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TOPOLOGICALLY STABLE LINEAR OPERATORS

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ABSTRACT. — In this study, we establish the equivalence of topological, structural, and strong structural stability for invertible linear operators on finite-dimensional Banach spaces. Furthermore, we demonstrate that every strongly structurally stable bilateral weighted shift also exhibits topological stability. As a consequence, there exist topologically stable operators that are not hyperbolic.

RÉSUMÉ (*Opérateurs linéaires topologiquement stables*). — Dans cette étude, nous établissons l'équivalence entre la stabilité topologique, structurelle et la forte stabilité structurelle pour les opérateurs linéaires inversibles sur des espaces de Banach de dimension finie. De plus, nous démontrons que chaque décalage pondéré bilatéral fortement structurellement stable présente une stabilité topologique. Par conséquent, il existe des opérateurs topologiquement stables qui ne sont pas hyperboliques.

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1. Introduction

This paper was driven by two main motivations. The first one revolves around the concept of *structural stability*, introduced by Andronov and Pontryagin [1], which has been extensively explored by numerous authors. Notable results include Peixoto's theorem concerning structurally stable flows on surfaces [21], Anosov's theorem [2] on the structural stability of U-systems (nowadays referred to as Anosov systems), Palis's theorem on the structural stability of Morse–Smale systems in higher dimensions [19], the structural stability of Axiom A systems with strong transversality condition proposed by Palis and Smale [20], and the solutions to the C^1-stability conjectures by Mañé [17] (for diffeomorphisms) and Hayashi [9] (for flows).

The second motivation centers on *topological stability*, introduced by Walters [27]. Here, remarkable results include Nitecki's theorem concerning the topological stability of Axiom A diffeomorphisms with a strong transversality condition [18], Walters's stability theorem on the topological stability of expansive homeomorphisms with the shadowing property in compact metric spaces, Thomas's stability theorem for flows [25], and the Chung and Lee stability theorem for finitely generated group actions [6].

The literature has explored the comparison between these two concepts. For instance, the C^1 stability conjecture and Nitecki's result [18] imply that every C^1 structurally stable diffeomorphism of a closed manifold is topologically stable. Although examples of topologically stable diffeomorphisms that are not structurally stable can be found, they are all topologically conjugated to structurally stable ones. Hurley [12] conjectured that this is always the case. For circle homeomorphisms, Yano [29] has proved it, and it is known that every smooth topologically stable flow on a closed surface is topologically equivalent to a Morse–Smale one [7]. In the case of diffeomorphisms of closed manifolds, Hurley [11] established that the combination of topological and structural stability is equivalent to the Axiom A property along with the strong transversality condition. Finally, it is worth mentioning that the structural stability of Anosov diffeomorphisms [2] follows as a consequence of Walters's stability theorem [14].

In this paper, we aim to compare these two significant concepts in the context of bounded linear operators on Banach spaces. While structural stable linear operators have been extensively studied by many authors over the last decades, topological stability has not received as much attention in the linear dynamics literature. Our objective is to demonstrate that both topological and structural stability are equivalent for invertible operators on finite-dimensional Banach spaces. Notably, in finite dimensions, these concepts are also equivalent to another crucial notion in linear dynamics, namely hyperbolicity. Additionally, we establish that every strongly structurally stable bilateral weighted shift exhibits topological stability. Consequently, topologically stable operators that are not hyperbolic do, indeed, exist. Let us state our results in a precise way.

Hereafter X will be a (complex) Banach space. We say that $W : X \to X$ is *Lipschitz* if there is $K > 0$ such that

$$\|W(x) - W(y)\| \leq K\|x - y\|, \qquad \forall x, y \in X.$$

The infimum of those K's is the Lipschitz constant denoted by $Lip(W)$. Given $r : X \to X$ we define

$$\|r\|_\infty = \sup_{x \in X} \|r(x)\|.$$

This is a norm except that it may take ∞ value (and so it is an ∞-*norm*). Denote by id_X the identity map of X.

