turns out to be a major, and evolving, issue. How were knowledge and know-how produced by underground surveyors? What did authorship mean in circles where methods were constantly transmitted and improved? Mathematical practitioners

⁴ [Lempe 1782, p. 10–11], introduction by J.F.W. Charpentier: "Es blieb also die Markscheidekunst immer noch bey denen, die sie ausüben sollten, in der gewöhnlichen handwerksmäßigen Behandlung, bis durch die preißwürdigsten Anstalten, der im Jahr 1765 hier errichteten Bergwerksakademie die Gelegenheit allgemein wurde, wodurch sich ein jeder, der Fähigkeiten und Lust zum Denken hatte, durch Erlernung mathematischer und anderer Hülfswissenschaften, nicht nur von den Gründen der Markscheidekunst, sondern auch von ihrem ganzen Umfange, und auf wie mancherley Fälle sie beym Bergbau brauchbar anzuwenden ist, und in was für genauer Verbindung sie mit der Mathematik steht, selbst überzeugen konnte." All translations, unless otherwise stated, are from the author of the present article.

had to reconcile two fundamental sets of values: while they had to ensure practicability and to cope with very specific problems, they also looked for tested methods that could easily be reproduced and systems of representation that would ensure an easy and unambiguous communication with their fellow surveyors.

To address these questions, we present a material and intellectual biography of August Beyer's Geometria subterranea.⁵ We will first introduce Beyer's work in the context of the early 18th century technical world of the mines, describing the scope and methods of subterranean geometry. We then focus on the manuscripts, studying the development and structure of Beyer's first Geometria subterranea, its diffusion and the various copies that were made. The influence of printing, the genesis of the first published version and the differences with the original manuscript have then to be analyzed. Beyer's decision reflected an evolution of the discipline, while its adaptation to a new readership had a direct influence on both its presentation and its content. We finally study the last edition, published some thirty years after Beyer's death and twenty years after the creation of mining academies. Analyzed as an academic textbook, this Geometria subterranea reveals precious information about the specific dynamics of mathematical practices as well as about the inherent difficulties associated with the institutionalization of engineering training.

2. AUGUST BEYER (1677-1753), MATHEMATICAL PRACTITIONER AND MINING EXPERT

2.1. Subterranean geometry in the German mining states

In 1708, a mining official named August Beyer (1677-1753) started writing a manuscript describing the use of geometry in the silver mines of the Ore Mountains (*Erzgebirge*), in the Electorate of Saxony. Beyer was a mining expert and wore the official title of subterranean geometer (*Markscheider*). He was therefore a mathematical practitioner of a rare kind, combining his geometrical knowledge with technical skills to play an important legal

⁵ Recent works in the history of early modern mathematics have seriously attempted to write biographies of scientific or technical works. [Métin 2016] studies numerous manuscripts written and compiled by military engineers of the sixteenth and seventeenth centuries, while [Joffredo 2017] presents a meticulous analysis of the genesis and reception of Gabriel Cramer's *Introduction à l'analyse des lignes courbes algébriques* (1750). See the bibliographies of both theses for further references.

role. ⁶ Despite its importance for the history of practical geometry and for the history of mining, Beyer's biography has not yet been written, as very little printed information is available about him. ⁷

Equally unfamiliar is the discipline he was excelling at: *Markscheidekunst* (lit. "the art of setting limits"), often referred to as *geometria subterranea*, was an art including all geometrical operations and surveying methods related to mining. It was in principle similar to surveying activities above ground, relying on elementary geometry and trigonometry, but its peculiar settings led to very specific instruments and methods.⁸ It was thus considered as an important mathematical discipline, albeit a very specific one, in the German states of the early modern period. Its wide scope will be illustrated in this paper with several examples, but we can at first rely on a definition provided by Christian Wolff (1679-1754), a leading scholar and mathematician of his time, professor at the university of Halle. In his *Mathematisches Lexicon*, whose first edition appeared in 1716—Beyer was already a seasoned *Markscheider* in Freiberg by that time—he gave the following definition:

Geometria subterranea, the *Markscheide-Kunst*, is a science aiming at measuring all fissures and veins in mining operations, and not only putting them on paper, but also marking them out above ground. They name the first *in Grund bringen*; and the other one *an Tag bringen* [*bring to daylight*]. This art has always

The term mathematical practitioner, popularized by several works about (mostly English) instrument-makers (see among others [Taylor 1966], [Johnston 1996] and about France [Daumas 1953]) is used here for its commodity rather than for its precision. In this article, the terms "mathematical practitioners" or "practical geometry" are used in an intentionally broad sense, including technical and economic activities that heavily rely on mathematics (the so-called 'civil life' or 'bürgerliches Leben'), and for which specific methods, formulas or tables are developed. We therefore consider a mathematical practitioner any person dealing intensively with mathematics for professional, technical or commercial purposes, even if not engaging with the development or teaching of mathematics per se. This often overlooked category of mathematicians is all the more interesting to study, since they played a key role in transmitting useful knowledge about their calculating practice before the creation of mining academies and modern engineering schools [Morel et al. 2016]. It is a substantial departure from other definitions addressing either closed social groups [Taylor 1966] or a specific corpus of texts [Raynaud 2015] and comes close to what Hélène Vérin defines as "la science des ingénieurs" [Vérin 1993, p. 246]. We don't engage in this article with the concept of "mixed mathematics" since it was mainly used by scholars to describe their study of natural phenomena. See [Brown 1991].

⁷ Classical biographical sources such as Meusel's *Lexicon* or the *Neue Deutsche Biographie* barely mention him, and their information is drawn from [Benseler 1843, p. 1161-1162].

⁸ About the history of surveying in the German states at the time, see [Torge 2007, p. 44-94].

been kept secret by subterrane an geometers: this is why they haven't published anything about it. 9

This short definition roughly sums up the main activities of a subterranean geometer, although it understandably doesn't say a word about their methods. The daily routine of a *Markscheider* was devoted to recording data about length and angles underground, and drawing maps and setting property limits above ground, although he was regularly asked to perform other engineering tasks, such as planning new mining operations and water galleries, or building water ponds.¹⁰

This discipline had an obvious economic interest in Germany, where metal mining was very developed. ¹¹ This helps explain both the secrecy surrounding this mathematical discipline and the importance given to it by various scholars. In a later edition of his best-selling dictionary, Christian Wolff even highlighted on the title page that his book "describe[d] the expressions of subterranean geometers". ¹² Wolff went so far as to include in his *Lexicon* specific sine tables as well as a guide for calculation with the non-decimal length units used in the mines. ¹³

2.2. A short biography of August Beyer

When August Beyer began his career as mining official at the turn of the 18th century, subterranean geometry was therefore an intriguing mathematical discipline whose apparent secrecy was matched by its economic role. Beyer was born in 1677 and raised in Freiberg, at the time a thriving mining city, reviving in the aftermath of the Thirty Years War. Freiberg was

⁹ [Wolff 1716, p. 672]: "Geometria Subterranea, die Markscheide-Kunst, ift eine Wissenschaft aller Klüffte und Gänge in den Bergewercken abzumessen und nicht allein auf dem Papiere in Grund zu legen, sondern auch oben im freyen abzustecken. Das erste nennen sie *in Grund bringen*; das andere aber *an Tag bringen*. Diese Kunst ist von den Marckscheidern jederzeit geheim gehalten worden: Daher auch von ihnen nichts davon heraus gegeben worden."

¹⁰ Another interesting study concerning the profession of subterranean geometer around the middle of the 17th century, is the biography of Balthasar Rößler included in the *Kommentarband* of [Meixner et al. 1980], especially p. 23-54 and 78-80.

¹¹ In Saxony, mining was the single biggest contributor to the State's treasury. About the general and economic history of mining, see [Bartels & Slotta 2012].

¹² See for example the 1747 edition, title page: "endlich auch die Redens-Arten der Markscheider [...] beschrieben worden". *Markscheidekunst* was the only mathematical discipline explicitly mentioned on the title page.

¹³ The sine tables were specifically conceived for mining in that they were computed for various subparts of a *Lachter* (fathom), allowing for faster computations. See in the second volume of the *Lexicon* the *Tabula IV Extrahirte Tabulae Sinuum der Markscheider*, p. 219-243. Wolff also gave tables to convert the non-decimal *Lachter* into its subparts.

the capital of the Ore Mountains of Saxony and its biggest mining district with a population of about 8 000 [Jobst & Schellhas 1994, p. 14]. A biography claims that he belonged to a family of mining officials, which was frequent at the time ¹⁴: the administration recruited among a few powerful families, creating dynasties of technicians and officials. Another source claims that his father, Andreas Beyer, was the rector of the Latin school (*Gymnasium*) in Freiberg.¹⁵

The first reliable information about him is that he was appointed Markscheider in 1697 shortly before turning 20 years old. It may be surprising to see such a young man hired as the official surveyor of the biggest mining district and city of Saxony, a position with tremendous responsibilities where a lot of experience was necessary. Reconstructing the chronology of his predecessors can help us explain this: the surveyor Martin Hörnig had died in 1692,¹⁶ and Johann Berger-who had been Beyer's teacher-was recruited to replace him but died within a couple of years [Jobst & Schellhas 1994, p. 45]. Johann Martin Liebel was then hired in 1695 at the already notably young age of 23, but died in the following year.¹⁷ This means that three surveyors had died within five years, and there was likely little other choice left than Beyer to fill this important position, given that training a subterranean geometer was a long process.¹⁸ Beyer was hired in 1697, but the head of the mining administration (Oberberghauptmann) Abraham von Schönberg (1640-1711) was skeptical about his abilities:

Since we nevertheless doubt that the appointed Beyer has acquired in that little time such *Fundamenta*, and *habilited* himself through the indispensible *Exercitium*, so that he does not provoke useless costs to the investors and irretrievable damage; it is then our desire that you will first investigate if it would be possible to recruit a *Subjecta*, either in Freyberg or in the Obergebürge, that would have more experience in this science, who would already have an experience in this

¹⁴ Neue Deutsche Biographie (NDB) [1955] Author: Pieper, Wilhelm.

¹⁵ [Jobst & Schellhas 1994, p. 45] without evidence to support this claim.

¹⁶ See the State Archive of Saxony in Freiberg (in the following text SächsBergAFG) 40001 – Oberbergamt Freiberg – 3477, f. 29r-v.

 ¹⁷ In early December 1696. SächsBergAFG 40001 – Oberbergamt Freiberg – 3477,
 f. 30v-31r. His name Liebel is sometimes written 'Siebel'. About Liebel, see also [Grübler 1731, vol. 2, p. 209].

¹⁸ This is corroborated by the fact that another subterranean geometer had been recruited in 1694 with the authorization of working in every district of Saxony—an uncommon decision suggesting that several of the smaller districts had nobody left to fulfill the task.

profession and already have shown [that he masters] its fundamenta by executing some measurements. $^{19}\,$

This means Beyer had not even finished his training and had very little experience. He was at first nominated temporarily in order to avoid a vacancy for this important and dangerous position. Since no other able candidate was found to be available, he was finally appointed by the Elector Friedrich August on the third of February, 1697, despite his young age.²⁰ While his predecessors had died very quickly, Beyer would serve as a *Markscheider* for more than half a century. Over five decades, he would refine the mathematical methods that were used and train new generations of surveyors, gaining the nickname "old Beyer" (*der Ältere*).

