Revue d'Histoire des Mathématiques

The Gaṇitapañcaviṃśī *attributed to Śrīdhara*

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Tome 19 Fascicule 2 2013

SOCIÉTÉ MATHÉMATIQUE DE FRANCE

Publiée avec le concours du Centre national de la recherche scientifique

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Périodicité : La *Revue* publie deux fascicules par an, de 150 pages chacun environ.

Tarifs : Prix public Europe : 80 €; prix public hors Europe : 89 €;
prix au numéro : 43 €.
Des conditions spéciales sont accordées aux membres de la SMF.

Diffusion : SMF, Maison de la SMF, Case 916 - Luminy, 13288 Marseille Cedex 9
Hindustan Book Agency, O-131, The Shopping Mall, Arjun Marg, DLF Phase 1, Gurgaon 122002, Haryana, Inde
AMS, P.O. Box 6248, Providence, Rhode Island 02940 USA

Maquette couverture : Armelle Stosskopf

Revue d'histoire des mathématiques
19 (2013), p. 245–332

THE *GAṆITAPAÑCAVIṂŚĪ* ATTRIBUTED TO ŚRĪDHARA

Takao Hayashi

Abstract. — This paper provides a detailed study of the *Gaṇitapañcaviṃśī* attributed to the famous eighth-century mathematician Śrīdhara with a view to restore it to its original form. It consists of a revision of the text edited by David Pingree, an English translation of the whole text, and a mathematical commentary.

Résumé (Le *Gaṇitapañcaviṃśī* attribué à Śrīdhara). — Ce document présente une étude détaillée de la *Gaṇitapañcaviṃśī* attribuée au célèbre mathématicien Śrīdhara du huitième siècle en vue de lui redonner sa forme originale. Il se compose d'une révision du texte édité par David Pingree, d'une traduction en anglais de l'ensemble du texte, et d'un commentaire mathématique.

INTRODUCTION

David Pingree discovered a manuscript of a small arithmetical text named *Gaṇitapañcaviṃśī* (hereafter GP) in the library of the Wellcome Institute for History of Medicene, London. The manuscript consists of three folios, but the second folio is missing. The first and the penultimate verses mention the name of the author as Śrīdhara; the last verse here

Texte reçu le 5 janvier 2012, révisé et accepté le 18 avril 2013.

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2000 Mathematics Subject Classification : 01A32, 01A35.
Key words and phrases : Śrīdhara, gaṇita-pāṭī, Sanskrit arithmetical text, restoration.
Mots clefs. — Śrīdhara, gaṇita-pāṭī, texte arithmétique en sanscrit, restitution.

occurs also at the end of most of the extant manuscripts of the *Triśatikā* (Tr) written by the eighth-century mathematician Śrīdhara. Pingree has published an edition based on this single manuscript under the title 'The *Gaṇitapañcaviṃśī* of Śrīdhara' [Pingree 1979].

In an earlier paper [Hayashi 1995], I expressed doubts about Śrīdhara's authorship of the GP. My main arguments were: (1) the GP contains more verses (estimated to be about eighty-seven) than its title ('Mathematics in Twenty-Five ⟨Ślokas⟩') and the introductory stanza (*ślokānāṃ pañca-viṃśatyā* 'by twenty-five ślokas') claim; (2) it contains no stanzas of the previously known two works in the same field by Śrīdhara, namely the Tr and the *Pāṭīgaṇita* (PG), which in turn have a number of verses in common (these two points had already been briefly mentioned by Pingree 1979, 888); (3) it contains half a verse of Śrīpati's *Gaṇitatilaka* (GT, 11th century) and eight and a half verses of Bhāskara II's *Līlāvatī* (L, AD 1150) and some of these fit metrically better in the GT and in the L than in the GP; (4) it contains half a verse prescribing a formula for the area of a circle-segment, which Gaṇeśa in his commentary (AD 1545) on the L attributes to his father Keśava and which causes inconsistency in the GP with regard to π; and (5) it uses, in addition to the ordinary numerals, the word-numerals which Śrīdhara avoids in the Tr and PG. My conclusion in that paper was that 'the present GP is not the original work itself (GP_0) of Śrīdhara, although we cannot of course deny the possibility that the GP_0 existed' [Hayashi 1995, 247].

Table 1 shows the number of verses in the Tr, PG, and GP as well as a conjectural number of verses to be added to, or removed from, each chapter or section of the GP in order to get GP_0. This has been determined as follows.

First, I regarded the 'twenty-five ślokas' as referring to the number of the verses for the *sūtras* (rules) only, because it is too small to include the *saṃjñāḥ* (terminology) and the *udāharaṇāni* (examples) or even one of them. Note that the anonymous *Pañcaviṃśatikā* ('⟨Mathematical Text⟩ Consisting of Twenty-Five ⟨Verses⟩', PV) is so called without terminology and examples.

Second, I estimated the number of verses in the chapters and sections which must have been contained in the lost folio as follows: half a verse each for the barter and the selling of living beings in the lost portion of the chapter on 'Three-quantity operation etc.'; one verse each for the interest, the purity of gold, and the investment in the lost chapter on Mixture; one verse each for the sum, the first term, the common difference, and the number of terms of an arithmetical progression in the lost chapter on Mathematical Series; and one verse each for equi- and inequi-perpendicular quadrilaterals in the lost portion of the chapter on Plane Figures. Thus, the total number of the verses to be restored for GP_0 is twenty, the average being ten verses per page. On the other hand, the extant three pages contain fifty-one verses, the average being seventeen verses per page. The gap can be explained by interpolations in those lost chapters and sections.

Third, I omitted from the extant portion of the GP those verses which have obviously been borrowed from other works. They are designated by 'F + serial number' in Table 2. They are half a verse (F1) of the chapter on Classes (from GT), four (F5-8) of the chapter on Plane Figures (from Keśava and L), and one (F9-10) of the chapter on Shadow (from L). When F7-8 and FE6 are omitted, the GP_0 would be without the rules and examples related to the Pythagorean Theorem which is one of the most basic propositions of geometry. But this would not be so strange as it appears to be at first sight, because the same is also the case with the PV. I also omitted three more verses (F2-4) in the chapter on Classes because the problems they treat are too sophisticated to be included in a small text like GP.

The total number of the verses for *sūtras* of the GP_0 thus obtained is exactly twenty-five without the introductory and concluding verses (S1, 26). The GP_0 still contains poetic meters other than *śloka* in its narrow sense, that is, the Anuṣṭubh meter, (Āryā in 23cd and 26 and Upajāti in 12-3ab), but this difficulty is resolved by taking the word in its broader sense, that is, 'a verse in general.'

The possibility, however, still remains that even the GP_0 is not the original work of Śrīdhara but a counterfeit. The following are the several points unfavorable to Śrīdhara's authorship of GP_0. For the details see the relevant places in the Commentary.

(1) The terminology of the weights and measures of the GP_0 shows affinities with the L, rather than with the Tr and PG, in the use of the second weight system (S3ab), of the monetary unit *dramma* (S5), and of the area unit *nivartana* (S7bcd). (2) The sum and difference in the GP_0 (1ab) is the ordinary sum and difference of positive integers by means of the decimal place-value notation while those in the Tr and PG are the sum of a finite series of the natural numbers and the difference between two of them. (3) In the inverse problem class (10cd–11ab), the GP_0, like the L, includes the case where the unknown number is increased or decreased by its own part, but the PG does not do so (the Tr does not deal with this class at all). (4) The Tr and PG do not contain the optional-quantity operation or the so called *regula falsi*, but the GP_0 (11cd), like the L, does. Moreover,

the way the GP_0 treats the four examples under that rule is very close to that of the L. (5) The comprehensive treatment of the 'three-quantity operation etc.', which prescribes a vertical double-column arrangement of the given terms even for the direct and inverse three-quantity operations, is unique to the GP_0 (12-3ab). The *Brahmasphuṭasiddhānta* (BSS, 12.11cd-12) also prescribes the same arrangement for the direct three-quantity operation but not for the inverse one. (6) The GP_0 still contains word-numerals. (7) Śrīdhara prefers the Āryā meter in the Tr and PG but the GP_0 prefers the Anuṣṭubh.

It is, however, also true that none of the above points is a definitive reason that completely denies Śrīdhara's authorship of the GP_0.

In Table 1, the sign '–' and a pair of parentheses ( ) indicate respectively that the chapter or section is missing and that it exists but incomplete.

Table 1: Number of Verses in the Tr, PG, and GP and the Conjectural Number of Verses to be added or removed

Chapters and	Number of verses					Topics to be
sections	Tr	PG	GP	±	GP_0	restored
Terminology	8	8	9	0	9	
Eight fundamental operations						
sūtras	$26\frac{1}{2}$	29	8	0	8	
udāharaṇas	22	24	3	0	3	
Classes						
sūtras	$\frac{1}{2}$	5*	7	−4	3	
udāharaṇas	7	6*	6	−3	3	
Three-quantity operation etc.						
sūtras	4	4	$(1\frac{1}{2})$	+1	$2\frac{1}{2}$	barter, selling of
udāharaṇas	27	27	(3)	+2	5	living beings
Mixture						
sūtras	6	27**	–	+3	3	interest, purity of
udāharaṇas	16	45**	–	+3	3	gold, investment
Mathematical series						
sūtras	3	29	–	+4	4	sum, first term, etc.
udāharaṇas	2	19	–	+1	1	of arithm. progr.

*PG treats the class problems in Mixture.

**Except the class problems.

Chapters and sections	Number of verses Tr	PG	GP	±	GP_0	Topics to be restored
Plane figures						
Rectilinear figures						
sūtras	5	11	(2)	$-2, +2$	2	tri- and quadri-
udāharaṇas	5	12	(2)	$-1, +4$	5	laterals
Circular figures						
sūtras	4	–	$2\frac{1}{2}$	-2	$\frac{1}{2}$	
udāharaṇas	2	–	2	$-1\frac{1}{2}$	$\frac{1}{2}$	
Excavations						
sūtras	6	–	$\frac{1}{2}$	0	$\frac{1}{2}$	
udāharaṇas	8	–	$\frac{1}{2}$	0	$\frac{1}{2}$	
Brick-piling						
sūtras	1	–	$\frac{1}{4}$	0	$\frac{1}{4}$	
udāharaṇas	4	–	$\frac{1}{2}$	0	$\frac{1}{2}$	
Timber-sawing						
sūtras	2	–	$\frac{1}{4}$	0	$\frac{1}{4}$	
udāharaṇas	3	–	$\frac{1}{2}$	0	$\frac{1}{2}$	
Heaped-up grain						
sūtras	4	–	$\frac{1}{2}$	0	$\frac{1}{2}$	
udāharaṇas	4	–	$\frac{1}{2}$	0	$\frac{1}{2}$	
Shadow						
sūtras	1	–	$1\frac{1}{2}$	-1	$\frac{1}{2}$	
udāharaṇas	2	–	$2\frac{1}{2}$	-2	$\frac{1}{2}$	
Concl. rem.	0	–	2	-1	1	
Total number of verses						
sūtras	63	(105)	(24)	$+1$	25	
udāharaṇas	102	(133)	$(20\frac{1}{2})$	$+2\frac{1}{2}$	23	

Table 2 shows the contents of the GP and parallel verses in the Tr, PG, and L. In Pingree's edition of the GP (designated P in the table), the consecutive verse numbers for the examples are given separately from those for the rules and printed, like the texts of the examples, in smaller letters. They are put in a pair of parentheses in Table 2. In my numbering, 'E' is attached to the verse numbers for the examples, and 'F' to the borrowed verses. The abbreviations 'S' in the first column and 'Pb' under Tr stand respectively for *saṃjñā* and for *paribhāṣā* (both meaning terminology). Other abbreviations: cl. (class), decr. (decrease), diff. (difference), frac.