Definition 1.1. — An invertible bounded linear operator of a Banach space $L : X \to X$ is:

- *Structurally stable* if there is $\delta > 0$ such that for any Lipschitz continuous map $g : X \to X$ satisfying
$$\|L - g\|_\infty < \delta \quad \text{and} \quad Lip(L - g) < \delta.$$
There is a homeomorphism $h : X \to X$ such that
$$L \circ h = h \circ g.$$
- *Strongly structurally stable* if for every $\varepsilon > 0$ there is $\delta > 0$ such that for any Lipschitz continuous map $g : X \to X$ satisfying
$$\|L - g\|_\infty < \delta \quad \text{and} \quad Lip(L - g) < \delta,$$
there is a homeomorphism $h : X \to X$ such that
$$\|h - id_X\|_\infty < \varepsilon \quad \text{and} \quad L \circ h = h \circ g.$$
- *Topologically stable* if for every $\varepsilon > 0$ there is $\delta > 0$ such that for any homeomorphism $g : X \to X$ with
$$\|L - g\|_\infty < \delta,$$
there is a continuous $h : X \to X$ such that
$$\|h - id_X\|_\infty < \varepsilon \quad \text{and} \quad L \circ h = h \circ g.$$

The notion of a structurally stable operator is due to Palis and Pugh [13, 5]. Topological stability for operators can be derived from strong structural stability by allowing arbitrary C^0-perturbation and weakening the resulting conjugacy. Every strongly structurally stable operator is structurally stable. The converse is unknown as far as we know.

We will prove that these definitions are equivalent on finite-dimensional Banach spaces. More precisely, we have the following result.

Theorem 1.2. — *The following properties are equivalent for every invertible operator of a finite-dimensional Banach space $L : X \to X$:*

1. *L is structurally stable.*
2. *L is strongly structurally stable.*
3. *L is topologically stable.*

(It is worth mentioning that all these equivalent conditions are equivalent to the hyperbolicity of L).

The Hurley conjecture [12] motivates us to ask if this theorem holds for general Banach spaces. We can also ask if Hurley result [11] holds for linear operators as well. More precisely, if an invertible bounded linear operator of a Banach space is hyperbolic if and only if it is both topologically and structurally stable.

Now, we compare topological and strongly structural stable operators in another important case: bilateral weighted shifts.

Fix $p \in [1, \infty)$ and let $X = l_p(\mathbb{Z})$ be the set of complex number sequences $\xi = (\xi_i)_{i \in \mathbb{Z}}$ such that

$$\sum_{i \in \mathbb{Z}} |\xi_i|^p < \infty.$$

This is a Banach space if equipped with the norm

$$\|\xi\| = (\sum_{i \in \mathbb{Z}} |\xi_i|^p)^{\frac{1}{p}}.$$

Given $n \in \mathbb{Z}$ we define $e_i \in X$ by $(e_n)_i = 1$ (if $i = n$) or 0 (otherwise). Then, we have

$$\xi = \sum_{i \in \mathbb{Z}} \xi_i e_i, \qquad \forall \xi \in X.$$

Choose a sequence of complex numbers $(w_i)_{i \in \mathbb{Z}}$ such that

$$0 < \inf_{i \in \mathbb{Z}} |w_i| \leq \sup_{i \in \mathbb{Z}} |w_i| < \infty. \tag{1}$$

Then, $B_w : X \to X$ defined by

$$B_w(\xi) = \sum_{i \in \mathbb{Z}} w_{i+1} \xi_{i+1} e_i, \qquad \forall \xi \in X,$$

is an invertible bounded linear operator of X. It is called a *bilateral weighted shift.* Based on [4] and [3] we shall prove the following result.

Theorem 1.3. — *Every strongly structurally stable weighted shift is topologically stable.*

2. Proof of the theorems

The proof is based on four lemmas. The first one (which seems to be well known) is motivated by Theorem 1.8, p. 39 in [10]. Given a Banach space X, $x \in X$ and $a \geq 0$ we denote by $B[x, a]$ the closed a-ball centered at $x \in X$.

LEMMA 2.1. — *For every finite-dimensional Banach space X and $\gamma > 0$ there is $\rho > 0$ such that if $h : X \to X$ is continuous and $\|h - id_X\|_\infty < \rho$, then $B[0, \gamma] \subset h(X)$.*

Proof. — Since all Banach spaces of the same (finite) dimension are linearly homeomorphic, we can assume that $X = \mathbb{R}^n$ with the Euclidean norm, for some $n \in \mathbb{N}$. Fix $\gamma > 0$. Denote $B = B[0, 2\gamma]$ and ∂B the boundary of B.

Choose $\rho > 0$ small enough such that for every $z \in B[0, \rho]$, every $x \in \partial B$ and $y \in X$ with $\|x - y\| < \rho$ it is true that the line traced from z to y intersects ∂B at some point u with $\|u - x\| < 4\gamma$.