As was common at the time, Beyer had to sign an *Instruktion*, i.e. a description of his duties and rights as a subterranean geometer.²¹ This document helps us understand the role of this profession. An underground surveyor had to obey his hierarchy, meaning especially that he was not allowed to perform any kind of measurement without telling the administration. As the official *Markscheider* for Freiberg, Beyer had a monopoly on all underground surveying activities in the district. His annual salary (*fixum*) amounted to 52 taler, a relatively low sum supplemented by a fee charged for every measurement he made. The price of each operation was precisely given and counted according to the number of angles and the length the geometer had to record, taking into account danger, technical difficulty and the amount of time needed for the operation.²² He had to plan, or at least review all future mining operations and oversee existing galleries and

¹⁹ SächsBergAFG 40001 – Oberbergamt Freiberg – 3477, f. 30v: "Dieweile Wir aber insorgen stehen, ob izt gewalter Beyer in der wenigen Zeit solche *Fundamenta* erlanget, und durch das unumgängliche *Exercitium* sich soweit *habilitir*et, damit bey ein und andern Zug die Gewercken nicht in vergebliche Kosten, und unwiederbringlichen Schaden gesezet werden möchten; Als ist hiermit Unser Begehren, Ihr wollet zuförderst, ob nicht sowohl in Freyberg, als Obergebürge mehrern dieser Wißenschaft erfahrene *Subjecta*, welche bey derglei-*profession* schon geübt, und bereits durch einige verrichtetn Züge und Proben ihre *fundamenta* erwiesen, zu erlangen seyn wollen, Erkündigung einziehen."

²⁰ SächsBergAFG 40001 – Oberbergamt Freiberg – 3477, f. 31v.

²¹ The *Instruktion* was signed on February 27th, 1697 (SächsBergAFG 40001 – Oberbergamt Freiberg – 3477, f. 31v-33r). It follows a very codified pattern, and is for example very similar to the contract signed seventy years earlier by his predecessor Elia Morgenstern (?-1649).

 $^{^{22}}$ See the description given by A. von Schönberg, who had appointed Beyer, in [Schönberg 1693, p. 112]. The price should not exceed nine *Pfennig* per *Lachter* (about two meters) or three *Groschen* per angle, to which a daily sum of half a taler was added.

shafts, to avoid any crumbling or clogging. Finally, he had to draw maps of every mining operation in duplicate, one belonging to the investors and one conserved by the administration. ²³ The *Instruktion* emphasized the importance of using clean and precise surveying instruments without going into details.

Beyer realized many operations and maps in his first years as subterranean geometers, thus proving his skills in spite of his young age and short training. When he began the first draft of his *Geometria subterranea* in 1708, he was therefore a well-established surveyor and mining official. Before studying this work in detail, let us briefly close Beyer's biography. From 1725 on, he received help from a *Vice-Markscheider*, relieving him from the most dangerous and laborious tasks, and outlived three of them. ²⁴ This is when he became *Bergkommissar* and counselor for the Elector of Saxony. ²⁵ He therefore participated to the meeting of the *Obergbergamt*, the highest mining administration, that enjoyed exceptional powers. In most German states, private investors were only source of funding of the operations, while technical decisions were discussed and implemented by officials under the command of the mining admistration. The *Oberbergamt*, to which Beyer belonged, had a monopoly on technical scientific knowledge in general, and subterranean cartography in particular.

2.3. Dating and comparison of the three manuscripts

Let us now focus on Beyer's subterranean geometry manuscript, entitled *Geometria Subterranea oder Marckscheide Kunst, das ist Meβ-Kunst unter der*

 $^{^{23}}$ $\,$ Many maps drawn by August Beyer have been conserved, mostly in the Bergarchiv Freiberg.

²⁴ This *Vice-Markscheider* was from 1725 to 1730 Friedrich Richter, from 1730 to 1746 Johann Adam Friedrich Zacharia, from 1746 to 1751 Georg Friedrich Seybt and from 1752 on Carl Ernst Richter. A third geometer, Gottfried Müller, was appointed (*Markscheider Extraordinarius*) in the years 1752-1753, suggesting that August Beyer was at this point completely unable to work. This fact is corroborated by a letter from 1751 explaining that "because of his old age, the titular subterranean geometer Beyer has now not been able to perform measurements in a long time" ("Nun hat der würkliche Marckscheider August Beyer wegen seines hohen Alters bereits eine gute Zeit nichts mehr verrichten können") (see HStA Dresden, 10036 Finanzarchiv Loc. 41789 *die Ersezung der Marckscheider Functionen zu Freyberg betr*, f. 5v).

²⁵ SächsBergAFG, 40006 – Bergamt Altenberg – 275, Besetzung der Stelle des Bergkommissars im Obergebürge. Although Beyer decided to stay in Freiberg, many *Markscheider* went on to become mining directors (*Bergmeister*) in smaller mining districts, for example Beyer's Vice-Markscheider F. Richter, or P.C. Zeidler, subterranean geometer in the 1690's and later mining director in Johanngeorgenstadt.

Erden, of which at least three handwritten copies exist. ²⁶ As in many practical mathematical sciences, knowledge about subterranean geometry was mainly circulated in manuscripts [Morel 2015]. The diffusion of knowledge seems to have taken a codified form around the middle of the 17th century. This is at least what the relative abundance of manuscripts from that period seems to imply. Analyzing Beyer's manuscripts and their diffusion in the early 18th century therefore means describing what had by then become standard practices.

A first and major question is the exact date at which Beyer's first draft was written. He had been trained as a *Markscheider* in the mid-1690's in the Ore Mountains of Saxony. This training likely included copying the manuscript of his master Berger who died in 1695, which is not known to us. ²⁷ Nevertheless, Beyer's *Geometria subterranea* must have been substantially different from his. Indeed, the copy now held in Freiberg bears a title page that is both informative and puzzling.

The title page (see fig. 1), as is the case in other subterranean geometry manuscripts, bears a title, an author name and a date, here: "Augustus Bayern p.t. Marckscheider etc: Anno Christi 1708". But a second piece of information, written in small characters in the bottom-right corner, immediately seems to contradict this: "Describendi initium cepi d: 8 nov : 1727 Adolph Beyer". The most plausible interpretation is that this document is a copy made in 1727 by Adolph Beyer from an original text written by August Beyer in 1708. Several pieces of evidence support this claim. First, it is known that Adolph Beyer (1709-1768) was eighteen at that time and began his training with his uncle August (eventually becoming an important mining scholar and Bergschreiber in Schneeberg). Towards the end of the manuscript one finds the annotation "d. 5 Martij 1728", possibly indicating that Adolph finished the main copy of his uncle's original on that day. Most of the content that comes afterward are observations and measurements realized in the late 1720's and 1730's. These were in all likelihood made by Adolph (not August) and later bound together with the main text.²⁸

²⁶ When referring to this manuscript, we usually use the Freiberg exemplar (TU BAF – UB XVII 12, *Geometria subterranea*), unless stated otherwise. The two other copies are conserved in Gotha (Landesbibliothek Chart A 972) and Bochum (Bergbaumuseum, Bergbauarchiv Sign. 875).

 $^{^{27}\,}$ About the training of the subterranean geometers, see [Sennewald 2002] and [Morel 2015, p. 19-23].

 $^{^{28}}$ The second date (05.03.1728) is to be found TU BAF – UB XVII 12, f. 141v. The rest of the manuscript (f. 142r-188v) contains only succinct notes and many observations.

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FIGURE 1. August Beyer's *Geometria subterranea* (Freiberg), title page. Source: TU BAF – UB XVII 12.

Second, we can use the data given in the manuscript to date its genesis. To solve their geometrical problems, August Beyer and his fellows geometers would record the positions of successive points of a gallery (*Zug*) and then process them to visualize the relative position of the galeries or shafts. The biggest part of the manuscript is a compilation of *Propositiones*, i.e. concrete mining problems that the geometer has to solve. Each situation is then exemplified by a real case, for which Beyer gives both the data he has

collected, the map he has drawn and the solution of the problem. Over the course of his manuscript, the first proposition for which a data set is available is dated from April 6th, 1697.²⁹ If we consider that Beyer had been appointed in February that year, this tends to show that his manuscript was based on his own experience and data, not his master's.

More than twenty data sets were used, all of them but one recorded between 1696 and 1706. ³⁰ It is important to note that the order is not chronological. For example, the sixth problem ("How to find the position above ground of a point in the mine") is based on data recorded in 1702, while the seventeenth ("From the direction of an underground ore vein, find where to dig above ground to find it") uses an observation from 1697.

From the observations and computations recorded by August Beyer, the following hypothesis can be made: in February 1697, Beyer was appointed subterranean geometer after three of his colleagues had died. Collating observations he made in the following years and choosing in each case the clearest or most remarquable situation he had encountered, he decided in 1708 to produce a new manuscript in order to train coming generations. This original manuscript has in all likelihood disappeared today, and what we have left are three pretty concording copies, therefore probably faithful, a first being the one we just described.

A second example is an anonymous copy of Beyer's manuscript, conserved today in the Gotha archive (see fig. 2), with the following title: "Systematic instruction in mining sciences and primarily of Subterranean Geometry as Herr August Beyer, Berg-Commissarius of the King of Poland and Elector of Saxony, also subterranean geometer in Freyberg, not only practices himself, but also teaches in the following manner. 1718". This manuscript is obviously a working copy: no fancy illustration is to be found, figures and tables are clearly readable but without any unnecessary care. This is a very literal copy of the manuscript, using the same numerical examples and the same data. ³¹ We should note that the title given by the anonymous student is already very similar to the one that Beyer would later

²⁹ TU BAF - UB XVII 12, f. 58r.

 $^{^{30}}$ We are talking here about the first part of the manuscript (f. 1-141) excluding the later additions. Five of the sets were recorded in 1697 (prop. 2, 17, 19, 21 and 37), one the year before (prop. 22), two in 1698 (prop. 8 and 28), two in 1699 (prop. 24 and 25), three in 1700 (prop. 7, 29 and 31), one in 1702 (prop. 6), three in 1703 (prop. 30, 27 and 30), three in 1704 (prop. 16, 23 and 26), one in 1705 (prop. 9) and one in 1706 (prop. 33).

³¹ The only minor exception is that chapter 28, f. 73r-74r of the Gotha copy, is absent from the Freiberg copy. It is important to note that ff. 142r-188v of the Freiberg manuscript are not to be found in the Gotha copy. This support our hypothesis that

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FIGURE 2. August Beyer's *Geometria subterranea*, anonym copy (Gotha), title page.

use in the 1749 printed version. A third copy (see fig. 7) was made in the late 1730s and will be analyzed in a later part of this article.