(fraction), incr. (increase), inv. (inverse), opt. (optional), qt. (quantity), rem. (remainder), and vis. (visible).

Abbreviations of Titles

GT	*Gaṇitatilaka* of Śrīpati.
GP	*Gaṇitapañcaviṃśī* of Śrīdhara.
GSK	*Gaṇitasārakaumudī* of Ṭhakkura Pherū.
Tr	*Triśatikā* of Śrīdhara.
PG	*Pāṭīgaṇita* of Śrīdhara.
PV	*Pañcaviṃśatikā* (anonymous).
BG	*Bījagaṇita* of Bhāskara.
BSS	*Brāhmasphuṭasiddhānta* of Brahmagupta.
L	*Līlāvatī* of Bhāskara.
L/ASS	Pune edition of the L.
L/VIS	Hoshiarpur edition of the L.

Table 2: Contents of the GP and the Parallel Verses in the Tr, PG and L

Verse No.		Meters	Contents	Tr	PG	L
New	In P					
Terminology (saṃjñā)						
S1	1	Anu 1	Homage	Pb1	1	1,9
S2	2	Anu 1	Units of weight	Pb5	10	4
S3ab		Anu 1	cont.	–	–	3
S3cd	3	Anu 1	Def. of *kuṭapa*. Cited by Sūrya.	–	–	–
S4	4	Anu 1	Units of volume	Pb6	11	8
S5	5	Anu 1	Units of money	Pb4	9	2
S6-7a	6	Anu 1	Units of length	Pb7	12	5-6a
S7bcd	7	Anu 1	Unit of area	–	–	6bcd
S8-9	8-9	Anu 1	Decimal names	Pb2-3	7-8	10-1
Eight fundamental operations (parikarmāṣṭaka)						
1ab	10ab	Anu $\frac{1}{2}$	Sum and diff.	–,8ab	–,21ab	12,45a, 46d
E1ab	(1ab)	Anu $\frac{1}{2}$	Exs.	–	–	13
1cd	10cd	Anu $\frac{1}{2}$	Multiplication	5-6ab	18-19ab	14
E1cd	(1cd)	Anu $\frac{1}{2}$	Exs.	E3-4	E3	17
2ab	11ab	Anu $\frac{1}{2}$	Division	9	22	18
2cd	11cd	Anu $\frac{1}{2}$	Square and cube	10,14-15a	24a,28b	19a, 24a
E2ab	(2ab)	Anu $\frac{1}{2}$	Exs.	E5-6	E4-5	21,27

Verse No.		Meters	Contents	Tr	PG	L
New	In P					
3-4ab	12	Anu $\frac{3}{2}$	Square-root	12-3	25-6	22
4cd-6ab	13-4	Anu 2	Cube-root	16-8	29-31	28-9
6cd-7ab	15	Anu 1	Part cl. and sum & diff. of frac.	23ab, 19	36cd, 32	30,37
E2cd	(2cd)	Anu $\frac{1}{2}$	Exs.	E13,7	E14,6-8	31,38
7cd	16ab	Anu $\frac{1}{2}$	Multi-part cl. and multiplication of frac.	23cd,20ab	38ab,33ab	32,39
E3ab	(3ab)	Anu $\frac{1}{2}$	Exs.	E14-5,9	E14-5,9	33,40
8ab	16cd	Anu $\frac{1}{2}$	Division of frac.	20cd	33cd	41
8cd	17ab	Anu $\frac{1}{2}$	Square etc. of frac.	21-2	34-5	43
E3cd	(3cd)	Anu $\frac{1}{2}$	Exs.	E11-2	E12-3	44
Classes (jāti)						
9ab	17cd	Anu $\frac{1}{2}$	Zero cl.	8cd	21cd	45b,46a
9cd-10ab	18	Anu 1	Part-incr./decr. cl.	24,–	39-40	34
E4ab	(4ab)	Anu $\frac{1}{2}$	Exs.	E16-7	E18-21	35
10cd-11ab	19	Anu 1	Inverse-op. cl.	–	78	48-9
E4cd	(4cd)	Anu $\frac{1}{2}$	Ex.	–	E102	50
11cd	20ab	Anu $\frac{1}{2}$	Optional-num. cl.	–	–	51
E5ab	(5ab)	Anu $\frac{1}{2}$	Ex. of opt.-num. cl.	–	–	52
E5cd	(5cd)	Anu $\frac{1}{2}$	Ex. of vis.-num. cl.	E23-9	E96	53
F1	20cd	Indr $\frac{1}{2}$	= GT 67: Rem. cl.	–	–	–
E6ab	(6ab)	Anu $\frac{1}{2}$	Ex. of rem. cl.	–	E97	54
E6cd	(6cd)	Anu $\frac{1}{2}$	Ex. of diff. cl.	–	E98	55
F2	21	Anu 1	Root cl.	–	75	65
FE1	(7)	Indr $\frac{1}{2}$	Ex.	–	E99	67-8
F3	22	Āryā 1	Seen-root-part cl.	–	76	66
FE2	(8)	Anu 1	Ex.	–	E100	69-72
F4	23	Indr 1	Tower cl.	–	–	–
FE3	(9-10)	Śār 1	Ex.	–	–	–
Three-quantity operation etc. (⟨trairāśikādi⟩)						
12-13ab	24	Upaj $\frac{3}{2}$	$(2n+1)$-qt. op. and inv. three-qt. op.	29-31	43,44cd,45	73,77,82
E7ab	(11ab)	Anu $\frac{1}{2}$	Ex. for 3-qt. op.	E30-9	E25-36	74-6
E7cd	(11cd)	Anu $\frac{1}{2}$	Ex. for 5-qt. op.	E44-50	E39-44	83-4
E8	(12)	Anu 1	Ex. for 7-qt. op.	E51	E45	85
E9	n.n.	Anu 1*	Ex. for inv. 3-qt. op.	E40-3	E34-8	80-1

* Only the first five syllables are extant.

Verse No.		Meters	Contents	Tr	PG*	L
New	In P					
.... Lost portion begins						
13cd		Upaj $\frac{1}{2}$	Barter	32ab	46ab	88
E10		Anu 1	Ex.	E54-5	E48-9	89
14ab		Anu $\frac{1}{2}$	Selling of living beings	32cd	46cd	78
E11		Anu 1	Ex.	E56	E50	79
Eight kinds of procedure (vyavahārāṣṭaka)						
Mixture (miśraka)						
14cd-15ab		Anu 1	Interest	33	47	90
E12		Anu 1	Ex.	E57-8	E52-3	91
15cd-16ab		Anu 1	Purity of gold	36	53ab	103ab
E13		Anu 1	Ex.	E63-4	E63	104
16cd-17ab		Anu 1	Investment	38ab	59ab	94
E14		Anu 1	Ex.	E67-9	E71-2	95
Mathematical series (śreḍhī)						
17cd-18ab		Anu 1	Sum of arithm. progr.	39	85ab	121
E15**		Anu 1	Ex.	E73-4**	E103-5**	122-3
18cd-19ab		Anu 1	First term	40ab	86ab	124
E15		Anu 1	Ex.	E73-4	E103-5	125
19cd-20ab		Anu 1	Common diff.	40cd	86cd	126
E15		Anu 1	Ex.	E73-4	E103-5	127
20cd-21ab		Anu 1	Number of terms	41	87	128
E15		Anu 1	Ex.	E73-4	E103-5	129
Plane figures (kṣetra)						
21cd-22ab		Anu 1	Equiperpendicular quadrilaterals	42	115	172-3
E16-9		Anu 4	Exs.	E75-9	E122-4	174-5
22cd-23ab		Anu 1	Inequiperpendicular quadrilaterals	43	117	169,171
E20***		Anu 1	Ex.	E80		170,186-7
.... Lost portion ends						
23cd	25ab	Āryā $\frac{1}{2}$	Circle	45		199,203
E21ab	(13ab)	Anu $\frac{1}{2}$	Ex.	E85		200,202

* The PG is available up to verse 118 which gives an approximation formula for the square-root to be used in Brahmagupta's formula for the area of a cyclic quadrilateral which is stated in verse 117.

** The same examples are used for the three rules that follow.

*** The verse of E20 is lost but a geometrical figure for it with a short prose sentence is extant.

Verse No.		Meters	Contents	Tr	PG	L
New	In P					
F5*	25cd	Anu $\frac{1}{2}$	Circle segment	47		–
FE4	(13cd)	Āryā $\frac{1}{2}$	Ex.	E86		–
F6	26	Indr $\frac{3}{2}$	Arrow and chord	–		= 204
FE5	(14)	Anu 1	Ex.	–		= 205
F7	27	Āryā 1	Def. of *bāhu* and *koṭi*	–		= 135
F8	28	Gīti 1	Sides of right-angled triangles	51		= 136
FE6	(15)	Anu 1	Exs.	–		= 137
Excavations (khāta)						
24ab	29ab	Anu $\frac{1}{2}$	Excavations	52-3		214
E21cd	(16ab)	Anu $\frac{1}{2}$	Exs.	E87-90		215-6
Brick-piling and timber-sawing (citi-krakaca)						
24cd	29cd	Anu $\frac{1}{2}$	Brick-piling & timber-sawing	58-60		220,223,225
E22ab	(16cd)	Anu $\frac{1}{2}$	Exs. for brick-piling	E95-8		221-2
E22cd	(17ab)	Anu $\frac{1}{2}$	Ex. for timber-sawing	E99-101		224,226
Heaped-up grain (rāśi)						
25ab	30ab	Anu $\frac{1}{2}$	Heaped-up grain	61-2		227
E23ab	(17cd)	Anu $\frac{1}{2}$	Ex.	E102		228
Shadow (chāyā)						
25cd	30cd	Anu $\frac{1}{2}$	Time from shadow	65		–
E23cd	(18)	Anu $\frac{1}{2}$	Ex.	E106-7		–
F9	31ab	Vas $\frac{1}{2}$	Shadow	–		= 234
FE7	(19)	Indr 1	Ex.	–		= 235
F10	31cd	Vas $\frac{1}{2}$	Height of lamp	–		= 236
FE8	(20)	Upaj 1	Ex.	–		= 237
Concluding remarks (⟨granthasamāpti⟩)						
26	32	Āryā 1	Merits of the GP	–		272
F11	33	Āryā 1	Praise of Śrīdhara	–		–

* Exactly the same half verse is ascribed by Gaṇeśa to his father Keśava.