Suppose by contradiction that there is $h : X \to X$ continuous such that

$$\|h - id_X\|_\infty < \rho \quad \text{and} \quad B[0, \gamma] \not\subset h(X).$$

Choose $z \in B[0, \gamma] \setminus h(X)$. Define $H : B \to \partial B$ by $H(x) = u$, where u is as above with $y = h(x)$. Then, H is continuous, and because $\|h(x) - x\| < 4\gamma$ ($\forall x \in \partial B$), we also have $H(x) \neq -x$ for every $x \in \partial B$. From this, we can construct a homotopy from $H|_{\partial B} : \partial B \to \partial B$ to $id_{\partial B}$ through the minimal circle arc in ∂B from $H(x)$ to x. This would imply that ∂B is a retract of B contradicting Brouwer's fixed point theorem. □

This proof does not work in infinite dimension since Brouwer's fixed point theorem fails in such a case [15].

On the other hand, a well-known theorem of Hartman [23] asserts that every structurally stable operator of a finite-dimensional Banach space is hyperbolic (and conversely). Our second lemma proves the same but replacing structural by topological stability.

LEMMA 2.2. — *Every topologically stable operator of a finite-dimensional Banach space is hyperbolic.*

Proof. — Recall that a homeomorphism of a metric space $f : Y \to Y$ is *expansive* [26] if there is $e > 0$ such that if $x, y \in Y$ and $d(f^n(x), f^n(y)) \leq e$ for every $n \in \mathbb{Z}$, then $x = y$. For invertible bounded linear operators $S : X \to X$ to be expansive is equivalent to the property that

$$\sup_{n \in \mathbb{Z}} \|S^n(y)\| < \infty \iff y = 0. \tag{2}$$

(See [4]).

Now, let $L : X \to X$ be a topologically stable operator of a finite-dimensional Banach space. Suppose that L is not hyperbolic. Then, there are $x \in X$ and $\lambda \in \mathbb{C}$ such that $\|x\| = |\lambda| = 1$ and $L(x) = \lambda x$.

By Lemma 2.1 there is $0 < \epsilon < \frac{1}{8}$ such that every continuous map with $\|h - id_X\|_\infty < \epsilon$ satisfies

$$B[0, 2] \subset h(X). \tag{3}$$

For this ϵ we take δ from the topological stability of L. By Lemma 1 in [5] (say) we can also assume that $L + \varphi : X \to X$ is a homeomorphism for every $\varphi : X \to X$ with $\max\{\|\varphi\|_\infty, Lip(\varphi)\} < \delta$.

Since $dim(X) < \infty$, the hyperbolic operators are dense in the set of invertible operators with respect to the operator norm (see p. 937 in [23]). Then, there is a hyperbolic operator $S : X \to X$ such that

$$\|S - L\| < \frac{\delta}{8}.$$

On the other hand, by (the proof of) Lemma 7 in [4], there is $\varphi : X \to X$ such that

$$\varphi(y) = S(y) - L(y) \qquad (\forall y \in B[0, 1 + \epsilon]),$$

$$Lip(\varphi) < \frac{3\delta}{8} \quad \text{ and } \quad \|\varphi(y)\| \leq \frac{\delta}{4} \qquad (\forall y \in X).$$

Define

$$g = L + \varphi.$$

Then, $g : X \to X$ is a homeomorphism with $\|L - g\|_\infty < \delta$. So, by topological stability, there is $h : X \to X$ continuous such that

$$\|h - id_X\|_\infty < \epsilon \quad \text{ and } \quad L \circ h = h \circ g.$$

Denote by $O_L(x) = \{L^n(x) : n \in \mathbb{Z}\}$ the orbit of x under L and by $\overline{B}$ the closure of $B \subset X$. Since $\|h - id_X\|_\infty < \epsilon$, the choice of ϵ implies that h satisfies (3). Since $\|L^n(x)\| = |\lambda^n| = 1$ for all $n \in \mathbb{Z}$, $\overline{O_L(x)} \subset B[0, 1]$ thus (3) yields

$$\overline{O_L(x)} \subset h(X).$$

Define

$$\Lambda = h^{-1}(\overline{O_L(x)}).$$

Given $y \in \Lambda$, we have $h(y) \in \overline{O_L(x)}$ so $\|h(y)\| = 1$ thus

$$\|y\| \leq \|h(y)\| + \|y - h(y)\| + \|h(y)\| \leq 1 + \|h - id_X\|_\infty < 1 + \epsilon$$

proving

$$\Lambda \subset B[0, 1 + \epsilon].$$

It follows that $g(y) = L(y) + \varphi(y) = S(y)$ for all $y \in \Lambda$.