3. CONTENT OF BEYER'S GEOMETRIA SUBTERRANEA

Let us now turn to the content of the main part of the manuscript, i.e. the part common to all three versions. We will first briefly sketch its general structure before describing in some detail the content of the various parts. The *Geometria subterranea* is composed of thirty-one chapters of varying length, writing style, scope and content. There is no formal introduction, preface or dedication, although the first four chapters describe the scope and content of the discipline. Chapters five to eighteen present very practical knowledge and know-how related to mining and geology, introducing numerous terms and distinctions. This typology is then used to describe mining problems and model them in mathematical terms. Chapter

they are later additions from Adolph Beyer and do not belong to the original *Geometria subterranea*.

eighteen deals with the various instruments used in underground surveying and how to use them to record data: the miner's compass (*Hängekompass*), the semi-circle (*Waßer Wage*), the protractor (*Transporteur*) and so on. Theoretical considerations about arithmetic and geometry are then introduced (resp. chapters 19-26 and 27-30). The last chapter is about twice as long as all previous ones taken together: it consists in the solution of forty-eight problems any geometer has to know. Many of these problems are illustrated with a concrete situation and a data set recorded by August Beyer himself and drawn into a map.³²

The manuscript's first chapter gives a concrete definition of what *Markscheidekunst* is about:

Subterranean geometry, in latin *Geometria subterranea*, consists or is no more than surveying underground, helping bring onto paper the structure of every mine; where and in which direction the work is going, and what is concealed under ground and cannot be seen; and to delimit on demand on the ground, as if the thereupon existing soil could be lifted or suppressed, and what is hidden underneath could be uncovered and seen, and [it is] therefore little or not at all be avoided if one wishes to establish useful operations.³³

It then lists its purposes and etymology while highlighting the importance of mining maps. As could be expected, Beyer emphasizes its usefulness. More interestingly though, he strongly insists on the relationship between subterranean geometry and other sciences in a chapter entitled "Is subterranean geometry grounded firmly enough so one can rely upon it?" The answer presents mathematics as a guarantee for the efficiency of the discipline: "This art is not only based on arithmetic, but as all similar arts, sciences and crafts it is based on mathematics, and will be clearly and extensively proven from trigonometry". ³⁴ In the whole manuscript, Beyer seems

 $^{^{32}}$ TU BAF – UB XVII 12, chapters 1 to 30 fill the folios 2r to 55v while chapter 31 stretches from f. 55v to 141v.

³³ TU BAF – UB XVII 12, chapter 1 f. 2r: "Die Marckscheider-Kunst in *Latein Geometria Subterranea* genannt, bestehet, oder ist so viel als Meßn unter der Erden, da durch einer jeder Zeche Beschaffenheit wo, wohin, und zu gebauet wird, und daß das, was unter der Erden verborgen und nicht zu sehen ist zu Pappier gebracht, auch auff Begehrn an Tage so abgestecket werden kann, als wenn das darauff liegende Erdreich abgeschautten oder abgehoben, und das darunter verborgene aufgedecket und gesehen werden kann und dannenhero wenig od[er] gar nicht bey den Bergwercke zu entbehren, wenn anders nüzliche Gebäude sollen angestellet werden.".

³⁴ TU BAF – UB XVII 12, chapter 4 f. 3r: " Ob die Markscheider Kunst so gegründet daß man sich darauf zu verlaßen hat?", "Diese Kunst ist nicht allem in der Arithmetica, sondern gleich allen andern dergleichen Künsten Wissenschaften, und Handwerken in der Mathematic fundiert, und soll [...] deut[lich] und weitläuffig aus der Trigonometria bewiesen werden."

to resort to pure mathematics to ensure the reliability of the *Markscheide-kunst* in two regards. Firstly, mathematics as an exact science ensures the accuracy of the results, compared to other competing methods such as the divining rods. ³⁵ Secondly, using mathematics should confer to the surveyor its inherent qualities, making him trustworthy and unbiased.

Using mathematics to solve technical problems implies a serious effort of modelling. Not only are ore veins very different from perfect Euclidian lines, but one has also to possess an extensive knowledge about mountains and geology. This is what Beyer presents in chapters five to seventeen. It would be misleading to think of these chapters as outside knowledge introduced to help understand the methods of subterranean geometry. This knowledge about mining, including a precise typology describing various veins according to their angular orientation, truly belongs to the discipline at the beginning of the 18th century. As in other practical mathematical sciences, the act of establishing an explicit and distinctive vocabulary as well as well-defined concepts makes an integrant part of the mathematical activity, for these choices are the frame in which arithmetic and geometrical methods will take place.³⁶ In this sense, it is perfectly legitimate to claim that, despite an inherent and legitimate tendency to see them mainly as men of action, "knowledge is the principal object and commodity of the practitioners" [Bennett 1998, p. 196].

In the tenth chapter, for example, Beyer explains the difference between "*Klufft*" and "*Gang*". A *Klufft* is a fissure ³⁷ filled with ore, but it is not a proper *Gang* (vein). Only the second kind is suitable for a proper operation, because it usually follows a more regular path, while the fissures are unpredictable and therefore unsuited for mathematical investigation. Once the subterranean geometer has ensured that he is dealing with a proper vein, one has to investigate its characteristics. The three main features of a vein are its direction (*Streichen*), its inclination (*Fallen*) as well as its width (*Breite*). A discussion on geological properties thus directly leads to an explicit geometrization of the physical space:

³⁵ The superior efficiency of mathematics for a proper understanding of nature would only be definitely recognized over the course of the 18th century [Lowood 1990, Introductory Essay]. About divining rod and their use in mining, see [Dym 2008].

³⁶ The research of Johann Heinrich Lambert about hygrometry is a good example from the 18th century. Lambert had first to define its object (humidity) in order to set up his hygrometer. See [Bullynck 2010].

 $^{^{37}}$ $\,$ In mining operations, one can also use the English term *cleft* borrowed from German.



FIGURE 3. Left: suspended compass (*Hängekompass*). Right: traditional mining compass (*Setzkompass*), from August Beyer's *Geometria subterranea*, Freiberg copy, ff. 15v, 16v.

Its direction extends in the length, the inclination in the depth and the width in the breadth; [the direction] can be observed for every vein according to the hours of the compass, the inclination according to the degree of the quadrant and the level and its width with the surveying chain [...] The determination and cognizance of veins consist of these three things that are direction, inclination and thickness and their equivalents". ³⁸

The next chapter contains a detailed description of a dozen of surveying instruments (including pictures), most of which were fairly common in the German practical geometry of the time [Lindgren 1989]. The most original is probably the suspended compass (*Hängekompass*) that can be seen in figure 3, left. Although similar instruments already existed elsewhere (especially in England for navigation purposes), the suspended compass might have been independently discovered by subterranean geometers in the middle of the 17th century. ³⁹ It likely represents an evolution of simple mining compasses from the 16th (see fig. 3, right), since it adopted the same division of the circle in twice twelve hours, with indication of cardinal

³⁸ TU BAF – UB XVII 12, chapter 10, f. 8v-9v: "Sein *Streichen* erstrecket sich nach der Länge, das *Fallen* in die Teuffe und Breite in die quere und wird ein jeder Gang nach der Stunde des *Compasses* das Fallen nach den *Grad* der Zirckelbogens und Waßerwage od[er] nach der Donlege deßen Breite aber nach dem Lachter Maß *observir*et [...] aus solchen dreyen Stücken als Streichen, Fallen und Dicke und darbey angeführten Gleichnißen bestehet die Be- und Erkänntniß derer Gänge".

³⁹ About the evolution of geometrical instruments, see [Daumas 1953], and about surveying instruments in England see [Richeson 1966]. It is very likely that the *Hängekompass* was first used in Saxony. Balthasar Rösler (1605-1673) is often presented as the "inventor" of this instrument, but never published about it, making a precise datation impossible ; only part of his manuscripts were published after his death ([Rösler 1700], and did not contain much about subterranean geometry. See [Meixner et al. 1980, p. 55-60].

points. ⁴⁰ The suspension in a circle allowed for a more precise recording of horizontal angles (what Beyer calls "direction") and made the instrument very useful in the mines until well into the 19th century, when it was replaced by the modern theodolite.

The surveying chains used in the German mining states where graduated in *Lachter* (fathoms). One *Lachter* (about two meters) was composed of eighth *Achtel* (*eighth*), each of which could itself be divided in ten *Zollen* (inches).⁴¹ Units were represented using a formal system close to what Simon Stevin's had introduced in his *Disme*: "one Lachter six Achtel two Zollen zero Primen and 9 Secunden" was written "1 \bigcirc 6 \bigcirc 2 \bigcirc 0 \bigcirc 9 ④".⁴² However, most cases did not need such precision, and surveyors usually indicated the number of *Lachter, Achtel* and *Zoll* in the following way: "one Lachter six Achtel two Zollen" as "1.6.2.".

With a non-decimal system, even basic operations needed some explanation. We should keep in mind that these manuscripts were also used by surveyors to train their successors, young practitioners who had generally enjoyed no specific education. The need for an elementary teaching of arithmetic and geometry was blatant. Just as previous chapters gave all required information about veins, shafts and rock layers and how to present them mathematically, chapters nineteen to twenty-six explained measurement units, the rule of three and square roots as well as some elements of trigonometry. Once again, Beyer seems to rely on Simon Stevin (1548– 1620), for he adapted Stevin's sine tables. ⁴³

It should be noted that Beyer generally avoids using mathematical terms such as *sinus rectus* and *sinus versus*. Since the right triangle used by surveyors always lies in the same prototypical position, measured angles being always the same and known side being always the *hypothenusa*, he directly refers to the *sinus rectus* as being the *Seigerteuffe*, i.e. the vertical depth or

⁴⁰ About older mining compasses, see [Michel 1956].

⁴¹ The inches could then be divided in ten *Primen*, each *Primen* in ten *Secunden* and so on. The *Lachter* unit was used almost exclusively in the mining context, but was also related to the civil life: one *Lachter* theoretically corresponded to three and a half ells (*Leipziger Ellen*), although each mining city factually had its own *Lachter* standard.

⁴² See for example TU BAF – UB XVII 12, chapter 19 f. 26r. Despite this, the *Lachter* unit system was not decimal, thus strongly limiting the interest of using Stevin's representation method such as described in [Stevin 1585]. Moreover, the fact that the *Primen* were represented by ((and not ())) and the *Secunden* by ((and not ())) did not help make it any clearer.

⁴³ TU BAF – UB XVII 12, chapter 23, Extractum Tabularum Sinuum In. Simonis Stevini, ff. 39r-39v.

vertical side of the triangle, while he uses *Sohle* for *sinus versus*, the *Sohle* being the basis of the triangle, or the "sole" of the mining gallery. ⁴⁴

4. AUTHORSHIP AND THE CRAFTING OF PRACTICAL GEOMETRY MANUSCRIPTS

Before turning to the core of the manuscript, i.e. the concrete problems of underground surveying, a few remarks should be made on these thirty chapters. It is possible to distinguish several parts or clusters of chapters, as we did in the introduction of this section. But within each part, the order of presentation is somewhat surprising. For example, the sine table is presented in chapter 23, followed by instruction for its use in chapter 24, while the vocabulary of trigonometry is only presented in chapter 25, and the proper description of the right triangle, names of its sides and general logic of trigonometry appears in chapters 28 to 30. This perplexing inner structure can only be explained if one considers this manuscript in the realm of subterranean geometry manuscripts that existed at the time. Take the beginning of chapter 28, "On the foundation and origin of subterranean geometry":

The true foundation of this art relies on a *Triangulo rectangulo* that is called master of mathematics [*Magister Matheseos*], this one being always given by the suspended semi-circle or quadrant with his *Perpendiculo* [...] such a triangle is now solved using the *Tabulas Sinuum* mentioned above. ⁴⁵

This paragraph is in fact an amalgamation of two paragraphs belonging to a *Markscheidekunst* manuscript written by another geometer, Adam Schneider (1634–1707), thirty years before. ⁴⁶ This borrowing should not be considered as a mere plagiarism. Let us for example consider the sine table given by Beyer and adapted from Simon Stevin. This exact table already appeared in Schneider's manuscript in 1669, but Schneider had not

⁴⁴ A clearer explanation of his system is presented in chapters 28 and 29, ff. 51r-54v.
⁴⁵ TU BAF – UB XVII 12, chapter 28 f. 51r: "Von Fundament und Ursprung der Marckscheider Kunst. Der wahre Grund dieser Kunst beruhet in einen *Triangulo Rectangulo* welcher *Magister Matheseos* genannt wird, solchen gibt mir allezeit die angehängte Waßerwaage od[er] *Quadrant* mit seinem *Perpendiculo* [...] solcher *Triangul* nun wird durch obbeschriebene *Tabulas Sinuum resolvir*et".