1. TEXT

. . . . Notation . . . .

B2	A Baroda manuscript of the Tr. See Bibliography.
E + v. no.	A verse for an example or examples.
F + v. no.	A verse borrowed from other work.
P	The text of the GP edited by Pingree.
S + v. no.	A verse for terminology (*saṃjñā*).
W	The Wellcome Institute manuscript of the GP on which P is based.
(A)	A has been added by Pingree.
⟨A⟩	A is added by me. This is not applied for the verse numbers, which are all mine. In P, they have been supplied by the editor. See Table 2 above.
A T1] B T2	A in T1, which is accepted here, reads B in T2. T1 is omitted if it is P.

The verses conjectured here to be of the original GP (i.e., GP_0) are printed in *italic face* and the passages regarded here as interpolations are printed in smaller letters within the delimiters,

(Interpolated passage)

. . . . Text . . . .

⟨p. 890⟩ (f. 1r) ओं॥

⟨अथ संज्ञाः⟩

शिवं प्रणम्य स्वकृतपाट्याः सारं प्रवक्ष्यति।
श्लोकानां पंचविंशत्या श्रीधरः प्रकटार्थया॥S1॥
पंचगुंजो भवेन्माषो माषैः कर्षो नृपप्रमैः।
स सुवर्णः सुवर्णस्य तैश्चतुर्भिः पलं स्मृतम्॥S2 ॥
वल्लस्त्रिगुंजस्तैर्गद्याणकः षोडशभिः स्मृतः।
सार्द्धं वेधो ऽङ्गुलं त्रीणि कुटपे दैर्घ्यविस्तृतिः॥S3॥[1]
चत्रुभिः कुटपैः प्रस्थः प्रस्थैश्चतुर्भिराढकः।
चतुर्भिराढकैर्द्रोणः खारिर्द्रोणैर्नृपप्रमैः॥S4॥
वराटकानां विंशत्या काकिणी तच्चतुष्टयम्।
पणस्ते षोडश द्रम्मस्ते षोडश च निष्ककः॥S5॥

1 ऽङ्गुलं] ऽङ्गुल P. This hemistitch is cited by Sūryadāsa, who also reads ‘ऽङ्गुलं’. See Hayashi 1995, 246.

त्रयो ऽङ्गुलं यवा ऊर्ध्वास्तच्चतुर्विंशतिः करः।
दण्डश्चतुष्करः क्रोशस्तत्सहस्रद्वयं भवेत्॥S6॥
योजनं स्याच्चतुःक्रोशं वंशो हस्ता दशाष्ट वा।
निवर्त्तनं तद्विंशत्या क्षेत्रं बद्धं चतुर्भुजम्॥S7॥
⟨p. 891⟩ **एकं दश शतं तस्मात्सहस्रमयुतं ततः।**
लक्षं च प्रयुतं कोटिरर्बुदं पद्ममेव च॥S8॥
खर्वं निखर्वं च महापद्मं शङ्कुः सरित्पतिः।[2]
अन्त्यं मध्यं परार्द्धं स्युः संज्ञा दशगुणोत्तराः॥S9॥[3]

१००००००००००००००००० ⟨॥⟩

इति संज्ञाः॥

⟨अथ परिकर्माष्टकम्⟩

सङ्कलितव्यवकलितयोः सूत्रम्॥

योगान्तरे यथास्थान⟨नं⟩ चेत्खेनाविकृतो भवेत्।1ab।[4]

उदाहरणम्॥

षष्टिर्नवार्काः किं युक्ताः स्युः शताद्वा विशोधिताः।E1ab।

न्यासः॥ ६०। ९। १२।योगे जातं ८१।एतच्छताद्वा विशोध्य शेषं। १९॥इति सङ्कलितव्यवकलिते॥

गुणने सूत्रम्।⟨।⟩

गुण्यस्यान्त्यादिकानङ्कान्गुणयेद्गुणकेन तु॥1cd॥

उदाहरणम्॥

जिनैर्नगार्काः किं निघ्नास्त्र्यर्कैः किं दन्तभूनृपाः॥E1cd॥

न्यासः॥गुण्य १२७ गुण २४⟨।⟩ गुण्य १६१३२ गुण १२३ ।गुणनाज्ज्ञाते फले।[5] 3048। १९८४२३६॥इति गुणकारः॥

⟨p. 892⟩ भागहारे सूत्रम्॥

भागे भाज्याद्धरो येन हतः शुध्येत्फलं तु सः।2ab।[6]

---

2 पद्मं] पद्मे W.
3 परार्द्धं] परार्द्धे W.
4 स्थानं] स्थान P.
5 गुणनाज्] गुणना W.
6 हतः] हृतः P.

उदाहरणम्॥ प्राग्लब्धगुणफलस्वगुणच्छेदयोर्भागार्थं न्यासः॥ [7] भाज्य ३०४८ हर २४⟨।⟩ भाज्य १९८४२३६ हर १२३।भाजनाल्लब्धौ गुण्यौ १२७। १६१३२।⟨।⟩ इति भागहारः॥

वर्गघनयोः सूत्रम्॥

समद्विघातो वर्गः स्यात्समत्रयहतिर्घनः॥2cd॥

उदाहरणम्॥

कौ नवानां त्रिसूर्याणां स्यातां वर्गघनौ सखे।E2ab।

										घ								
[8]न्यासः॥	१	२	३	४	५	६	७	८	९	१	२	३	४	५	६	७	८	९
	१	४	९	१६	२५	३६	४९	६४	८१	१	८	२७	६४	१२५	२१६	३४३	५१२	७२९

न्यासः॥ ९। १२३। जातौ वर्गौ ८१। १५१२९। घनौ च ७२९। १८६०८६७॥ इति वर्गघनौ॥

वर्गमूले सूत्रम्।

त्यक्त्वान्त्याद्विषमाद्वर्गं द्विघ्नमूलहृते समे।
लब्धवर्गं तदाद्यस्थात्त्यक्त्वाप्तं द्विगुणं न्यसेत्॥ ३॥ [9]
पंक्त्यां भक्ते समे त्यक्त्वेति मुहुस्तद्दलं पदम्।4ab। [10]

उदाहरणम्॥प्राग्वर्गयोर्मूलार्थं न्यासः॥ [11] लब्धे पदे। ९। १२३॥इति वर्गमूलम्॥

⟨p. 893⟩ घनमूले सूत्रम्॥

आद्यं घनो ऽघनद्वन्द्वमित्यन्ताद्घनतो घनम्॥4cd॥
त्यक्त्वा पृथक्पदं कृत्या त्रिघ्न्याद्यस्थं भजेत्फलम्। [12]
पंक्त्यां न्यसेदस्य वर्गं तदाद्यात्त्र्यन्त्यताडितम्॥ ५॥ [13]
जह्याद्घनं च तत्पूर्वान्मूलायैवं पुनर्विधिः।6ab।

उदाहरणम्॥प्राग्घनयोर्मूलार्थं न्यासः। [14] लब्धे घनमूले। ९। १२३॥इति घनमूलम्॥

7 भागार्थं] भागार्धम्॥) P.

8 Presumably this table was added later by someone who regarded 'नवानां' in E2ab as referring to 'the ⟨first⟩ nine ⟨natural numbers⟩.'

9 तदाद्यस्थात्] तदाद्योनात् P.

10 त्यक्त्वेति] भक्त्वेति P.

11 वर्गयोर्मूलार्थं] वर्गौ।मूलार्थं P. ८१। १५१२९।

12 त्रिघ्न्याद्यस्थं] त्रिघ्न्या स्थाप्यं P.

13 तदाद्यात्त्र्यन्त्य] तदाद्या अन्त्य P.

14 घनयोर्मूलार्थं] घनौ।मूलार्थः P. 729| 1860867|

इति श्रीधराचार्यविरचितं परिकर्माष्टकं समाप्तम्॥

⟨अथ भिन्नपरिकर्माष्टकम्⟩

भागजातिभिन्नसङ्कलितव्यवकलितयोः सूत्रम्॥

समच्छेदार्थमन्योन्यं छिद्भ्यां हन्याद्धरांशकौ॥6cd॥
समच्छिदो ऽंशा युतोनाः स्याद्रूपमहरे हरः।7ab।[15]

उदाहरणम्॥

अर्द्धत्र्यंशचतुर्थांशाः किं युक्ता वा त्रयाच्च्युताः॥E2cd॥

न्यासः[16]

१	१	१
२	३	४

[17]जाताः समच्छेदाः

६	४	३
१२	१२	१२

योगे जातम्

१३
१२

एतत्त्रयाद्विशोध्य शेषम्

२३
१२

इति भिन्नसङ्कलितव्यवकलिते॥

प्रभागजातिभिन्नगुणने सूत्रम्॥

भागप्रभागे गुणने ऽंशघातो हरघातहृत्॥7cd॥

उदाहरणम्।⟨।⟩

अंघ्रेरर्द्धतृतीयांशः कः किं त्र्यंशहतं दलम्।E3ab।

न्यासः

१	१	१
४	२	३

प्रभागे जातम्

१
२४

गुण्य

१
३

गुणक

१
२

गुणनाज्जातम्

१
६

इति भिन्नगुणकारः॥

भिन्नभागहारे सूत्रम्॥

परिवर्त्य लवच्छेदौ हरस्य गुणनं ततः।8ab।

⟨p. 895⟩ उदाहरणम्॥प्राग्लब्धभिन्नगुणफलस्वगुणच्छेदयोर्भिन्नभागहारार्थं न्यासः॥
[18]भाज्य

१
६

हर

१
३

लब्धम्

१
२

इति भिन्नभागहारः॥

भिन्नवर्गादिषु सूत्रम्॥

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15 रूपमहरे] रूपप्रहरे P.

16 न्यासः in margin W.

17 Here and hereafter, I supply the bottom line of each box, which is missing in P.

18 प्राग्लब्ध] प्राल्लब्ध W.