On the other hand, we can check that Λ is invariant for g, so $g^n(y) \in \Lambda$ and, thus, $g^n(y) = S^n(y)$ for all $y \in \Lambda$ and $n \in \mathbb{Z}$. Now, $x \in O_L(x)$, so $x \in h(X)$, and, thus, there is $y_* \in X$ such that $x = h(y_*)$, whence $y_* \in \Lambda$. It follows that

$$L^n(x) = L^n(h(y_*)) = h(g^n(y_*)) = h(S^n(y_*)), \qquad \forall n \in \mathbb{Z}.$$

But S is hyperbolic (hence expansive), so by (2) one has either $y_* = 0$ or $\|S^n(y_*)\| \to \infty$ when $n \to \infty$ or $n \to -\infty$, $\forall y \in \Lambda$. If $y_* = 0$, then $S^n(y_*) = 0$ for all $n \in \mathbb{Z}$ and, thus,

$$\lim_{n\to\infty} L^n(x) = h(0).$$

Since $\|h(0)\| = \|h(0) - 0\| \leq \epsilon < \frac{1}{8}$ and $\|L^n(x)\| = 1$ for every $n \in \mathbb{Z}$, we get a contradiction. The second case is also absurd since Λ is a compact (hence bounded) invariant set of g, $S^n(y_*) = g^n(y_*)$ for all $n \in \mathbb{Z}$. This completes the proof. □

It is also known that every hyperbolic operator of a Banach space is structurally stable (this is Palis–Pugh extension [13, 5] of the Hartman theorem [8]). Our third lemma proves the same but replacing structural by topological stability. A sketch of the proof of this lemma in $\mathbb{R}^n$ was given by Robbin [22].

Lemma 2.3. — *Every hyperbolic operator of a Banach space is topologically stable.*

Proof. — Let $L : X \to X$ be a hyperbolic operator of a Banach space. Then, L is *uniformly expansive in the linear sense*, i.e. there is $N \in \mathbb{N}$ such that $\max\{\|L^N(x)\|, \|L^{-N}(x)\|\} \geq 2$ for all $x \in X$ with $\|x\| = 1$ (see, for instance, Theorem 1 in [4]).

Next recall that a homeomorphism of a metric space $f : Y \to Y$ is *uniformly expansive* if there is $e > 0$ such that for every $\varepsilon > 0$ there is $N \in \mathbb{N}$ such that if $x, y \in Y$ and $d(f^n(x), f^n(y)) \leq e$ for all $-N \leq n \leq N$, and then $d(x, y) \leq \varepsilon$. This is not exactly the original definition (by Sears [24]) but an equivalent one (see Lemma 1 in [16]).

Since L is uniformly expansive in the linear sense, L is uniformly expansive by Lemma 3 in [16]. On the other hand, every uniformly expansive homeomorphism with the shadowing property of a metric space is topologically stable (this is implicit in Walters [28] and explicit in Theorem 5 of [16]). Since L is hyperbolic, L also has the shadowing property, and then L is topologically stable. This ends the proof. □

Combining Lemma 2.2 and Lemma 2.3 we obtain the following corollary proving the Hartman theorem [23] for topologically stable operators.

Corollary 2.4. — *An invertible linear operator of a finite-dimensional Banach space is topologically stable if and only if it is hyperbolic.*

Our last lemma is based on the proof of Theorem 9 in [4].

Lemma 2.5. — *Let $w = (w_i)_{i\in\mathbb{Z}}$ be a sequence of complex numbers satisfying* (1)*. If*

$$\lim_{n\to\infty}\sup_{k\in\mathbb{N}}|w_{-k}\cdots w_{-k-n}|^{\frac{1}{n}} < 1 < \lim_{n\to\infty}\inf_{k\in\mathbb{N}}|w_k\cdots w_{k+n}|^{\frac{1}{n}},$$

then the bilateral shift B_w is topologically stable.