⁴⁶ The manuscript is Adam Schneider, *Neu Markscheide-Buch* (c. 1669), TU BAF – UB XVII 18, f. 3v, 23v. Other borrowings from Schneider are to be found in chapters 23, 24 and 31.

made it himself either: he referred to it as the table made by the *Markscheider* Balthasar Rösler (1605-1673) in 1664. ⁴⁷ Since this table was not available in print until 1700, it was only natural for every geometer to have a copy of it. And B. Rösler was such an illustrious member in the profession that it was in all likelihood not even necessary to mention his name.

Das fecht Capitel. Die derrechtwincflig Triangel / der das fürnembft mittel ift / dardurch alles meffen gefchicht/gemacht/gebraucht/ vnnd verstanden foll werden. Ach dem nun der tunft Geometriæ art vnnd aigenschafft ift / eines jeden dings rechte boch ober tieff/leng oder weit / nabent oder ferne : wo die mit der fürgenommen vnnd verozoneten maß nit vberfchlagen tünden werden: durch andere Mathes matifche mittel / als durch Triangel vund fondere Inftrument / zu folchem meffen gemacht /fich der ers Eundigen und innen werden / Wöllen wir nun von ap:20 Wie Ser Derber Minstling

FIGURE 4. up: [Puehler 1563, p. 11] down: August Beyer's *Geometria subterranea*, Freiberg copy, f. 51v.

To build his manuscript Beyer used numerous other sources, sometimes unexpected. The 29th chapter of his *Geometria subterranea* presents the right triangle and introduces the Pythagorean theorem based on Euclid's *Elements* (I.47 and VI.31). Yet it is borrowed word-for-word from a mid-16th century *Introduction to Geometry* (*Kurtze und grundliche anlaytung zu dem rechten verstand Geometriæ*) [Puehler 1563, p. 11-12]. As can be seen

⁴⁷ See Adam Schneider, *Neu Markscheide-Buch*, TU BAF – UB XVII 18, f. 8r-9r. Although neither Beyer, nor Schneider or Rösler indicates where exactly this table comes from, by using computation one can trace the table they were using back to Stevin's *Tafel der houckmaten* presented in his trigonometry (*De Driehouckhandel*), printed in [Stevin 1608]. However, Rösler's table takes a *sinus totus* of 10 000 and a precision of a quarter of degree where Stevin used a *sinus totus* of 10 000 000 and a precision of one minute.

in figure 4, this chapter is quietly crafted into the text. Once again, it might not have been Beyer himself who went to read Puehler's *Geometria* and decided it was worth taking, for this very chapter can be tracked in several other, and sometimes older, manuscripts. It was probably first introduced into the surveying corpus by a *Markscheider* at the beginning of the 17th century. ⁴⁸ We have shown in a previous work how Puehler's chapter became a core part of this tradition, transmitted from one practitioner to another until Beyer's copied it in turn a century later. ⁴⁹

Seeing Beyer as part of a broad tradition of surveyors shows how inaccurate it would be to describe his writing process as plagiarism. When he began working on his manuscript at the turn of the 18th century, he was not thinking of publishing it, but only of creating his own vade mecum. To this aim he would, as was usual at the time, bring together pieces taken from previous authors, who had become common goods for all subterranean geometers, whenever he found them relevant. It is even likely that he inherited most of these not directly from their authors but through his own teacher Johann Berger (1649-1695). This work of collation and reordering was an achievement in itself. Beyer should thus be seen as the main actor in the writing process of this manuscript, although it certainly drew on an existing tradition. ⁵⁰ Another important activity for Beyer and his fellow underground geometers was the process of removing irrelevant components to replace them by newer procedures. This operation is hardly perceptible when reading an isolated manuscript, since surveyors rarely criticized their predecessors. At most they underlined the novelty or efficiency of a new method, and in many cases the assessment was silent, outdated practices being simply removed.

These remarks could plausibly be extended, *mutatis mutandis*, to many texts used in practical mathematics. These texts are hybrid works for

⁴⁸ This chapter of Puehler can be found for example in another *Geometria Subterranea: Unterirdische Erdmäβung, oder so genannte Marck-Scheide-Kunst,* probably from the mi-17th century, author unknown, Freiberg, TU BAF – UB XVII 11, f. 38r-39v.

⁴⁹ About the history of this transmission and more generally about the classical sources used by subterranean geometers, see [Morel 2017].

⁵⁰ As [Métin 2016, 408-416] notes, the process of reordering, collecting and ultimately appropriating pieces of knowledge was a central component of military engineers' professional activity: "Les ingénieurs de terrain rassemblent une documentation à laquelle ils donnent une forme personnelle" (p. 416).

which modern notions of authorship and plagiarism are not always appropriate. ⁵¹ A close equivalent seem to be the commonplace-books or copybooks used by early modern architects and often circulated in manuscripts which were not necessarily "conceived as an original invention intended for publication". ⁵² The very name Beyer gave to his manuscript (*Geometria subterranea*), clearly shows its affiliation with an existing tradition. From the early 17th century, a chain of manuscripts was used to circulate the geometrical knowledge necessary for mining operations. They shared a common structure, solved similar problems and bore titles such as "*De Geometria Subterranea*", "*Markscheidekunst*" or "*Neu Markscheide-Buch*". An analogous tradition can be seen, on a broader scale, for the *Geometria practica* of the early modern period, whose structures were strikingly close. ⁵³ If we provisionally discard the concept of authorship, it becomes clear that studying Beyer's text means studying the evolution of the discipline over the course of the 18th century.

5. PROPOSITIONS AND METHODS OF SUBTERRANEAN GEOMETRY

A similar process is at work in the last chapter, and main part of the book, i.e. the *Propositiones* or problems a subterreanean geometer had to be able to solve. These were transmitted from a generation to the next while being constantly improved and reshaped as each geometer brought his own methods. This is the part in which Beyer's originality and influence will understandably be felt the most, since he was active for sixty years, at a time when both the technical and the administrative sides of mining underwent deep evolutions. In the first decades of the 18th century, Beyer's prominent position rested more on its moral authority than on an asserted authorship.

It would be overly detailed to list the forty-eight propositions solved by Beyer, but it is nevertheless worth giving several examples to get a sense of

⁵¹ An interesting discussion about the notion of authorship in Renaissance technical and scientific texts can be found in [Long 2001, p. 7-8]. Noting that "copying two or more texts and putting them together may suffice" to claim authorship, Long explains that authorship "has been viewed variously in different historical ears" and was not always "a self-conscious authorial prescience within a given text".

⁵² See [Marr 2013, p. 435]. His analysis of Jacques Gentillâtre's copybook has been continued by F. Métin in [Métin 2016, 407-418].

⁵³ [Raynaud 2015, p. 12-16] enumerates the main features of the *Geometria practica* of this period, prefering the use of "genre" to collectively refer to these texts. The term "tradition" seems more appropriate for the subterranean geometry, since a tradition "suppose l'existence de transmissions directes et de conventions fortes liées à un contexte spécifique" [Raynaud 2015, p. 12].



FIGURE 5. Data table or *Gruben-Zug*, from August Beyer's *Geometria subterranea*, recorded on April 8th, 1697, Freiberg copy, f. 58r.

what subterranean geometry was about. The first problems deal with data recording and map drawing: their central object is to obtain a proper *Zug*, that is the measurement of a gallery: "How to perform a measurement"

(Prop. 1) and how to "inscribe" it (Prop. 2). ⁵⁴ Beyer introduces here a central object of subterreanean geometry: the "mine measurement" (*Gruben-Zug*), a term that referred not only to the action of recording data but also to the abstract polygonal chain it represented, and finally to the table that was obtained. Figure 5 shows an example given for the second proposition, a measurement made in a mining pit close to Müdißdorf (a dozen kilometers south of Freiberg) in 1697.⁵⁵

Each line of this table contains the coordinates of a "segment" of gallery: its length as well as the angles recorded with the *Quadrant* (vertical) and the suspended compass (horizontal). ⁵⁶ The vertical angle and the *hypotenusa* would then be processed to "solve" the triangle and obtain the other two sides, that is the perpendicular depth (*Seiger-Teuffe* or *cathetus*) and the sole (*Sohle* or *basis*). ⁵⁷ Most problems dealt with concrete practices of a *Markscheider* and were solved using intricated variations of this general principle. Prop. 6 presented a very common task, "to bring a common point to the day", that is to locate above ground the perpendicular to a given point in the mine. ⁵⁸ This procedure was commonly used to ensure that concession limits were respected. With the *basis* and the horizontal angle, it was then possible to draw a ground map of the pit, which could be interpreted as the measurement "above ground" (*Tages-Zug*) and show precisely where a limit was located.

The ninth proposition is particularly interesting since it provided a challenge for a mathematical practitioner: "To find the principal or horizontal direction to direct a gallery or a drift, assuming that the vein is falling steeply". ⁵⁹ It illustrates how geometry and geology were intertwined: it is obvious that ore veins do not follow perfect linear paths and this is especially true for veins that are falling steeply. The surveyor nevertheless

⁵⁴ Respectively "Wie der Zug verrichtet wird" and "Nun folgt wie man die Züge in der Grube und an Tage verrichtet, einschreiben soll".

⁵⁵ TU BAF – UB XVII 12, chap. 31, prop. 2, f. 57v-58v.

⁵⁶ One can see that this table has seven main columns plus one for commentaries. Columns two and three give the vertical angle (orientation and degrees), the fourth and five the length (number of *Lachter* with its *Achtel* and inches, while the first and the two last columns indicate the horizontal angle (cardinal point, number of hours and its subparts). It is worth noting that horizontal and vertical angles used different measurement units.

 $^{^{57}\,}$ This operation was named "Resolutio" of the measurement and was shown in a second table.

⁵⁸ "Eine gemeine Orthung am Tage einzubringen".

⁵⁹ TU BAF – UB XVII 12, chap. 31, prop. 9 "Eines Ganges Haupt oder Horizontal Streichen worauf ein Stolln oder Strecke getrieben, anzugeben so der Gang Sayger fallt", ff. 67r-68r.



FIGURE 6. Mining plan representing how to find the principal direction of an ore vein, from August Beyer's *Geometria subterranea*, Freiberg copy, f. 68r.

needed to find the "principal direction", i.e. the line that fits best a given set of points. This was mandatory if one wanted to extend a concession, settle a dispute or connect the vein to existing galleries. To solve this problem, Beyer proposed to record the *Zug* and then draw a ground map (reproduced in fig. 6), before asking its reader:

Take your protractor, or a ruler if the mining map is very long, and lay it out so that it covers or crosses most of the angles, and then draw a line, like here D.E., lay your compass out and look, when the needle has rested, what hour it shows, like here 5.4. hour, and this is the horizontal or principal direction of the vein. 60

This example illustrates how subterranean geometry worked in the early 18th century. Despite the lack of general mathematical training or mining academy, an efficient companionship system allowed for the transmission of a well-developed and coherent body of useful knowledge. Surveying operations were standardized thanks to the general use of the suspended compass, and so were the tables used to record data. However, data processing, that is the methods used to actually solve problems, still resorted to piecemeal solutions or *ad hoc* methods.