छेदांशयोः कृती वर्गे घने घनौ पदे पदे॥8cd॥

उदाहरणम्॥

सार्द्धत्रयस्य कौ वर्गघनौ ताभ्यां च के पदे॥E3cd॥

न्यासः	३	जातौ वर्गौ	४९	घनौ च	३४३	[19] जाते (f. 1v) पदे	७	७
	१		४		८		२	२
	२							

⟨इति भिन्नवर्गादि॥⟩

इति भिन्नपरिकर्माष्टकं समाप्तम्॥

⟨अथ जातिः⟩

शून्यजातौ सूत्रम्॥

शून्यं खेन वधे स्यात्खं खस्य वर्गघनादिषु।9ab।

इति शून्यजातिः॥

भागानुबन्धभागापवाहजातौ सूत्रम्॥

छेदघ्नरूपेष्वंशः स्वमृणं चैकस्य चेल्लवः॥9cd॥[20]
हतो हरो ऽधश्छिदांशस्तया स्वांशयुतोनया।10ab।[21]

⟨p. 896⟩ उदाहरणम्॥

त्र्यंशाढ्योनास्त्रयः किं किं
स्वत्र्यंशाढ्योनितास्त्रयः।E4ab।

न्यासः	३	३	[22] सवर्णिते जाते	१०	८	स्वांशपक्षे	३	३	[23] अस्य जाते ४।	
	१	१̊		३	३		१	१		
	३	३					१	१̊		
							३	३		

२॥इति भागानुबन्धभागापवाहजातिचतुष्टयम्॥

व्यस्तोद्देशकजातौ सूत्रम्॥

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19 ८] ∅ W.

20 अंशः] अंशाः P; लवः] लवानाम् P, लवानम् W.

21 हतो हरो] ह्रतो हरो P, हतो ह्ररो W; छिदांशस्] छिदंशस् W.

22 १̊] १ W.

23 १̊ W] 1 P.

व्यस्ते स्वर्णे गुणहारौ दृष्टे वर्गपदे अपि॥10cd॥[24]
स्वर्णे स्वांशे सति स्वांशाढ्योनरूपेण भाजयेत्।11ab।

उदाहरणम्।⟨।⟩

द्व्यूनस्त्रिघ्नो वर्गितः कः
स्वत्र्यंशाढ्यस्त्रिहृन्नृपाः॥E4cd॥

न्यासः॥२̊ गुण ३ वर्ग १ स्वत्र्यंश $\begin{array}{|c|}१\\३\\\hline\end{array}$ भाग ३ दृश्य १६।[25] जातो राशिः ४॥इति व्यस्तजातिः॥

इष्टजात्यादिषु सूत्रम्॥[26]

आलापवद्धतहृतांशयुतोनैकहृद्दृश्यः॥11cd॥[27]

उदाहरणम्॥

कस्त्रिघ्नः पंचहृद्राशेः
पंचांशाढ्यो ऽब्धिहृद्द्वयम्।E5ab।[28]

न्यासः॥गुण ३ भाग ५ राश्यंश धनं $\begin{array}{|c|}१\\५\\\hline\end{array}$ भाग ४ दृश्य २॥जातो राशिः १०॥ इति इष्टजातिः॥[29]

अथ दृश्यजात्युदाहरणम्॥

खं षष्ठो ऽगाज्जलं त्र्यंशः
षड्दृष्टाः कति ते शुकाः॥E5cd॥[30]

न्यासः $\begin{array}{|c|c|}१̊ & १̊\\६ & ३\\\hline\end{array}$ दृश्य ६।जाताः शुकाः १२॥इति दृश्यजातिः॥

अथ शेषजातौ सूत्रम्॥

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24 वर्गपदे] वर्णपदे P.

25 गुण] गुणा W; स्वत्र्यंश] स्वत्रयम् P; $\begin{array}{|c|}१\\३\\\hline\end{array}$] $\begin{array}{|cc|}१ & ४\\३ & \\\hline\end{array}$ W.

26 इष्ट W] दृष्ट P.

27 दृश्यः] दृशः W. Metrically the latter is better but the word form is difficult to explain unless the word दृश् has the passive meaning 'seen'. Presumably the syllable श्य was here either pronounced like 'श' or regarded as a single-consonant syllable.

28 हृद्] हृ W.

29 इष्ट] दृष्ट P.

30 षष्ठो ऽगाज्] षष्ठो ऽंगाज् P; षड्] षट् W.

छिद्घातभक्तेन लवोनहार-
घातेन भाज्यः प्रकटाख्यराशिः॥F1॥[31]

⟨शेषजात्य्⟩उदाहरणम्॥[32]

षष्ठे शेषार्द्धे तच्छेषत्र्यंशे याते शुका दश॥E6ab॥

न्यासः | १̊ ६ | शेष | १̊ २ | शेष | १̊ ३ |[33] दृश्य १०॥जातो राशिः ३६॥

अथवा भागापवाहविधिना वल्लीसवर्णनभागाः॥न्यासः | १ १ १̊ ६ १̊ २ १̊ ३ |[34]

हतो हरो ऽधश्छिदांश[35] इत्यादिकरणेन सवर्णनाज्जातम् | ५ १८ | एतेन दृष्टं भक्तं जाताः शुकाः ३६॥[36] इदं विलोमसूत्रेण वा सिध्यति॥इति शेषजातिः॥

अथ विश्लेषजात्युदाहरणम्॥

षडंशार्द्धान्तरं नष्टं दृष्टाः षड्गोयुतिः कति॥E6cd॥

न्यासः | १̊ ६ | १̊ २ | अस्यान्तरं | १̊ ३ | दृश्य ६।[37] यथोक्तकरणे जाता गोयुतिः ९॥इति विश्लेषजातिः॥

मूलजातौ सूत्रम्॥

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31 = GT 67 (p. 44, line 11). प्रकटाख्य GT] प्रकठाख्य P.
32 उदाहरणम्॥] ∅ W.
33 १̊] ०̊ १ W.
34 १̊ २] १̊° २° W.
35 Cited from GP 10a.
36 हतो हरो] ह्वतो हरो P, ह्वतो ह्वरो W.
37 अस्यान्तरं] अस्य अन्तरं W; १̊ (above '३')] १ W.

मूलाद्योनाच्चतुर्निघ्नान्मूलवर्गयुतात्पदम्।[38]
मूलोनयुतं दलितं वर्गितं जायते फलम्॥F2॥

उदाहरणम्॥

मूलोनितः शेषषडंशहीनः कः शेषमूलेन युतः खरामाः॥FE1॥

न्यासः॥१̊ śeṣa

१̊
६

शेषमूल धन १ दृश्य ३०।जातो राशिः ३६॥इति मूलजातिः॥

दृष्टमूलांशजातौ सूत्रम्॥

अथ लवयुतोनितोक्तौ लवान्वितोनैकभाजिताद्दृश्यात्।
तस्यानयनं प्राग्वन्मूलादपि यदि लवो राशेः॥F3॥

उदाहरणम्॥

यो भांशत्र्यंशनन्दांशैर्निजैर्युक्तः पदेन च।
राशिर्नवार्कतुल्यः स्यात्तं द्राक्कीर्त्तय कोविद॥FE2॥

न्यासः

१	१	१
२७	३	९

मू १ दृश्य १२९॥[39] जातो राशिः ८१॥इति दृष्टमूलांशजातिः समाप्ता॥

⟨p. 900⟩ माडजातौ सूत्रम्॥[40]

माडस्य भूमौ सदृशाङ्कपंक्तिरिष्टैः सरूपैः क्रमशो विभक्ता।[41]
इष्टघ्नलब्धं सहितं तदूर्ध्वे सूत्रं विचित्रं शृणु नार्मदोक्तम्॥F4॥[42]

उदाहरणम्॥

शिष्याकर्णय गोपुरे क्षितिभृतः सप्तक्षणे सुन्दरे
तत्रासन्मनुजाः क्षणं क्षणमधस्तिष्ठन्जगादोच्चगीः।[43]
यावन्तो वयमत्र पातनभयात्तावन्त एते क्षणाद्
आयाताः प्रतिभूमिकां वद सखे संख्यां नराणां पृथक्॥FE3॥[44]

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38 वर्गयुतात्] वर्गायुतात् P.

39 मू १] मूल P, मू ४ १ W.

40 माडजातौ सूत्रम्॥] ∅ W. The same rule (F4) and example (FE3) have been noted down on the first page of a Baroda manuscript (B2) of the Tr.

41 माडस्य] माडस्था B2.

42 तदूर्ध्वे] तदूर्द्धे B2; नार्मदोक्तम्॥ (२३॥)] नार्मदोक्तं॥१ B2.

43 य of आकर्णय in margin B2; जगादोच्चगीः।] जगादोच्चगीः॥P; जगादोच्चकैः।B2.

44 वयमत्र] यवमत्र B2 (corrected by putting '१' and '२' above 'व' and 'य' respectively); संख्यां B2] संख्या P; पृथक्॥१ B2] समा॥ (१०॥) P.

न्यासः॥पृथक्क्षणे क्षणे जाता नराः समाः

६४
६४
६४
६४
६४
६४
६४

[45] ⟨यथोक्तकरणेन जाताः प्रतिभूमिकाः संख्याः⟩

१२७
६३
६२
६०
५६
४८
३२

[46] इति माडजातिः॥

इति श्रीधराचार्यविरचिता जातिः समाप्ता॥

⟨अथ त्रैराशिकादि⟩

⟨p. 901⟩ त्रैराशिकादिषु सूत्रम्॥

त्रिपंचसप्तादिकराशिके स्तः
प्रमाणमिच्छा च समानजाती।
अन्योन्यपक्षे फलहारकाणां
विधाय यानं न फलस्य वामे॥ १२॥[47]
बह्वंशपक्षाङ्कवधो ऽल्पकांश-
पक्षाङ्कघातेन हृतः फलं स्यात्।13ab।

उदाहरणम्॥

सार्द्धैः षड्भिः पणैराम्रशतं द्रम्मेण ते कति।E7ab।

---

45 न्यासः … समाः | ६४ ⋮ ६४ |] न्यासः … समाः ६४। ६४॥P, सदृश्य ६४ B2.

46 | १२७ ⋮ ३२ |]  B2.

47 ।] । १ W.