Proof. — As in p. 971 of [4], by Lemma 19 in [4], there are $s\in(0,1)$ and $\beta>1$ such that

$$(4)\qquad |w_{-j}w_{-j-1}\cdots w_{-j-k+1}|\leq\beta s^k \quad\text{and}\quad \frac{1}{|w_jw_{j+1}\cdots w_{j+k-1}|}\leq\beta s^k,$$

$\forall j,k\in\mathbb{N}$. Now fix $\epsilon>0$ and define

$$\delta=\frac{1-s}{\beta}\epsilon.$$

Let $g: l_p(\mathbb{Z})\to l_p(\mathbb{Z})$ be a homeomorphism such that

$$\|B_w-g\|_\infty<\delta.$$

Define $\alpha=B_w-g$. Hence $\|\alpha\|_\infty<\delta$. Denote by $\{\alpha_l : l_p(\mathbb{Z})\to\mathbb{C}\}_{l\in\mathbb{Z}}$ the coordinates of α. Define the sequence of maps

$$\{v_i : l_p(\mathbb{Z})\to\mathbb{C}\}_{i\in\mathbb{Z}}$$

by $v_0=0$,

$$v_i=\sum_{j=0}^{i-1}\frac{\alpha_j\circ g^{(i-j-1)}}{w_{j+1}\cdots w_i}\quad\text{and}\quad v_{-i}=-\sum_{j=1}^{i}w_{-i+1}\cdots w_{-j}\alpha_{-j}\circ g^{(-i+j-1)},$$

$\forall i\in\mathbb{N}$.

The argument in p. 972 of [4] shows that the map $v: l_p(\mathbb{Z})\to l_p(\mathbb{Z})$ defined by

$$v(\xi)_i=v_i(\xi),\qquad(\forall\xi\in l_p(\mathbb{Z}))$$

is continuous. And the argument on p. 973 of [4] shows

$$v\circ g-B_w\circ v=-\alpha.$$

This implies

$$B_w\circ h=h\circ g\qquad\text{with}\qquad h=id_X+v.$$

Since v is continuous, h is also. Finally, by rearranging the series defining v as on p. 11 of [4], one gets

$$\|v\|_\infty\leq\frac{\beta}{1-s}\|\alpha\|_\infty.$$

Then

$$\|v\|_\infty\leq\frac{\beta}{1-s}\|B_w-g\|_\infty<\frac{\beta}{1-s}\cdot\frac{1-s}{\beta}\epsilon=\epsilon,$$

and so $\|h - id_X\|_\infty = \|v\|_\infty < \epsilon$. Therefore, B_w is topologically stable, and we are done. □

From this lemma we have the following example.

EXAMPLE 2.6. — There are topologically stable operators that are not hyperbolic.

Proof. — Every bilateral weighted shift B_w as in Lemma 1 is topologically stable but not hyperbolic (by Theorem 9 in [4]). □

Proof of Theorem 1.2. — The equivalence between topological and structural stability for invertible linear operators on finite-dimensional Banach spaces follows from Corollary 2.4 and the Hartman theorem [23]. The one between structural and strong structural stability in finite dimensions is well known. □

Proof of Theorem 1.3. — Let B_w be a bilateral weighted shift with w satisfying (1). If B_w is strongly structurally stable, then Bayart Theorem [3] and Theorem 18 in [4] imply that one of the following properties hold:

(A) $\lim_{n\to\infty} \sup_{k\in\mathbb{Z}} |w_k w_{k+1} \cdots w_{k+n}|^{\frac{1}{n}} < 1$;
(B) $\lim_{n\to\infty} \inf_{k\in\mathbb{Z}} |w_k w_{k+1} \cdots w_{k+n}|^{\frac{1}{n}} > 1$;
(C)

$$\lim_{n\to\infty} \sup_{k\in\mathbb{N}} |w_{-k} \cdots w_{-k-n}|^{\frac{1}{n}} < 1 < \lim_{n\to\infty} \inf_{k\in\mathbb{N}} |w_k \cdots w_{k+n}|^{\frac{1}{n}}.$$

Properties (A) and (B) imply that B_w is hyperbolic (p. 976 in [4]) and so topologically stable by Lemma 2.3. Then, we are done since (C) implies that B_w is topologically stable (by Lemma 2.5). □

The comparison between topologically stable and structurally stable operators can be extended using Theorem 6 in [4]. According to this theorem, any structurally stable (resp. positively) expansive operator in a Banach space also qualifies as uniformly expansive (resp. hyperbolic). Moreover, applying the arguments presented in [4], we can demonstrate that every topologically stable (resp. positively) expansive operator in a Banach space is uniformly expansive (resp. hyperbolic).

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