⁶⁰ TU BAF – UB XVII 12, f. 67r: "So nimm deine Zulage von *Compass*, od[er] so der Riß sehr lange ist, nimm ein Linial und lege es also an, daß das *Linial* die meisten Winckel bedecket, od[er] durchschneidet, als dem ziehe eine Linie als hier D.E und lege deinen Zuleg-*Compass* daran, *observier* wann sich das *Magnet* Zünglein zur Ruhe begeben hat, was es dir vor eine Stunde zeiget allhier ist 5.4. Uhr dieses ist also das *Horizontal* od[er] Haupt-Streichen des Ganges."

Beyer himself called this a means to solve a problem "geometrically", ⁶¹ meaning that one had to draw on a mining map to solve the problem. This method had several good qualities: one needed more care and experience than theory, since it considerably reduced the amount of calculation. Moreover, it probably had a superior convincing power for third parties and legal issues. Among its numerous shortcomings, the most important was probably its lack of precision. This is precisely what would change over the course of the 18th century: while precision of instruments only modestly improved, important efforts were directed towards the use of data. Replacing ruler-and-compass solutions by analytical methods (in Beyer's word solve a problem "arithmetically", "*Arithmetice zu* finden") was a joint endeavour of several subterranean geometers, and the evolution of Beyer's *Geometria subterannea* will testify to these efforts.

6. PRINTING A PRACTICAL GEOMETRY IN THE MID-18TH CENTURY

August Beyer was in his seventies when in 1749 he published a book closely based on his 1708 manuscript and entitled *Systematic instruction in mining sciences, preceded by an introduction to subterranean geometry.* ⁶² Despite his eminent position, he had published only a couple of minor administrative brochures over the course of his long career. ⁶³ Why did Beyer suddenly decide to print a manuscript he had written more than four decades earlier? Answering this question will help us understand the transformations practical geometry was undergoing around the middle of the century.

Was there a single good reason for publishing his *Geometria subterranea*? Beyer had already a brilliant career behind him. At 72, he was too old to hope for a promotion. If we remember the first section of this paper, where we described subterranean geometry as an esoteric knowledge, a plausible hypothesis would be that he wanted to reveal it to a general audience. While not entirely false, the argument of secrecy is only partially convincing and should be seen in a broader context.

In the middle of the 18th century, subterranean geometry was not anymore as secret as it used to be when Beyer began his career in 1693.

⁶¹ See for example TU BAF – UB XVII 12, f. 43r: "Geometrice zu finden".

⁶² Gründlicher Unterricht von Berg-Bau, nach Anleitung der Marckscheider-Kunst, [Beyer 1749]. Regardless of its title the whole book, and not only the introduction, deals with subterranean geometry.

⁶³ See for example [Beyer 1732], containing the dividends given to shareholders in mining entreprises over two centuries. Another text, unrelated to subterranean geometry, has recently been published as [Beyer et al. 1998].

Not only had Voigtel's *Geometria subterranea* [Voigtel 1686] often been reprinted, most recently in 1713 and 1714, but J.F. Weidler (1691–1755), professor of mathematics at the University of Wittenberg, had also published his *Institutiones geometria subterranae* in 1726 [Weidler 1726]. L.C. Sturm, professor in Frankfurt-an-der-Oder and J.G. Jugel, mining director for the Prussian state, had published on that subject respectively in 1743 and 1744 [Sturm 1743; Jugel 1744]. Even Christian Wolff, whom we quoted in 1716 explaining how secret the art of underground surveying was, had to update his *Mathematisches Lexicon* in 1747, now listing and comparing various existing publications [Wolff 1747, p. 843-844].

But all books are not equal, and each of the authors had different audiences and purposes in mind. University professors were writing, sometimes in Latin, for prospective students, not mining officials. J.G. Jugel was indeed a professional, but he tried to encourage the development of mining in Prussia. His intended audience was therefore those "who have a wrong idea, or no idea at all, about mining, and who would be inclined to try their luck". ⁶⁴ These books show the influence of the cameralist movement, which had a deep influence on German universities and policies, but they did not present a real transformation of the discipline.

The existing literature (perhaps excepting Voigtel's contribution) was therefore hardly comparable to what Beyer had to offer: his intended audience was mining officials or young people who already had some knowledge of mining and wanted to learn the practical, concrete operations of subterranean geometry. He marked his difference with university professors by proudly claiming to be an "unscholarly technician" (*Ungelehrter*) and asserted in his introduction: "we don't have a practical textbook written in German to teach young people subterranean geometry". ⁶⁵ He insisted on his five-decades long experience and numerous improvements he had brought to his manuscript over the years, emphasizing his unique position.

His aim was thus not exactly to disclose well-kept secrets, since other books were indeed available, but to give a broader access to the state of the art of actual mathematical practices. In doing so Beyer consciously broke with the established attitude who gave a complementary role to

⁶⁴ [Jugel 1744, p. 12]: "die entweder gar keinen, oder doch schlechten Begriff, von dem Bergbau haben, und sich gerne zum Versuch ihres Glücks zu demselben wenden wollen".

⁶⁵ [Beyer 1749, introduction]: "wir kein teutsches *practisches* Buch haben, darnach wir junge Leute in der Marckscheider-Kunst [...] unterrichten könnten".

printed books and manuscript. Printed books about practical mathematics were by no means the natural evolution of handwritten knowledge. They were about delivering a different content for a more learned audience, for which usefulness and practicability (*Brauchbarkeit*) played a minor role. They generally presented coherent systems and insisted on general concepts, minimizing the dull series of problems and lists of exceptional cases [Vérin 1993, 12]. Printed sources in practical geometry usually gave a sense of what the work of a surveyor could be, but were not explicit enough to actually carry that work out. In her analysis of technical literature of the Renaissance, P. Long rightfully noted that printed books did not replace manuscripts, at least not in patronage relationships [Long 2001, p. 181-182, 191]. Our analysis further shows that manuscript books could play a major role until well into the 18th century, not only in a courtly setting, but to circulate practical geometry and useful knowledge.

The publications of Voigtel and Rösler at the turn of the 18th century were certainly important events, but the habit of circulating and updating manuscripts went on, as Beyer's example illustrates, so that printed book did not replace the handwritten knowledge. To take this important decision, Beyer must have had a good reason. Was it then to assert his authorship? This is somewhat puzzling, for we have shown how inaccurate it is to apply the modern concept of authorship in the late 17th century. And yet Beyer seems to explicitly say so in his preface, where he describes his writing process from 1708 to the final version:

In this long time, it could not fail that I collected many things, so that I had to change or add, and in this way this treatise went through the hands of many hundreds of people, and always in a different form. I also came to know that some of them slightly changed the content of this treatise and had it circulated as their own work. I had thus to fear, as it happened to some university professors with their lessons, that after my death one would as well present this treatise [originally written] for my students as his own work, or otherwise hand it over incomplete to be printed. 66

⁶⁶ [Beyer 1749, introduction]: "Es kunte nicht fehlen, durch die Länge der Zeit kame mir manches für die Hand, so ich ändern oder hinzusezen muste, auf welche Art dieser Aufsaz viel hundert Personen und immer einem anders als dem andern in die Hände gekommen ist. Ich habe auch erfahren müssen, daß mancher solchen Aufsaz in etwas nach seinen Sinn geändert, und hernach als seine eigene Arbeit weiter ausgegeben hat. Wie es nun manchem Lehrer auf Hohen Schulen mit seinen Vorlesungen gegangen, so muste ich ebenfalls befürchten, daß ein anderer diesen Aufsaz für meine Scholaren nach meinen Todte für seine Arbeit anziehen, oder sonsten unvollkommen zum Druck übergeben möchte."

This account is both informative and problematic. Beyer shows here the wide circulation of practical knowledge and gives further evidence that "manuscript" did not mean confidential or secret. Subterranean geometry was indeed available, as long as one was ready to pay for the training, just as one could learn the art of smelting. The "hundreds of people" mentioned were "mining enthusiasts, Saxon as well as foreigners, that went to high or low offices" [Beyer 1749, introduction], and Saxons (*Inländer*) could even ask for a grant to finance their study [Sennewald 2002]. On the other hand, precisely because Beyer was such a well-known figure among subterranean geometers, one wonders if the threat was real, or if it was a rhetorical artifice to justify his publication.

One piece of evidence tends to support Beyer's claim: the third manuscript of his Geometria subterranea, written in 1739, today held in the Bergbaumuseum Bochum (see figure 7).⁶⁷ The structure of the text is rigorously identical, with the same thirty-one chapters and forty-eight propositions. The only difference is to be found on the gorgeous title page where the name of the author has been changed to Johann Gabriel Beer, without stating any official position, which was very unusual at the time. This is a very beautiful exemplar, especially concerning the colored mining plans. The situation here is very different from the Gotha copy of the manuscript (compare fig. 7 and fig. 2), which not only was anonymous but explicitly stated on the cover that it was a study copy of Beyer's Geometria subterranea. We can probably talk about plagiarism here, since the title page clearly implies that Beer wrote the text, which is untrue: there were neither additions nor improvements. Such a manuscript might indeed have been used to find a position in a mining administration (outside Saxony) after Beyer's death, or put into print as Beyer's feared.

How did J.G. Beer manage to copy the manuscript? As explained, Beyer was a well-known teacher and his handwritten textbook was widely circulated. Luckily, we can be more precise about the circumstances. The Beers were a family of mining officials in Johanngeorgenstadt (in the Ore Mountains of Saxony). In one of Freiberg's archives, we found the following certificate from Beyer's hand:

That, following the gracious order from Dresden, dated from August 2nd, 1738, Mr. Johann Gabriel Bähr, knowledgeable about mining and born in Johann Georgenstadt, has come to Freyberg to be examined as a royal stipendiary from Johann Georgenstadt for having learnt subterranean geometry; that he was able to indicate and discuss the principal and most important *Propositiones*, and that [I] went with him over everything, that he has become very skilled in

⁶⁷ Bergbaumuseum, Bergbauarchiv Sign. 875.



FIGURE 7. Title page from August Beyer's *Geometria subterranea* (1739) bearing J.G. Beer's name, Bochum copy.

the aforementioned art so that he is well capable to serve in this office, this is what I want to attest here on request. Freyberg, August 29th 1740, August Beyer, *Markscheider*. ⁶⁸

This was a standard procedure at the time: Johann Gabriel Beer had obtained a grant to learn smelting and subterranean geometry in his hometown in the late 1730s, ⁶⁹ and went to Freiberg to pass the examination in 1738. His version of the manuscript is dated from 1739 and Beyer's certificate from 1740, so he must have made the copy while he was in Freiberg, either directly from Beyer or from one of his students. Since he was still a student at the time, it becomes clear why no profession was stated on the title page.

No evidence indicates that Beyer was directly referring to Beer when talking about plagiarism, or if he had one or several other cases in mind. The mere existence of this copy nevertheless shows that an independent publication without the master's approval was a possibility.⁷⁰ Justified or not, Beyer's fear distinctly shows that the question of authorship, that did not really exist in the previous century when many manuscripts about *Markscheidekunst* were anonymous and most geometers did not feel the need to assert their authorship, was becoming increasingly relevant.

Maybe the fact that the mining administration was then a small and connected milieu, without much contact to the outside world, made it

⁶⁸ Universitätsarchiv Freiberg (UAF)—OBA 5, f. 122r: "DASS AUF ERGANGENEN ALLERGNÄDIGSTEN BEfehl *Sub Dato* Dreßden am 2. Aug[ust] 1738 Herr JOHANN GABRIEL BÄHR des Bergwercks Befließener bürtigens Johann Georgenstadt, wegen *Examinir*ung in dem, als ein Königl. *Stipendiate* zu Johann Georgenstadt erlernten Marckscheiden sich alhier in Freyberg eingestunden. Die in gedachter Marckscheider Kunst vornehmsten und Wichtigsten *Propositiones* zu erörthern auf- um anzugeben vorgelegt, und alles ingesamt mit Ihm durchgegangen, daß er in gedachter Kunst gantz *habil* erkunden worden so daß er Dienste darinnen zu leisten und zu *praesti*ren ganz wohl vermögent ist, Solches habe auf Verlangen hiermit *attesti*ren wollen. Freyberg, den 29. Aug[ust] 1740 Augustus Beyer, Markscheider".