न्यासः॥

६	१००	१६
१		
२		

[48] लब्धमाम्राः

२४६
२
१३

इति त्रैराशिकम्॥

पंचराशिके उदाहरणम्॥[49]

सार्द्धे मासे शते पंच षष्टेर्वर्षे फलं तु किम्॥E7cd॥

न्यासः

३	१२
२	१
१००	६०
५	०

[50] लब्धं कलान्तरे २४॥इति पंचराशिकम्॥

सप्तराशिके उदाहरणम्॥

दशायामा त्रिविस्तारा पटी द्रम्माष्टकेन चेत्।
लभ्यते द्वे त्रिविस्तारे अर्कायामे किमाप्नुतः॥E8॥

न्यासः

१	२
१०	१२
३	३
८	०

[51] लब्धं द्रम्माः

१९
१
५

[52] इति सप्तराशिकम्॥

⟨p. 902⟩ व्यस्तत्रैराशिके उदाहरणम्॥

दत्त्वाष्टवर्ण (here ends f. 1v) **…॥E9॥**[53]

… … …

⟨अथ व्यवहाराष्टकम्⟩

⟨मिश्रक- and श्रेढी-व्यवहार and most part
for rectilinear figures in क्षेत्र-व्यवहार are missing.⟩

… … …

---

48 १००] १०० १ W.

49 Here and hereafter, I leave the irregular sandhi (°के उ°) as is in P.

50 १] ∅ P; ० W] ∅ P. In this and the next enclosures, the zero signs, which must have been a dot or a small circle called bindu, signify that there is no numerical figure in the respective places.

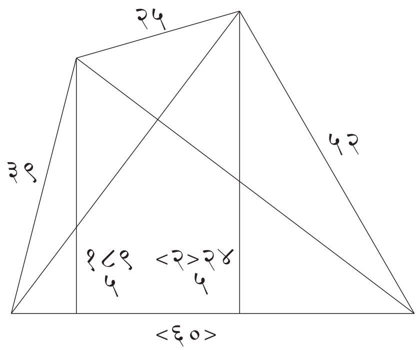
51 १२] ३ P; ३ (below '१२')] १३ P; ० W] ∅ P. For the zero sign see the footnote for the previous enclosure.

52 १९] १८ P.

53 दत्त्वाष्ट] दत्तेष्ट P.

⟨… ॥ E20 ॥⟩

… … … (f. 3r) अथ दर्शनक्षेत्रम् ॥



⟨क्षेत्र १⟩[54]

अस्य फलं १७६४ ॥[55] इति चतुरस्रक्षेत्रज्ञानं समाप्तम् ॥

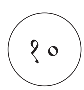
वृत्तक्षेत्रज्ञाने सूत्रम् ॥

व्यासव्यासार्द्धकृती परिधिफले नगहृते द्विदस्रघ्ने ॥ 23cd ॥

उदाहरणम् ॥

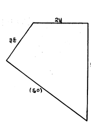
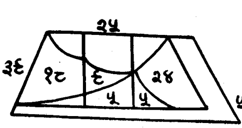
परिधिः को दशव्यासे वृत्ते स्पष्टफलं तु किम् । E21ab ।

न्यासः ॥



⟨क्षेत्र २⟩[56]

परिधिः	३१ ३ ७	अस्य फलं	७८ ४ ७	इति वृत्तक्षेत्रम् ॥

54  P, 

W.

55 End of the prose comment on the missing E20 for a quadrilateral.

56  P, 

W.

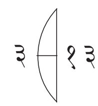
⟨p. 903⟩ धनुषः क्षेत्रानयने सूत्रम्॥

चापे फलं शरो ज्येषुयोगार्द्धघ्नो नखांशयुक्॥F5॥[57]

उदाहरणम्॥

चापे त्रयोदशज्ये किं फलं स्पष्टं त्रिसायके॥FE4॥

न्यासः॥



⟨क्षेत्र ३⟩[58]

अस्य फलं

२५
१
५

इति धनुषः फलानयनम्॥

शरानयने सूत्रम्॥

ज्याव्यासयोगान्तरघातमूलं व्यासस्तदूनो दलितः शरः स्यात्।
व्यासाच्छरोनाच्छरसङ्गुणाच्च मूलं द्विनिघ्नं भवतीह जीवा।[59]
जीवार्द्धवर्गे शरभक्तयुक्ते व्यासप्रमाणं प्रवदन्ति वृत्ते॥F6॥[60]

उदाहरणम्॥

दशविस्तृतिवृत्तान्ता यत्र ज्या षण्मिता सखे।[61]
तत्रेषुं वद बाणाज्ज्यां ज्याबाणाभ्यां च विस्तृतिम्॥FE5॥[62]

न्यासः

१०	६
१	१

[63] लब्धा बाणमितिः १॥ ज्ञाते बाणे लब्धा ज्या ६॥ ज्याबाणयोर् ज्ञातयोर्लब्धा वृत्तविस्तृतिः १०॥इति शरानयनम्॥

⟨p. 904⟩ आयतत्र्यस्रे सूत्रम्॥[64]

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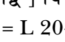
57 Cited, and ascribed to his father Keśava, by Gaṇeśa of Nandigrāma in his commentary on L 213.

58 

P,

W.

59 द्वि] वि W.

60 = L 204.

61 वृत्तान्ता] वृत्तान्तर् L/ASS, वृत्ते च L/VIS.

62 = L 205. तत्रेषुं P, L/ASS] तत्रेष्टं L/VIS; बाणाज् P, L/ASS] बाण L/VIS.

63 १ (below '१०')] ∅ P, ० W.

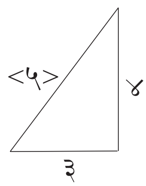
64 त्र्यस्रे] त्र्यस्त्रे P.

इष्टाद्बाहोर्यः स्यात्तत्स्पर्धिन्यां दिशीतरो बाहुः।[65]
त्र्यस्रे चतुरस्रे वा सा कोटिः कीर्त्तिता तज्ज्ञैः॥F7॥[66]
तत्कृत्योर्योगपदं कर्णो दोःकर्णवर्गयोर्विवरात्।[67]
मूलं कोटिः कोटिश्रुतिकृत्योरन्तरात्पदं बाहुः॥F8॥[68]

उदाहरणम्॥

कोटिश्चतुष्टयं यत्र दोस्त्रयं तत्र का श्रुतिः।
कोटिं दोःकर्णतः कोटिश्रुतिभ्यां च भुजं वद॥FE6॥[69]

न्यासः



⟨क्षेत्र ४⟩[70]

यथोक्तकरणे जातः कर्णः ५॥इति त्र्यस्रायतम्॥

इति श्रीश्रीधराचार्यविरचितगणितपंचविंशत्यां
क्षेत्रव्यवहारः समाप्तः॥[71]

खातव्यवहारे सूत्रम्।⟨।⟩

घनहस्ताः क्षेत्रफलं खाते वेधेन ताडितम्।24ab।

उदाहरणम्॥

त्रिवेधे घनहस्ताः के प्राक्त्र्यस्रे चतुरस्रके॥E21cd॥[72]

⟨p. 905⟩ न्यासः॥

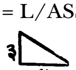
65 इष्टाद्बाहोर् P, L/VIS] इष्टो बाहुर् L/ASS; यः L/ASS, L/VIS] यत् P; तत्स्पर्धिन्यां L/ASS] पार्श्वे ऽन्य् (आय्) आं P, स्पार्द्धे ऽन्यां W, तत्समतिर्यग् L/VIS.

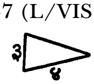
66 = L 135. कोटिः L/ASS, L/VIS] कोतिः P.

67 कर्णो L/ASS, L/VIS] कर्णौ P; दोःकर्णवर्गयोर् L/ASS, L/VIS] दोःकर्णयोर् P.

68 = L 136.

69 = L/ASS 137 (L/VIS 138). कोटिं P, L/VIS] कोटिः W, कोटि L/ASS.

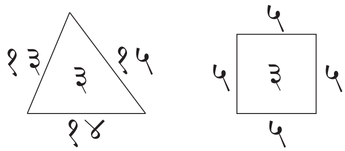
70 

P, 

W.

71 समाप्तः in margin W.

72 त्र्यस्रे] त्र्यस्त्रे P.



⟨क्षेत्र ५ क्षेत्र ६⟩[73]

लब्धं घनहस्ताः २५२। ७५॥[74] इति खातव्यवहारः॥

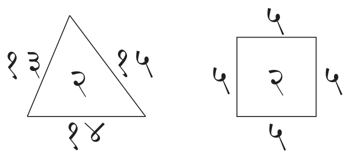
चितिक्रकचव्यवहारे सूत्रम्॥

उच्छ्रयेण चितौ मार्गैः क्रकचे फलमाहतम्॥24cd॥[75]

उदाहरणम्॥

प्रात्त्र्यस्रे चतुरस्रे च द्व्युच्छ्रये किं फलं चितौ।E22ab।[76]

न्यासः॥



⟨क्षेत्र ७ क्षेत्र ८⟩[77]

लब्धं घनहस्ताः १६८। ५०॥[78]

क्रकचव्यवहारे उदाहरणम्॥

दशदैर्घ्यत्रिविस्तारे मार्गैः षड्भिः फलं तु किम्॥E22cd॥

न्यासः॥

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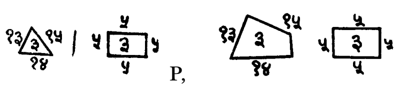
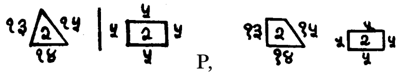
73  P, W.

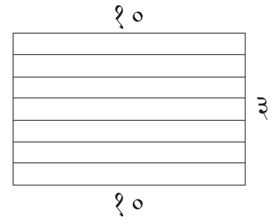
74 लब्धं W] लब्धे २ P.

75 मार्गैः] मार्गे P; आहतम्] आहृतम् P.

76 प्रात्त्र्यस्रे] चितौ त्र्यस्त्रे P.

77  P, W.

78 P puts 'लब्धं' before the figures.



⟨क्षेत्र ९⟩[79]

मार्गाः ६ ।[80] अस्य फलं घनहस्ताः १८० ॥ इति चितिक्रकचव्यवहारः ॥

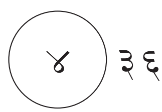
⟨p. 906⟩ राशिकाव्यवहारे सूत्रम् ॥

राशौ नवांशः परिधेः षड्भागकृतिताडितः । 25ab ।[81]

उदाहरणम् ॥

षट्त्रिंशत्परिधौ राशौ कति स्युर्घनबाहवः । E23ab ।[82]

न्यासः ॥



⟨क्षेत्र १०⟩[83]

लब्धं घनहस्ताः १४४ । एतावन्तो मागधखार्यः ॥[84] इति राशिव्यवहारः ॥

छायाव्यवहारे सूत्रम् ॥

सशङ्कुनाप्ते शङ्कुवर्द्धे गतशेषं दिनं भवेत् ॥ 25cd ॥

उदाहरणम् ॥

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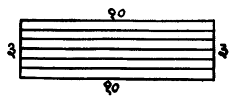
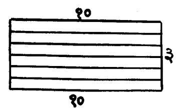
79 

P, 

W.

80 मार्गाः] मार्गः W.

81 कृति] युति P.

82 कति] केति P.

83 

P, 

W.

84 P puts this sentence 'एतावन्तो …', together with numerical figures '३६' at the beginning and 'रवार्या' instead of 'खार्यः' at the end, between the figure and 'लब्धं'.

सिद्धा भाष्टाङ्गुले शङ्कौ के द्युशेषगते तदा॥E23cd॥[85]

न्यासः॥[86] शङ्क्वङ्गुल ८ छायाङ्गुल २४।[87] लब्धं दिनगतशेषं $\boxed{\begin{matrix}१\\८\end{matrix}}$ इति दिनानयनम्॥

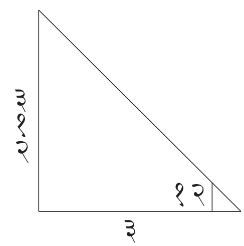
⟨p. 907⟩ छायाज्ञाने सूत्रम्॥

शङ्कुः प्रदीपतलशङ्कुतलान्तरघ्नश्
छाया भवेद्विनरदीपशिखौच्च्यभक्तः॥F9॥[88]

उदाहरणम्॥

शङ्कुप्रदीपान्तरभूस्त्रिहस्ता दीपोच्छ्रितिः सार्द्धकरत्रया चेत्।[89]
शङ्कोस्तदार्काङ्गुलसंमितस्य तस्य प्रभा स्यात्कियती वदाशु॥FE7॥[90]

न्यासः॥



⟨क्षेत्र ११⟩[91]

लब्धानि छायाङ्गुलानि १२॥इति छायाज्ञानम्॥[92]

दीपौच्च्यज्ञाने सूत्रम्॥[93]

छायोद्धृते तु नरदीपतलान्तरघ्ने
शङ्कौ भवेन्नरयुते खलु दीपकौच्च्यम्॥F10॥[94]

85 सिद्धा भा] सिद्धभा P.

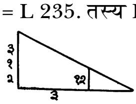
86 W repeats the last part of 25cd, 'शेषं दिनं भवेत्', before 'न्यासः'.

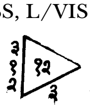
87 शङ्क्वङ्गुल] शङ्कु अङ्गुल W; छायाङ्गुल] छाया अङ्गुल W.

88 = L 234. शङ्कुः P, L/ASS, L/VIS] शङ्कु W; शङ्कुतला P, L/ASS, L/VIS] शङ्क्वतला W.

89 शङ्कु L/ASS, L/VIS] शङ्क P.

90 = L 235. तस्य L/ASS, L/VIS] तस्याः P.

91 

P, 

W.

92 ज्ञानम्॥] ज्ञानम्॥1 W.

93 दीपौच्च्य] दीपोच्च्य P.

94 = L 236. छायोद्धृते P, L/VIS] छायाहृते L/ASS; दीपकौच्च्यम् P, L/ASS] दीपकोच्चम् L/VIS.

उदाहरणम्॥

प्रदीपशङ्क्वन्तरभूस्त्रिहस्ता छायाङ्गुलैः षोडशभिः समा चेत्॥
दीपोच्छ्रितिः स्यात्कियती तदास्याः प्रदीपशङ्क्वन्तरमुच्यतां मे॥FE8॥[95]

न्यासः॥शङ्कु १२ छायाङ्गुलानि १६ शङ्कुप्रदीपान्तरं हस्ताः ३।लब्धं दीपौच्च्यं ११/४
इ(f. 3v)ति दीपौच्च्यम्॥

⟨p. 908⟩ इति छायाव्यवहारः॥⟨इति व्यवहाराष्टकम्॥⟩

प्रतिकंचुकरचनापटुपाशनपतनं न ते भवत्वररे।[96]
श्रीधररचितं पठ पठ गणिताध्यायं रसिकरम्यम्॥२६॥

उत्तरतः सुरनिलयं दक्षिणतो मलयपर्वतं यावत्।
प्रागपरोदधिमध्ये नो गणकः श्रीधरादन्यः॥F11॥

इति श्रीश्रीधराचार्यविरचिता गणितपंचविंशी समाप्ता॥[97]

2. TRANSLATION

This is a literal English translation of the text of the GP edited by me in the foregoing section and neither of the manuscript nor of Pingree's edition. A pair of parentheses, (A), indicates that A is either an explanation or the original Sanskrit word(s) for the preceding word(s). A pair of angular brakets, ⟨A⟩, means that A has been added by me for smoothing the English translation. For E, F and S attached to the verse numbers, see the Notation for the Text.

Now, the terminology (*paribhāṣā*) ⟨begins⟩.

95 = L 237. तदास्याः] वदाऽऽशु L/ASS, तथाऽऽभ्यां L/VIS.
96 कंचुक] कुञ्चुक W; पाशन] पाश P.
97 विंशी समाप्ता] विंशिः समाप्त W.

Having bowed to Śiva, Śrīdhara teaches the essence of the book of algorithms (*pāṭī*) composed by him, in twenty-five verses (*ślokas*) with clear import. /S1/

Five *guñjās* shall constitute one *māṣa* and 'kings' (16) *māṣas* one *karṣa*. The same ⟨when used⟩ for gold (*suvarṇa*) is ⟨called⟩ *suvarṇa*. Four of these (*karṣas*) are laid down to be one *pala*. /S2/

Three *guñjās* constitute one *valla*; sixteen of these are laid down to be one *gadyāṇaka*. /S3ab/

In ⟨a measuring cup called⟩ *kuṭapa*, the depth is one and a half *aṅgulas* and the length and width are three ⟨*aṅgulas* each⟩. Four *kuṭapas* make one *prastha*, four *prasthas* one *āḍhaka*, four *āḍhakas* one *droṇa*, and 'kings' (16) *droṇas* make one *khāri*. /S3cd–S4/

Twenty *varāṭakas* (cowry-shells) make one *kākiṇī*, four of these one *paṇa*, sixteen of these one *dramma*, and sixteen of these make one *niṣka*. /S5/

Three corns of *yava* placed vertically (i.e. lengthwise) shall constitute one *aṅgula*, twenty-four of these one *kara* (cubit), four *karas* one *daṇḍa*, two thousand of these one *krośa*, and four *krośas* one *yojana*. /S6–S7a/

Ten or eight *hastas* (cubits) make one *vaṃśa* (bamboo rod). One *nivartana* is ⟨the area of⟩ the plane figure bound by four ⟨equal⟩ arms (sides) which are made up of twenty of these (*vaṃśas*). /S7bcd/

Eka, daśa, śata, sahasra, ayuta, and then *lakṣa, prayuta, koṭi, arbuda, padma, kharva, nikharva, mahāpadma, śaṅku, saritpati, antya, madhya,* and *parārddha* (10^{17}): these shall be the names ⟨of numbers as well as of places⟩, each of which when multiplied by ten becomes the succeeding term. /S8–S9/

100000000000000000.

Thus the terminology.

Now ⟨begins⟩ the eightfold fundamental operation (*parikarma-aṣṭaka*).
Rule for the sum (*saṅkalita*) and the difference (*vyavakalita*).

> The sum or difference ⟨of the digits⟩ at each ⟨notational⟩ place ⟨should be taken⟩. In the case of ⟨the sum or difference⟩ with zero, there shall be no change./1ab/

Example.

> What will be sixty, nine and 'suns' (12) when added together? Or, ⟨what will be the remainder when the same are⟩ subtracted from one hundred? /E1ab/

Setting-down: 60, 9, 12. What is produced in the summation is 81. When one has subtracted this ⟨result⟩ from one hundred, the remainder is 19. Thus the sum and the difference.

Rule for the multiplication (*guṇana*).

> One should multiply the digits of the multiplicand, beginning from the last ⟨place⟩, by the multiplier. /1cd/

Example.

> What are 'mountains-suns' (127) multiplied by 'Jaina sages' (24)? What are 'teeth-earth-kings' (16132) ⟨multiplied⟩ by 'three-suns' (123)? /E1cd/

Setting-down: multiplicand 127, multiplier 24; multiplicand 16132, multiplier 123. The results known from the multiplication: 3048 and 1984236. Thus the multiplication.

Rule for the division (*bhāga-hāra*).

> In division, that which, multiplied by the divisor, can be cleared (i.e., subtracted) from the dividend is the result (quotient). /2ab/

Example. Setting-down, for division, of the result of multiplication previously obtained and of its own multiplier as the divisor: dividend 3048, divisor 24; dividend 1984236, divisor 123. The multiplicands obtained from the division: 127 and 16132. Thus the division.

Rule for the square (*varga*) and the cube (*ghana*).

> The product of two equal ⟨numbers⟩ shall be the square, and the product of three equal ⟨numbers⟩ the cube. /2cd/

Example.

O friend, what will be the square and cube of nine and of 'three-suns' (123)? /E2ab/

										cu								
Setting-down:	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
	1	4	9	16	25	36	49	64	81	1	8	27	64	125	216	343	512	729

Setting-down: 9, 123. The two squares produced are 81 and 15129; and the two cubes 729 and 1860867. Thus the square and the cube.

Rule for the square-root (*varga-mūla*).

When one has taken away the ⟨largest possible⟩ square from the last odd ⟨place⟩, the even ⟨place next to it on the right⟩ being divided by twice the root ⟨which is placed in a line for the root⟩, and when one has taken away the square of the quotient from its preceding place, one should place the quotient multiplied by two in the line ⟨for the root⟩. ⟨The two operations, viz.⟩, 'the even ⟨place next to it on the right⟩ being divided' and 'when one has taken away ⟨the square of the quotient⟩,' are repeated ⟨and each time twice the quotient obtained is placed in the line for the root⟩. Half of that ⟨number which is obtained in the line for the root⟩ is the ⟨square-⟩root. /3–4ab/

Example. Setting-down, for the root, of the two previous squares: 81, 15129. Two roots obtained are 9 and 123. Thus the square-root.

Rule for the cube-root (*ghana-mūla*).

⟨In each triplet of notational places⟩, the first is a cube ⟨place⟩ and a pair of non-cube ⟨places follow⟩. When one has taken away the ⟨largest possible⟩ cube from the last cube ⟨place⟩, its root being separately ⟨placed in a line⟩, one should divide its preceding place by thrice the square ⟨of the root⟩ and place the quotient in the line ⟨of the root⟩. One should subtract the square of that ⟨quotient⟩ multiplied by three and by the last ⟨root⟩ from its preceding ⟨place⟩, and also the cube ⟨of that quotient⟩ from its preceding ⟨place⟩. This operation is further ⟨repeated⟩ for obtaining the ⟨cube-⟩root. /4cd–6ab/

Example. Setting-down, for the root, of the two previous cubes: 729, 1860867. Two cube-roots obtained are 9 and 123. Thus the cube-root.