⁶⁹ UAF—OBA 3, f. 335r, 341r-345v and UAF—OBA 6, f. 245r. In this other file, his name is properly spelled (Beer instead of Bähr). It is explicitly stated that he had received a hundred taler to study both smelting (*Probierkunst*) and subterranean geometry (*Markscheidekunst*). In the same file, according to another document written in the late 1750s, Beer was said to be still working in Johanngeorgenstadt.

⁷⁰ Maybe August Beyer had also been instructed of his nephew Adolph's misadventures. Adolph had published in 1748 (one year before August) a book about smelting sciences based on his own manuscripts, and wrote in the introduction that he had printed them because "one or the other had against my will been handed to outsiders, who might have published it without my permission" ("Ein und das andere aber ist wieder meinen Willen in fremde Hände gekommen, die es ohne meinen Danck öffentlich herausgeben dürfften"). Moreover, this second case shows that Beyer's move was part of a broader movement.

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pointless at the time. When professional recognition happens foremost in a small closed group, publishing or asserting authorship has little interest compared to, say, the reputation gained for concrete accomplishments. On the contrary, the first half of the 18th century had seen a great increase of institutionalization and contacts, among other due to the new grant system.⁷¹ In a more open system, where no global overview existed, printing one's own knowledge was obviously more important.

To sum up, a close study of why Beyer did have his manuscript printed shows that there is no single or simple explanation. When talking about practical mathematics, a work does not exists *in abstracto* but is embedded in its social context. Beyer's *Geometria subterranea* belonged to the teaching and administrative tradition of the German mining states (*Bergstaaten*). His decision to have this manuscript put in print could also be seen as a harbinger of institutional upheaval. This interesting hypothesis can be developed: in the second half of the 1740's, propositions for creating a mining academy were gaining traction, and Beyer himself drew a parallel between his teaching activity and the duty of a university professor.⁷² These plans did not immediately materialized, mostly because of the wars of the Austrian Succession (1740-1748) and the Seven Years War (1756-1763). Important steps were nevertheless taken: a little known example is the introduction in Saxony of an admission test for the training in subterranean geometry around 1750.

This leads us to an unexpected conclusion: August Beyer had grown up and been taught in the late 17th century, when knowledge about subterranean geometry was collective and its diffusion very codified. He received this common good, borrowing from his masters and adding his own contribution that was in turn circulated to many. Half a century later, the context had changed and the discipline enjoyed a much wider diffusion. This is precisely when Beyer decided to assert his authorship. In order to do this, however, he had to use the conventions of the printed and scholarly world. Ironically, this led him to modify his own text: while Beer's plagiarism of

⁷¹ About the grant system introduced in 1702, see [Sennewald 2002] and [Morel 2013, p. 146-151].

⁷² See for example [Zimmermann 1746] and [Oppel 1749]. Although these propositions were receiving more attention at the time, they were not exactly knew. Already in 1695, Beyer himself was saying (in a funeral speech): "Surely, here is the university / of all mining and smelting arts / here other towns are getting advices / as well as other countries / about how mining should be" (German version to be found in [Herrmann 1953, p. 26-27]). In 1749, he then wrote that subterranean geometry "should be held as the cornerstone of a systematic instruction in mining sciences" ("für den Grund-Stein eines Gründlichen Unterrichts vom Berg-Bau zu halten") [Beyer 1749, p. 1].

1739 was an exact copy of the original manuscript, his published textbook of 1749 was not.

6.1. Differences between the manuscript and the 1749 printed version

Publishing a manuscript about practical mathematics in the mid-18th century was not a mere technical change of medium. The *Geometria sub-terranea* written in 1708 by Beyer was a work produced for a closed milieu where all mining officials shared not only a common technical language (the miner's language, or *Bergmannsprache*), but specific values and goals as well. Moreover, it was meant to be used in the context of a long teaching process, where a student would go into the mines with his master and directly learn the know-how. This is why, for example, the original text did not have an introduction. Once printed, it would be public and therefore be reviewed and criticized by the community of German scholars (*Gelehrte*), to which the author did not belong. Beyer was well aware of this, and discussed the issue in his new introduction: "I could not perform this work like a scholar, with mathematical order, erudite remarks and a pleasant style", trying to preemptively answer:

And I am quite ready to admit that a scholar would have presented many things better, especially from the natural and mathematical sciences. But is it not equally true that we would have waited in vain for a scholar [to write] a *practical book* of this kind? 73

With a rhetorical twist, Beyer then presented his manuscript as the "draft" (*Entwurff*) of the printed text, as if he had already planned to publish it forty years ago. The manuscript thus suddenly ceased to be considered as a commonplace-book, a *vade mecum*, or the ongoing medium on which experience was collected, to become a mere sketch. The author explained having decided to "completely recast this first draft for the students, to change and to augment, so that this work has grown and has taken a shape that bears little similarity to the first draft". ⁷⁴

⁷³ [Beyer 1749, introduction]: "ich gleich einem Gelehrten die mathematische Ordnung mit gelehrten Anmerckungen und einer angenehmen Schreib-Art in den Wercke nicht leisten möchte"; "Und ich will ganz gerne eingestehen; daß ein Gelehrter besonders aus der Natur-Lehre und Mathematischen Wissenschafften manches vielleicht besser fürgetragen haben könnte. Alleine ist es denn nicht auch wahr? Daß wir ein *Practisches Buch* in dieser Art vergeblich von einen Gelehrten erwartet hätten."

⁷⁴ [Beyer 1749, introduction]: "Ich habe also für einigen Jahren mich entschlossen, diesen ersten Entwurff für die Scholaren gänzlich umzuschmelzen, zu ändern und zu vermehren, wodurch dieses Werck erwachsen, und so beschaffen ist, daß es dem ersten Entwurff wenig mehr ähnlich sey wird."

Once again, the situation is more complicated that this bold claim seems to imply. Indeed, the *Systematic instruction in mining sciences* published in 1749 is not completely different, at least from a mathematical point of view, from the 1708 manuscript. It is true that there are several evolutions, but mostly in the structure and the presentation: Beyer had rewritten his text for a general audience, without truly changing its scientific content. The handwritten *Geometria subterranea* was thought as a compendium used in the companionship system of the mining state. It could afford to be directive or obscure precisely because it would be an object of discussion. The printed book, on the other hand, was an object that had to be self-contained, even when describing technical operation or mathematical methods.

Instead of a succession of many diverse paragraphs, that could be confusing for the unfamiliar or unaccompanied reader, Beyer introduced seven main parts. The introductory chapters and geological knowledge were somewhat pompously presented as *Praecognita*. The section about instruments was slightly expanded, as well as the considerations about arithmetic (*Pars III Von der Arithmetica*) and geometry (*Pars IV Von der Geometria*). Sine tables were expanded but still drew on the appropriation of Simon Stevin by the subterranean geometry tradition (neither were they recalculated nor improved in terms of precision). An important difference is the writing style: Beyer used the miner's language (*Bergmannsprache*) less often. He also gave more details about the context and assumed little familiarity with the context of the mines from his reader. When introducing the suspended compass, for example, he described at length the history of mining compasses and added some remarks about magnetism [Beyer 1749, p. 26-29].

The printed edition also brought more substantial improvements. The last part of the book deals with standard surveying, that is on the ground (*Pars VII Vom Feld-Messen*), while this topic was only briefly treated in the manuscript. ⁷⁵ Beyer's aim was to show that all common surveying operations could be performed with a greater precision using the miner's suspended compass. The author also added a few new propositions dealing with water ponds that were not to be found in the manuscript; it reflects the increasing use of artificial lakes to provide water for various mining machines [Beyer 1749, p. 206-214, p. 222-224].

To put Beyer's textbook and his evolution in perspective, it is useful to compare it briefly with a book published in the same year, von Oppel's

⁷⁵ See the *Geometria subterranea*, Freiberg copy, problems 41-44.

*Introduction to subterranean geometry.*⁷⁶ Its author, F.W. von Oppel (1720-1769) was in many respect the opposite figure to Beyer: much younger, he came from the nobility and had extensively studied at the university of Leipzig, publishing in 1746 a Latin trigonometrical work [Oppel 1746]. He directly entered the higher mining administration in Dresden and soon published his textbook about *Markscheidekunst.*⁷⁷ The *Introduction to subterranean geometry* was twice as long as Beyer's treatise. It was also much closer to usual geometry textbooks of the time, going successively through *longimetria, planimetria* and *stereometria*, and did not follow the structure of *Markscheider*'s manuscripts.⁷⁸ Von Oppel used more advanced mathematics and tools Beyer did not, such as logarithms. Moreover, he showed his knowledge of foreign works such as J. Picard's and P. La Hire's *Traité du nivellement* [Oppel 1749, p. 223, §512]. The work finally contained several criticisms of what geometers actually did as well as proposals to change the existing methods, who might have been aimed at Beyer.

7. THE MANY LESSONS OF A NEW EDITION

Let us now turn to the last edition of Beyer's work, published in 1785 (see fig. 8). It appeared more than thirty years after its author's death in a very different context. Twenty years before, in 1765, a mining academy had been created in Freiberg. This new institution is today widely praised for having opened a new chapter for mining sciences and their teaching [Taton 1964, p. 365-418; Morel 2013, p. 141-252]. In this section, we show how the history

⁷⁶ The full German title is Anleitung zur Markscheidekunst nach ihrem Anfangsgründen und Ausübungen kürzlich entworfen [Beyer 1749].

⁷⁷ [Oppel 1749]. The relationship between these two books, strikingly published in the same year, is difficult to ascertain. It is possible that Beyer rushed the publication of his manuscript to outpace Oppel (which it actually did by a few months). It is conversely possible that Oppel undertook this project to compel Beyer to publish his work. [Oppel 1749] mentions Beyer's book both in the preface (when it was not yet printed) and in the very last page. Oppel very politely notes that "one finds in [Beyer's] book the whole subterranean geometry, presented not only as it was presently known to us in the Ore mountains of Saxony, but also with some improvements" ("so findet man doch sonst in diesem Buche die ganze Markscheidekunst nicht nur in demjenigen Umfange, in welchem man sie bisher bey uns in auf denen Obersächsichen Gebürgen gekannt hat, sondern auch mit einigen Vermehrungen vorgetragen"), subtly criticizing it by adding that he wrote his own book because "all sciences can always be improved in many ways" ("Eine jede Wissenschaft aber auch immer zu mehrerer Erweiterungen fähig ist"), thus contrasting the existing tradition with his more scientific approach (p. 484).

 $^{^{78}}$ $\,$ About the classical structure of these practical geometries, see the introduction of [Raynaud 2015].

of practical geometry can benefit from complementary approaches, that is institutional history and history of textbooks and practices.