Thus the eightfold fundamental operation composed by the revered teacher Śrīdhara is completed.

Now ⟨begins⟩ the eightfold fundamental operation for fractions (*bhinna*).

Rule for the sum and the difference of fractions and the part class (*bhāga-jāti*).

> For the sake of equal denominators, one should multiply the denominator and the numerator ⟨of each fraction⟩ by mutual denominators. The numerators having the same denominator are added together or subtracted ⟨from one another⟩. In the case of an integer (*ahara*, lit. one that has no denominator), the denominator shall be unity. /6cd–7ab/

Example.

> What are half, one-third and one-fourth added together? Or, ⟨what is the remainder when the same are⟩ removed from three? /E2cd/

Setting-down: $\begin{array}{|c|c|c|}1 & 1 & 1 \\ 2 & 3 & 4 \\ \hline\end{array}$ ⟨The fractions⟩ obtained with equal denominators are $\begin{array}{|c|c|c|}6 & 4 & 3 \\ 12 & 12 & 12 \\ \hline\end{array}$ By addition is produced $\begin{array}{|c|}13 \\ 12 \\ \hline\end{array}$ When one has subtracted this from three, the remainder is $\begin{array}{|c|}23 \\ 12 \\ \hline\end{array}$ Thus the sum and the difference of fractions.

Rule for the multiplication of fractions and the multi-part class (*prabhāga-jāti*).

> In the case of the multi-part ⟨class⟩ and multiplication of fractions, the product of the numerators is divided by the product of the denominators. /7cd/

Example.

> What is one-third of a half of a quarter? What is a half multiplied by one-third? /E3ab/

Setting-down: $\begin{array}{|c|c|c|}1&1&1\\4&2&3\\\hline\end{array}$ By ⟨the operation called⟩ multi-part is produced $\begin{array}{|c|}1\\24\\\hline\end{array}$ Multiplicand $\begin{array}{|c|}1\\3\\\hline\end{array}$ multiplier $\begin{array}{|c|}1\\2\\\hline\end{array}$ By multiplication is produced $\begin{array}{|c|}1\\6\\\hline\end{array}$ Thus the multiplication of fractions.

Rule for the division of fractions.

> Having interchanged the numerator and the denominator of the divisor, ⟨one should perform⟩ a multiplication ⟨of the dividend⟩ by it. /8ab/

Example. Setting-down, for the division of fractions, of the result of multiplication of fractions previously obtained and its own multiplier as the divisor: dividend $\begin{array}{|c|}1\\6\\\hline\end{array}$ divisor $\begin{array}{|c|}1\\3\\\hline\end{array}$ Quotient $\begin{array}{|c|}1\\2\\\hline\end{array}$ Thus the division of fractions.

Rule for the square etc. of fractions.

> The squares, the cubes, and the ⟨square- and cube-⟩roots of the denominator and numerator ⟨are made⟩ respectively for the square, the cube, and the ⟨square- and cube-⟩roots ⟨of a fraction⟩. /8cd/

Example.

> What are the square and the cube of three and a half? What are the ⟨respective⟩ roots from them? /E3cd/

Setting-down: $\begin{array}{|c|}3\\1\\2\\\hline\end{array}$ The squares obtained $\begin{array}{|c|}49\\4\\\hline\end{array}$ and the cubes $\begin{array}{|c|}343\\8\\\hline\end{array}$ The roots obtained $\begin{array}{|c|c|}7&7\\2&2\\\hline\end{array}$ Thus the square etc. of fractions.

Thus the eightfold fundamental operation for fractions is completed.

Now ⟨begin⟩ the ⟨operations called⟩ class (*jāti*).

Rule for the zero class (*śūnya-jāti*).

> In the case of multiplication by zero, ⟨the result⟩ shall be zero. In the case of the square, cube, etc. of zero ⟨also, the result shall be⟩ zero. /9ab/

Thus the zero class.

Rule for the part-increase and the part-decrease classes (*bhāgānubandha-bhāgāpavāha-jāti*).

> The numerator is added to, or subtracted from, the integer multiplied by the denominator ⟨and the numerator is deleted⟩ if the fraction is of unity. ⟨But if the fraction is of the preceding quantity⟩, the denominator is multiplied by the lower denominator and the numerator by the same (i.e., the lower denominator) increased or decreased by its own numerator ⟨and the lower numerator and denominator are deleted⟩. /9cd–10ab/

Example.

> What is three increased or decreased by one-third? What is three increased or decreased by one-third of itself? /E4ab/

Setting-down: $\begin{array}{|c|c|} 3 & 3 \\ 1 & \overset{\circ}{1} \\ 3 & 3 \\ \hline \end{array}$ When these are made into the same color, ⟨the results⟩ produced are $\begin{array}{|c|c|} 10 & 8 \\ 3 & 3 \\ \hline \end{array}$

⟨Setting-down⟩ for the case of ‘its own numerator’: $\begin{array}{|c|c|} 3 & 3 \\ 1 & 1 \\ 1 & \overset{\circ}{1} \\ 3 & 3 \\ \hline \end{array}$ Produced for this are 4 and 2.

Thus the fourfold class of part-increase and part-decrease.

Rule for the inverse-problem class (*vyastoddeśaka-jāti*).

> The positive and negative, the multiplier and divisor, and also the square and root are inversely ⟨applied⟩ to the seen ⟨quantity⟩ (*dṛṣṭa*). When its own part has been added to, or subtracted from, ⟨the original quantity⟩, one should divide ⟨the seen quantity⟩ by unity increased or decreased by that part. /10cd–11ab/

Example.

> What is that ⟨number⟩ which, when decreased by two, multiplied by three, squared, increased by its own one-third, and divided by three, is ‘kings’ (16)? /E4cd/

Setting-down: −2, multiplier 3, square 1, its own one-third $\begin{array}{|c|}1\\3\end{array}$ divisor 3, and visible 16. The quantity produced is 4. Thus the inverse class.

Rule for the optional-number class (*iṣṭa-jāti*) etc.

> The visible ⟨quantity⟩ (*dṛśya*) is divided by unity which is multiplied, divided, or increased or deceased by part according to the statement ⟨of the problem⟩. /11cd/

Example.

> What is that ⟨quantity⟩ which, when multiplied by three, divided by five, increased by one-fifth of that quantity, and divided by 'seas' (4), is two? /E5ab/

Setting-down: Multiplier 3, divisor 5, part of the quantity, positive, $\begin{array}{|c|}1\\5\end{array}$ divisor 4, and visible 2. The quantity produced is 10. Thus the optional-number class.

Now, an example of the visible-number class (*dṛśya-jāti*).

> One-sixth ⟨of a flock of parrots⟩ went to the sky and one-third to the water; six are seen. How many are those parrots? /E5cd/

Setting-down: $\begin{array}{|c|c|}\overset{\circ}{1} & \overset{\circ}{1}\\ 6 & 3\end{array}$ visible 6. The ⟨number of⟩ parrots produced is 12. Thus the visible-number class.

Now, a rule for the remainder class (*śeṣa-jāti*).

> The quantity called visible (*prakaṭa*) should be divided by the product of the denominators decreased by the ⟨corresponding⟩ numerators which (product) is divided by the product of the denominators. /F1 = GT 67/

Example of the remainder class.

> When one-sixth ⟨of a flock of parrots⟩, a half of the remainder, and one-third of the remainder from that ⟨remainder⟩ are gone, ten parrots ⟨are seen⟩. /E6ab/

Setting-down: $\begin{array}{|c|}\overset{\circ}{1}\\ 6 \\ \hline\end{array}$ remainder's $\begin{array}{|c|}\overset{\circ}{1}\\ 2 \\ \hline\end{array}$ remainder's $\begin{array}{|c|}\overset{\circ}{1}\\ 3 \\ \hline\end{array}$ visible 10. The quantity produced is 36.

Or, otherwise, the fractions ⟨given here are regarded as those⟩ for homogenization of the creeper ⟨of fractions⟩ by means of the rule of the partial-decrease ⟨class⟩. Setting-down $\begin{array}{|c|}1\\ 1\\ \overset{\circ}{1}\\ 6\\ \overset{\circ}{1}\\ 2\\ \overset{\circ}{1}\\ 3\\ \hline\end{array}$ From the homogenization by means of the computation beginning with 'the denominator is multiplied by the lower denominator and the numerator ...' (10ab), what is produced $\begin{array}{|c|}5\\ 18\\ \hline\end{array}$ By this the seen ⟨quantity⟩ is divided. ⟨The number of⟩ the parrots produced is 36.

Or, otherwise, this is solved by means of the rule of inverse operations. Thus the remainder class.

Now, an example of the difference class (*viśleṣa-jāti*).

> The difference between one-sixth and a half ⟨of a herd of cows⟩ is lost and six are seen. What is the total number of the cows? /E6cd/

Setting-down: $\begin{array}{|c|c|}\overset{\circ}{1} & \overset{\circ}{1}\\ 6 & 2\\ \hline\end{array}$ difference of this ⟨pair⟩ $\begin{array}{|c|}\overset{\circ}{1}\\ 3\\ \hline\end{array}$ visible 6. The total number produced by means of the computation as told is 9. Thus the difference class.

Rule for the root class (*mūla-jāti*).

> ⟨The visible quantity that is the original quantity⟩ increased or decreased by ⟨a certain number of⟩ the root ⟨of the original quantity⟩, is multiplied by four and increased by the square of ⟨the number of⟩ the root; the square-root ⟨from the result⟩, when decreased or increased by ⟨the number of⟩ the root, halved, and squared, becomes the answer (*phala*). /F2/

Example.

> What is that ⟨quantity⟩ which, when decreased by the root, decreased by one-sixth of the remainder, and increased by the root of the remainder, is 'sky-Rāmas' (30)? /FE1/

Setting-down: ⟨root of the original quantity, negative⟩ $\overset{\circ}{1}$, remainder's $\begin{array}{|c|}\overset{\circ}{1}\\ 6 \\ \hline\end{array}$ root of the remainder, positive, 1, visible 30. The quantity produced is 36. Thus the root class.

Rule for the seen-root-part class (*dṛṣṭa-mūla-aṃśa-jāti*).

> Now, in the case of the statement in which ⟨the original quantity is⟩ increased or decreased by part ⟨of the quantity⟩, the visible is divided by unity increased or decreased by that part and the calculation of it (the original quantity) is as above ⟨in the root class⟩. ⟨The number of⟩ the root also ⟨is divided by the same⟩. This is when ⟨each⟩ part is of the original quantity. /F3/

Example.