Professors of the newly founded *Bergakademie* insisted on replacing the existing artisanal practices by the scientific approach delivered through academic teaching, in the spirit of the "economic Enlightenment" of the late 18th century.⁷⁹ The professor of mathematics and future director J.F.W. Charpentier (1738–1805), as we have seen in our introduction, claimed that the implementation of academic mathematics had instantly reinvigorated a fossilized body of practices. The first three sections of this paper have hopefully shown that issues were less simplistic than that. The tradition of subterranean geometry was not a static body of knowledge, but was evolving at its own rhythm. Moreover, professors of the newly funded *Bergakademie* had to cope with the existing situation and to collaborate with established engineers and surveyors. A close analysis of the archival material indeed shows that subterranean geometry continued to be taught primarily by the practitioners themselves until well in the 1790s, when they were finally replaced by professors [Morel 2016].

In this context, how can we make sense of the reedition of Beyer's textbook that happened in 1785? The enterprise was supervised by Johann Friedrich Lempe (1757-1801), a young professor of mathematics who had already extensively published on that subject, writing two books and several journal articles [Kaden 2013; Morel 2013, 161-179]. This fact in itself is puzzling: if mathematicians in Freiberg wanted to replace practitioners' methods by new ones, and if Lempe had extensively described how to do this, why republish a book that was already forty years old, based on a manuscript written at the beginning of the century? It is crucial here to distinguish between "reprinting" and "reediting". The title page bears the following fine print: "drafted by August Beyer, thoroughly augmented and improved" ("entworfen von August Beyern. Durchgängig vermehrt und verbessert"), without mentioning Lempe at all (see fig. 8). The new edition is indeed substantially bigger: almost five times longer, with 1176 pages versus 251 for the first edition (register, errata and figures not included).

⁷⁹ See [Popplow 2012, p. 415] about the concept of economic Enlightenment, whose adepts "were convinced that the application of 'scientific' standards as they were represented by practices established in contemporary academies of science with some adaptations, would also serve to create such sets of 'useful' knowledge."



FIGURE 8. Title page from August Beyer's *Anleitung*, second edition, 1785

In the new preface, the mathematician explains that he was asked by the publisher to prepare a new edition, since the first was out of print. Indeed, as Charpentier wrote in another publication, "given its completeness and the numerous problems [he contained], it has conserved its value among the practical subterranean geometers, who have used it for their teaching." ⁸⁰ This is a very polite way to admit that despite professors' efforts to build a theoretical basis allowing for more analytical methods, practical teaching of *Markscheidekunst* still relied on Beyer's 1749 textbook.

"However", Charpentier continued, "a closer look at this [Beyer's textbook] will easily reveal that its author did not dispose of enough auxiliary sciences, and especially of mathematics." ⁸¹ The lack of theory had a direct impact on the practices: "because of this, his practical works always stayed raft-oriented and inaccurate [*handwerksmäßig und fehlerhaft*]". Coming back to Lempe's introduction to the second edition, we find almost the same discourse, which proves that the two professors of mathematics shared the same goal of replacing Beyer:

"Maybe there are still some underground surveyors who are accustomed to perform subterranean geometry according to the first edition of Beyer's book, or at least in a similar way, and are reluctant to learn something new, even if it is better than what they already know. The current Beyer [sic] will certainly appear to them useless [*unbrauchbar*]; and yet it will bring them considerable benefits, if they only take the time to study it."⁸²

It is highly unusual to criticize a book precisely within the introduction to its new edition. Indeed, the "current Beyer" counted about 900 pages more than the original; talking of a new book would thus have been be more appropriate. Lempe himself implicitly recognized the fact, stating: "I allowed [myself] to follow Beyer's plan only in general and not in the details." ⁸³ In the long history of Beyer's *Geometria subterranea*, this second printed edition offers us a last paradox about the concept of authorship. The introduction of 1749 emphasized the differences between the

⁸⁰ [Lempe 1782, p. 8]: "Wegen der mehrern Vollständigkeit und der vervielfältigen Aufgaben, hat es sich auch noch bis itzt, bey den praktischen Markscheidern in seinem Werthe erhalten, die sich dessen zum Unterricht bedienet haben."

⁸¹ [Lempe 1782, p. 8-9]: "Man wird aber bey genauerer Uebersicht desselben leicht bemerken, daß der Verfasser bey nicht gnugsamen Hülfswissenschaft, besonders mathematischen Kenntnissen, nicht vermögend war"; "wie denn auch selbst seine praktischen Arbeiten aus eben diesen Ursachen immer noch handwerksmäßig und fehlerhaft geblieben sind."

⁸² [Beyer & Lempe 1785, introduction]: "Es giebt vielleicht noch einige practische Markscheider, die sich ganz an die erste Ausgabe des Beyerschen Buchs, oder wenigstens an eine der dort vorgetragenen ähnlichen Art, die Markscheidekunst auszuüben, gewöhnt haben, und nicht gerne was Neues lernen, wenn es auch besser ist, als was sie schon wissen. Diesen dürfte freylich der itzige Beyer unbrauchbar scheinen; allein er wird ihnen sehr viel Nutzen schaffen, wenn sie nur die Gewogenheit haben wollen, darinn zu studiren".

⁸³ [Beyer & Lempe 1785, introduction]: "Ich durfte daher Beyers Plan nur im Ganzen, nicht in seinen einzelnen Theilen befolgen".

manuscript and the printed version, although the author was the same and mathematical methods were at least roughly similar. This second edition, which largely constituted a new book—at least from a mathematical point of view—written by another person, chose to highlight the continuity in its introduction. Lempe did not put his name on the title page and insisted on referring to the book as "the current Beyer", while systematically criticizing Beyer's methods. Both printed editions presented the previous work they were based on as a mere draft (*Entwurff*), in order to legitimize themselves as the only valid work.⁸⁴

The most plausible hypothesis is the following: Lempe, and Charpentier before him, had been trained as mathematicians and wished to introduce analytical methods in subterranean geometry. But the more they boasted about the purportedly success and influence of their *Bergakademie*, the clearer it appeared that it was precisely not working the way they intended. Both were teaching elementary geometry, arithmetic and physics to the students, but the *Markscheider* who was in charge of the practical geometry did not seem to fully use this in practice and mainly relied on Beyer. Lempe's books and articles were fairly respected by engineers all over Europe, but his own academy still relied on manuscripts [Morel 2015, p. 24]. In his own backyard, an editor even asked Lempe to republish Beyer's textbook. Statements from foreign mining officials visiting Freiberg in the early 1780's support this interpretation:

"So Mr. Scheidhauer unveiled his invention in 1773 and presented it to some students. A couple of them already used it the following year to draw [maps]. But this method was not to the liking of the old underground geometers who had been trained according to Voigtel and Beyer." ⁸⁵

Lempe took the chance to influence practitioners, or at least to convince his students to use more modern mathematical methods, using Beyer's name to promote his own principles. In his new edition, or "the current Beyer" as he names it, Lempe therefore pursued two related goals: on the one hand, he wanted to convince current surveyors that the methods they used (often based on Beyer's book) were defective, and on the other hand he tried to promote his own theories. Over more than thousand pages, Lempe used the same method tirelessly. Whenever he

⁸⁴ Both claims were untrue: we have seen that the 1708 manuscript was by no mean a draft, while the edition of 1749 was obviously a well-written book.

⁸⁵ [Becher 1782, p. 344-345]: "so hat Herr Scheidhauer in 1773 seine Erfindung bekannt gemacht, und einigen Akademisten gezeigt. Das folgende Jar darauf bedienten sich einige ihrer schon zum Zulegen. Den alten Markscheidern, nach Voigtel und Bayer gebildet, behagte die Metode nicht".

found something useless, he wrote "If Beyer would write his book nowadays, he would certainly have left this chapter aside". When he disagreed with him, as was often the case, he claimed "Beyer would have proceeded more correctly if..." or "Beyer has here obviously wrongly measured and miscalculated." ⁸⁶ Having proven how inaccurate Beyer's procedures were, he could then introduce his own methods.

Firstly, he transformed Beyers' introduction to elementary mathematics into a four hundred page textbook on arithmetic, geometry and trigonometry. The main novelties were probably the introduction of logarithms, a long exposition of solid geometry and trigonometry. Lempe used the opportunity to clarify why formulas should be prefered to geometrical drawing procedures. A *Markscheider* looking for the *hypotenusa* of a triangle would usually "draw on paper a similar triangle" and then measure the wanting length.⁸⁷ Lempe compared the average error made using this method with the precise use of trigonometry, to show that the discipline was difficult, but worth learning.

The second part of the book contained two main chapters, the "preparation to subterranean geometry" (p. 406-672) and subterranean geometry itself (p. 673-1093). In the first, Lempe greatly improved the discussion about instruments. Not only was the vernier scale introduced, but he then analyzed the average error made using every instrument. This allowed him to directly compare competing surveying procedures on an objective basis. Lempe then substantially altered the old system of propositions. First of all, he introduced mathematical concepts such as sine-direction and cosine-direction (*Streich-sinus* and *Streich-cosinus*) in order to solve all problems with computation and completely avoid the use of geometry.⁸⁸ He then grouped together practical problems which could be mathematically treated in a similar way, and effectively solved them as mathematical problems.

Concretely, let us now analyze how Lempe dealt with one important problem, the determination of the main direction of a vein (analyzed

⁸⁶ [Beyer & Lempe 1785, p. 17, p. 888, p. 937]: "Dieses Kapitel würde Beyer, wenn er itzt sein Buch schriebe, gewiß weggelassen haben"; "Richtiger hätte Beyer verfahren, wenn er ..."; "Ohnstreitig hat sich hier Beyer verrechnet und vermessen."

⁸⁷ [Beyer & Lempe 1785, 339]: "indem man auf dem Papiere ein ähnliches Dreyeck verzeichnet, und durch den verjüngten Maaßstab sich die Hypothenuse mißt".

⁸⁸ A portion of mine gallery being given and represented by the segment [AB], J.F. Lempe calls sine-direction the distance between B and the meridian plane going through A and cosine-direction the distance between B and the equatorial plane going through A. See [Morel 2013, 224-227] for a detailed exposition of Lempe's approach.

above p. 230). Beyer's method was simply to record the polygonal chain (*Zug*), draw it on paper as a ground plan and try to find a straight line that kind of fitted between the dots. In that respect, F.W. von Oppel did not really go further, neither about defining what was meant by "principal direction" nor in the geometrical method to find it. ⁸⁹ Lempe first presented "Beyer's method" and explained several of its drawbacks. He then suggested improvements to the geometrical method, but concluded that even then "the principal direction is found to be very erratic and not as precise as its definition requires". ⁹⁰

Having shown that Beyer's "geometrical method" was beyond remedy, he then used the third-person singular to present his own ideas, introducing "Lempe's method to find the principal direction of a vein" ("Lempens Verfahren, das Hauptstreichen eines Ganges zu finden"). Replacing the common-sense definition of "direction" (*Streichen*) by a less ambiguous one, ⁹¹ Lempe introduced his analytical concepts of sine-direction and cosine-direction (*Streichsinus* and *Streichcosinus*).

This preliminary clarification allowed him to achieve two things. Firstly, he was able to give formulas to compute the principal direction of a mining vein, knowing the sine- and cosine-direction of all the segment of the polygonal chain. What he did was dividing the segments in two groups, finding their center of gravity, and then compute the direction of the straight line going through these two points. This idea was probably borrowed from the mathematician Johann Heinrich Lambert.⁹² This was an important departure from Beyer's method that was exclusively

⁸⁹ See [Oppel 1749, p. 245-246] and [Morel 2013, p. 239] for a short analysis.

⁹⁰ [Beyer & Lempe 1785, p. 888]: "das Hauptstreichen sehr schwankend gefunden wird, und nicht so bestimmt, wie dieser Begriff desselben erfordert".

⁹¹ [Beyer & Lempe 1785, p. 416 § 71]: "The direction (*Streichen*) of a plane or a line is the position of its extension in the horizontal direction (*Richtung*)" ("Das Streichen einer Ebne oder Linie ist die Lage ihrer Ausdehnung nach einer söhligen Richtung"). He then drew an important distinction between the *observed* direction (given by the compass) and the *reduced* direction that took into account the magnetic deviation.

⁹² [Morel 2013, p. 239-241]. The idea of breaking down a set of observations into subsets to mitigate errors and improve precision can be tracked to Tobias Mayer (1723–1762), who used it for studying the libration of the moon ([Mayer 1750, 52-183], the method itself is described p. 153-154). While Mayer simply used the method for his own purpose, J.H. Lambert wrote a more general article adressing explicitly the issue of measurement errors, entitled "Theory of the reliability of observations and trials" (*Theorie der Zuverlässigkeit der Beobachtungen und Versuche*). It is thus likely that Lempe relied on Lambert, although he does not explicitly mentions either of these mathematicians in that case. It should be noted that J.F. Lempe had a good knowledge of the mathematics of his time, since both Lambert and Mayer were cited several times in [Beyer & Lempe 1785] and in his other publications.

instrumental, using the imprecise mining plans. Secondly, now that he had given an analytical (one would say "arithmetical" at the time) solution to this problem, theoretical improvements could be discussed and would indeed be introduced by other engineers [Stoyan & Morel 2018]. But to reach the point where mathematical criticisms of a method could happen, this double work of definition and computation had to be done.

The new method was not only more precise and adaptable. It used analytical methods rather than a simple geometrical reasoning, and therefore required a solid comprehension of the underlying mathematical theory. The change Lempe was aiming at was similar to the contemporary evolution in the training of French engineers under the impulsion of Gaspard Monge.⁹³ To convince his readers that it was worth using his new procedure, Lempe might have presented cases where the difference between the two mattered. He chose instead to give a reference to an article he had written specially on that topic in the Leipziger Magazin für Naturkunde, Mathematik und Ökonomie, a scientific journal published by Karl Friedrich Hindenburg (1741-1808), professor of mathematics at the university of Leipzig and leader of the school of combinatorial analysis. He also referred to the book he had written about subterranean geometry in 1782. This means that Lempe used the new edition to draw the attention of practitioners to the many papers he had written on the subject. Reciprocally, he later mentioned his new edition of Beyer in subsequent publications such as his own scientific periodical, the Magazin für die Bergbaukunde.⁹⁴

The second edition finally included numerous new tables. Trigonometric tables of the first edition had been replaced by more precise tables computed for mining units by F.W. von Oppel in [Oppel 1749]. Lempe also added tables to convert hours in degrees, hoping to replace the old hours system used for recording horizontal angles. He nevertheless made clear that all these specific tables were only makeshifts, for *Markscheider*

⁹³ [Belhoste et al. 1990], in particular p. 108. The new geometry of Monge is said to "donne au dessin d'ingénieur une dimension universelle qui le distingue radicalement des techniques graphiques enseignées dans les anciennes écoles d'ingénieurs" and p. 109: "à l'ingénieur dessinateur, 'artiste' ou géomètre, du XVIII^e siècle, se substitue un ingénieur savant, féru d'analyse et de mécanique". These efforts indeed date back to the previous century. For a recent analysis of the French context, see [Troudet & Crépel 2016, p. 215-216].

⁹⁴ See for example the [Lempe 1786, p. 229]. Of course, he never wrote that he was the principal author of this new edition, only refering to "Beyer's Markscheidekunst, second edition, greatly expanded and improved."

should simply cease to rely on particular tables and use general trigonometric methods together with Johann Carl Schulze collection of tables instead. 95

The analysis of the figures at the end of the volume only confirms the general feeling about this new edition. Most of the plates were borrowed from Lempe's own textbook on the subject and even wear the inscription "Lempe Marksch." [Lempe 1782]. Altogether, this new edition of Beyer's *Geometria subterranea* was a successful hybridization between the first edition and Lempe's previous works on the subject. It was thus put at the center of a web of publications, books as well as articles, most of them written by J.F. Lempe. In doing so, Lempe was able to use the reputation of his predecessor, whose book had been read all over Europe.

This edition was reviewed not only in the major German *Rezensionszeitungen*, but in French and Swedish journals too. Most of them did not actually judge the new book by its cover and insisted that Lempe was responsible for most of the work, as for example the *Journal de médecine, chirurgie, pharmacie*: "Par le travail & les soins de l'éditeur, cet ouvrage peut être actuellement regardé comme un livre absolument neuf. Il traite particulièrement de la géométrie souterraine, de l'influence des connaissances arithmétiques sur la science des mines." ⁹⁶

Other reviews underlined that the new edition of Beyer was a new book written by J.F. Lempe. We have indeed shown several evidence that support this view. But the question is actually slightly more complicated than it seems, especially concerning the notion of authorship. We have shown above that even the first *Geometria subterranea* written by Beyer in 1708 borrowed on many sources. The first and second printed editions carry on this process of integrating new and more relevant procedures. And although Lempe was undoubtedly the editor of the 1785 version, many of the modifications he brought were not his owns. He acknowledged many times his debt to Johann Andreas Scheidhauer (1717-1784), Saxon mining director

⁹⁵ [Beyer & Lempe 1785], preface to the second edition: "Moreover, Schulze Collection of mathematical tables would be of great use to the underground surveyor" ("Überdieß werden dem Markscheider Schulzens Sammlung mathematischer Tafeln sehr gute Dienste thun"). About Schulze's tables, see [Bullynck 2010, 160, 170–171].
⁹⁶ [Bacher mai 1787, p. 372-373]. Not all reviewers saw through Lempe's misrepresentation. At least one took the book at face value, writing "someone looking for information about German subterranean geometry should read the *Gründlicher Unterricht vom Bergbau, nach Anleitung der Markscheidekunst* by August Beyer, printed in 1785 in Altenberg" ("Den som åstunder underrätelse om tyska Markscheideriet, behagar läsa AUG. VON BEIJERNS *Gründlicher Unterricht vom Bergbau, nach Anleitung 1785*") [Horneman 1802, introduction].

and mathematician whose unpublished work is of greatest importance for the history of subterranean geometry. ⁹⁷

Moreover, Lempe presented innovations made by numerous local scholars and technicians such as the machine master J.F. Mende [Beyer & Lempe 1785, p. 19, p. 722-730] or the instrument-maker G.F. Brander [Beyer & Lempe 1785, p. 451, p. 555]. He finally introduced and adapted methods developed by well-known mathematicians of his time: J.H. Lambert, T. Mayer, G.S. Klügel or A.G. Kästner. This second edition was in fact a huge compendium of all existing methods that could be used for underground surveying, adapted to the conventions used in Saxony. And although it departs from the first edition and was edited by Lempe, he cannot be seen as the author in a narrow sense of the word either. It would be very justified to consider that the *Gründlicher Unterricht vom Bergbau* was a collective work by professors and engineers of the *Bergakademie*, using the name of a famous *Markscheider* to build a bridge between mathematicians of the academy and practitioners.

8. CONCLUSION

Just as the ship of Theseus, the *Geometria subterranea* studied in this paper was a work in constant evolution. Its journey began in fact well before 1708, when Beyer wrote his first version, since it extensively drew on existing manuscripts that in turn borrowed from various sources. Over the course of the century, several authors, most notably Beyer and Lempe, worked on its structure to adapt the mathematical and technical content to new problems or new audiences. At the end of the day, there is objectively little in common between the 1708 and the 1785 texts besides their titles, just like a ship the parts of which would have been continuously replaced.⁹⁸

It seems futile to single out one version in this process and label it as a new work. When talking about the *Geometria subterranea*, the very notion of "authorship" often blurs. As in many fields of early modern practical mathematics, such as wine gauging, fortification, architecture or commercial arithmetic, writing a book often meant bringing together, arranging and

⁹⁷ Lempe explicitly thanks Scheidhauer in the preface, and quotes his methods at least fifteen times. About the contribution of J.A. Scheidhauer to practical geometry, see [Klinger & Morel 2018].

⁹⁸ It is indeed interesting to track original parts that are still present in the second printed version. For example, a description of the right triangle as *magister matheseos*, taken from an 17th century tradition of subterranean geometry and originally coming from the university of the middle ages, is still there: [Beyer & Lempe 1785, p. 225].

improving existing solutions to a shared set of problems. ⁹⁹ At first considered as a collective work and often used as commonplaces books, their importance and social role gradually shifted during the first half of the 18th century. In many cases, the institutionalization of training seems to have been a key factor.

In this precise case, it does not mean that this *Geometria subterranea* had no author, but rather that several individuals contributed to it and can be seen as authors, without any of them qualifying as the sole authority on its content. Thus relativising the pertinence of authorship does not imply that individual practitioners were interchangeable. Historical actors from the 18th century clearly identified the originality and quality of Beyer's *Markscheidekunst* over other books on the subject. This is indeed the very reason why J.F. Lempe decided to use it as a Trojan horse for his own theories. In 1708, Beyer wrote on the first title page: "brought together for the instruction of all amateurs and students of mining sciences, and especially this art". ¹⁰⁰ Three generations later, this description would fit Lempe's endeavour almost perfectly.

The evolution of the *Geometria subterranea* mirrors the evolution of the discipline over the course of the 18th century. In the first half of the century, Beyer exerted a deep influence on new generations, and his slowly-matured manuscript was circulated so widely that he decided to officially publish it. The textbook was then used until the very end of the century, although the geometrical and piecemeal approaches it contained had theoretically been outdated by Scheidhauer's and Lempe's early works.

This leads us to a major feature of practical mathematics: the outstanding value given to usefulness and practicality. Despite all the improvements brought by Lempe and other mathematics professors, Beyer's *Geometria subterranea* was sufficient for a skilled geometer to solve most problems of the daily praxis. Actual realizations from German subterranean geometers in the 1770's and 1780's were astonishing by their degree of precision [De Luc 1779, p. 623-625]. No scientific or technical frontier immediately blocked the development of subterranean geometry [Morel 2013, p. 151-154]. But the Saxon mining administration wanted to avoid relying on the

⁹⁹ The history of wine gauging methods (*Visirkunst* or *Stereometria doliorum*) shows similar processes concerning the notion of authorship or the circulation of useful knowledge; see [Folkerts 1974, p. 15-21, 34-35]. In an article about commercial arithmetic, Swetz underlines the "extensive use and reliance on problems in the instructional process" [Swetz 1992, p. 373].

¹⁰⁰ TU BAF – UB XVII 12, f. 1r: "Allen Bergwercks- und vornehml. dieser Kunst-Liebenden und Lernenden zum nöthigen Unterricht zusammen gebracht".

skills and experience of individuals. They wished to implement a unified curriculum, standardized measurement units, instruments and analytical methods. For their part, professors of mathematics valued general and analytical methods, rightly convinced of their long-term necessity for the evolution of practical geometry.

While this certainly turned out to be true in the 19th century, their efforts encountered understandable resistance from their contemporaries. The institutionalization of mathematics in newly created institutions was not always as smooth as *Festschriften* tend to present it [Bergakademie 1866, p. 204-205]. It was not enough to teach new methods and their underlying mathematical theories, assuming that practitioners would realize how "better" analytical methods are than existing procedures. Moreover, our analysis shows that underground surveyors had their own dynamic and values. Their practices were not set in stone and their early reluctance should not be mistaken for backwardness. The final success was due to Lempe's tenacity and willingness to cooperate with practitioners, as showed by his ambiguous embrace of Beyer's *Geometria subterranea*.

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