> A quantity, when increased by one-'lunar mansions'-th part (i.e., 1/27), one-third part, and one-'Nandas'-th part (i.e., 1/9) of itself as well as by the root, will be 'nine-suns' (129). O skilled one, tell it quickly. /FE2/

Setting-down: $\begin{array}{|c|c|c|}1 & 1 & 1\\ 27 & 3 & 9\\ \hline\end{array}$ root 1, visible 129. The quantity produced is 81. Thus the seen-root-part class is completed.

Rule for the tower class (*māḍa-jāti*).[98]

> A line of like digits corresponding to ⟨each⟩ floor (*bhūmi*) of a tower (*māḍa*) is successively divided by the desired number (*iṣṭa*) increased by one. The quotient ⟨of each division⟩ is multiplied by the desired number and, ⟨starting from the bottom, successively⟩ added to the above. Listen to ⟨this⟩ wonderful rule taught by Nārmada[99]. /F4/

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98 The word *māḍa* is not Sanskrit but either Malayalam or Kannada or Tamil (cf. also *mēḍa* in Malayalam, Tamil, and Telugu) and means a tower or a house with multiple floors or stories. It seems to be used here synonymously with *gopura* that occurs in FE3a.

99 It is not certain whether this Nārmada is Padmanābha's father of the same name or not. The rare name itself may suggest their identification, but the following consideration is unfavorable to it. According to Pingree 1976, 171b and 1994, 183a, Nārmada, father of Padmanābha, wrote a *Nabhogasiddhi* in the latter half of the fourteenth century; three manuscripts of the work are extant in Varanasi and in Bikaner. Padmanābha, too, wrote several books on astronomy and on astronomical instruments [?, 170a–172a] and [?, 205b]; many manuscripts of his works are extant, but their circulation is confined to North India. The family, therefore, seems to have belonged to North India. The present 'rule taught by Nārmada,' on the other hand, deals with a problem which is set in a building called *gopura*; the *gopura* is a style of architecture that has been developed in South India as a gateway tower of a Hindu temple. Moreover, the category of the problem treated here is expressed by the South-Indian synonym *māḍa*. This Nārmada, therefore, seems to have belonged to South India.

Example.

> O disciple, listen ⟨to me⟩. In a king's beautiful tower (*gopura*) with seven floors (*kṣaṇa*)[100] there were ⟨some⟩ men. On each floor, a man standing below was ⟨heard⟩ saying in a loud voice, 'As many of us are here, so many of them came from the floor ⟨above⟩, for fear of falling down.' O friend, tell the number of persons on each floor (*bhūmi*) separately. /FE3/

Setting-down: The same number of men produced separately for each floor is

64
64
64
64
64
64
64

The number ⟨of men⟩ of each floor produced by means of the said computation is

127
63
62
60
56
48
32

Thus the tower class.

Thus the ⟨chapter on the operations called⟩ class composed by the revered teacher Śrīdhara is completed.

Now ⟨begin⟩ the three-quantity operation (*trairāśika*) etc.

Rule for the three-quantity operation etc.

> In the three-, five-, seven-quantity operations, etc., ⟨the corresponding terms in⟩ the standard and requisite ⟨sides⟩ are of the same kind (*jāti*). When one has moved the fruit and the denominators ⟨of fractions⟩ to the mutually opposite sides, excepting the inverse ⟨three-quantity operation⟩ where there is no move of the fruit, the product of the digits of the side with more numerators divided by the product of the digits of the side with fewer numerators shall be the fruit. /12-3ab/

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100 The word *kṣaṇa* occurs four times in this verse and twice in the prose part that follows, and in all these cases the meaning 'floor' or 'story' fits well, although this meaning cannot be attested in Sanskrit dictionaries (where, usually, *kṣaṇa* stands for a very small unit of time). It may be either a Sanskritization of an unknown non-Sanskrit word or a corruption of *kṣamā* used synonymously with *bhūmi* of F4a and FE3d, or otherwise a word somehow related to the Sanskrit word *kṣoṇi*/*kṣoṇī* ('earth').

Example.

> One hundred mangos are ⟨obtained⟩ for six and a half *paṇas*. How many of them are ⟨obtained⟩ for one *dramma*? /E7ab/

Setting-down:

6	100	16
1		
2		

Obtained are

246
2
13

pieces of mango fruit.

Thus the three-quantity operation.

Example for the five-quantity operation (*pañcarāśika*).

> The fruit (interest) for one hundred in one and a half months is five. What is it for sixty in one year? /E7cd/

Setting-down:

3	12
2	1
100	60
5	0

Obtained for the interest is 24. Thus the five-quantity operation.

Example for the seven-quantity operation (*saptarāśika*).

> If a piece of cloth whose length and width are ten and three respectively is obtained for eight *drammas*, what do two pieces, each of which measures three in width and 'suns' (12) in length, obtain? /E8/

Setting-down:

1	2
10	12
3	3
8	0

Obtained is

19
1
5

drammas. Thus the seven-quantity operation.

Example for the inverse three-quantity operation (*vyasta-trairāśika*).

> Having given ⟨a piece of gold whose purity is⟩ eight *varṇas* … /E9/

⟨Here begins the lost portion, for which see Table 2 in the Introduction.⟩

… … …

Now ⟨begin⟩ the eight kinds of procedure.

… … …

> … /E20/

… … …

Now, an illustrating figure:

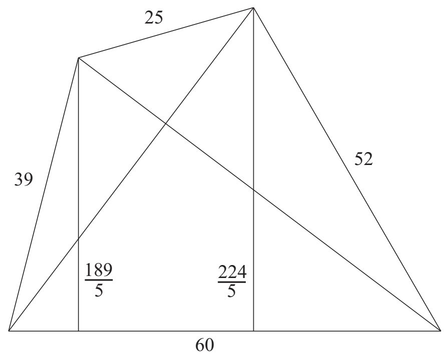

Figure 1t

The area (*phala*) of this is 1764.[101] Thus the knowledge of quadrilateral figures (*caturasra-kṣetra*) is completed.

Rule for the knowledge of circular figures (*vṛtta-kṣetra*).

> The diameter (*vyāsa*) and the square of the semi-diameter, each multiplied by 'two-twins' (22) and divided by 'mountains' (7), are respectively the circumference (*paridhi*) and the area (*phala*). /23cd/

Example.

> What is the circumference of a circle whose diameter is ten? What is its true area? /E21ab/

Setting-down:

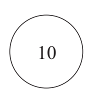

Figure 2t

The circumference ⟨obtained⟩ is $\begin{array}{|c|} 31 \\ 3 \\ 7 \\ \hline \end{array}$ and its area $\begin{array}{|c|} 78 \\ 4 \\ 7 \\ \hline \end{array}$ Thus the circular figure.

101 End of the prose comment on the lost E20.

Rule for the computation of the area of a bow (*dhanus*) (a segment of a circle).

> The area of a bow (*cāpa*) is the arrow (*śara*) multiplied by half the sum of the chord (*jyā*) and the arrow (*iṣu*) and increased by one-‘nails’-th part (i.e., 1/20) ⟨of itself⟩ /F5/[102]

Example.

> What is the true area of a bow whose chord and arrow are thirteen and three respectively. /FE4/

Setting-down:

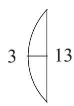

Figure 3t

The area of this is

25
1
5

Thus the computation of the area of a bow.

Rule for the computation of the arrow ⟨etc.⟩

> The square-root of the product of the sum and the difference of the chord and the diameter ⟨is taken⟩. The diameter decreased by that ⟨square-root⟩ and halved shall be the arrow. The square-root from the diameter decreased and multiplied by the arrow, when multiplied by two, becomes the chord. When the square of half the chord is divided and increased by the arrow, they tell ⟨the result as⟩ the length of the diameter of the circle. /F6 = L 204/

Example.

> O friend, tell the arrow when a chord ending with a circle[103] whose diameter is ten measures six. ⟨Also tell⟩ the chord from the arrow, and the diameter from the chord and arrow. /FE5 = L 205/

Setting-down:

10	6
1	1

The length of the arrow obtained is 1. When the arrow is known, the ⟨length of the⟩ chord obtained is 6. When the chord and the arrow are known, the diameter of the circle obtained is 10. Thus the computation of the arrow ⟨etc.⟩

Rules for oblong (*āyata*) and trilateral (*tryasra*).

> In a trilateral or a quadrilateral, against an optional arm (*bāhu*, one of the two sides orthogonal to each other), another arm (side) that lies in the direction competing with it (i.e., orthogonal to it) is called the edge (*koṭi*, upright) by

102 Gaṇeśa cites this half Anuṣṭubh verse in his commentary on L 213 and ascribes it to his father Keśava.

103 Instead of ‘a chord ending with a circle,’ L/ASS reads ‘a chord inside a circle,’ and L/VIS ‘a chord in a circle.’

experts. /F7 = L 135/ The square-root of the sum of the squares of the two is the ear (*karṇa*, hypotenuse); the square-root from the difference between the squares of the arm and the ear is the edge; and the square-root from the difference between the squares of the edge and the ear is the arm. /F8 = L 136/

Example.

What is the ear where the edge is four and the arm is three? Tell also the edge from the arm and ear, and the arm from the edge and ear. /FE6 = L 137/

Setting-down:

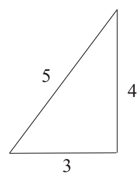

Figure 4t

The ear obtained by means of the computation as told ⟨above⟩ is 5, ⟨the edge 4, and the arm 3⟩. Thus trilaterals and oblongs.

Thus, the ⟨chapter called⟩ Procedure for Plane Figures in the *Mathematics in Twenty-Five Verses* composed by the revered teacher Śrīdhara is completed.

Rule for the procedure of excavations (*khāta*).

The ⟨horizontal⟩ plane area of an excavation multiplied by its depth is the volume in *hastas*. /24ab/

Example.

In the case of the previous trilateral and quadrilateral, when they have the depth of three, what is the volume in *hastas* ⟨for each⟩? /E21cd/

Setting-down:

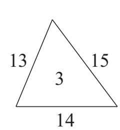

Figure 5t

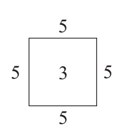

Figure 6t

Obtained are 252 and 75 solid *hastas*. Thus the procedure of excavations.

Rule for the procedure of brick-piling (*citi*) and timber-sawing (*krakaca*).

The area (*phala*) is multiplied by the height in the brick-piling and by the ⟨number of sawing⟩ routes in the timber-sawing. /24cd/

Example ⟨for brick-piling⟩.

In the case of the previous trilateral and quadrilateral, when they have the height of two, what is the fruit (*phala*) for piling? /E22ab/

Setting-down:

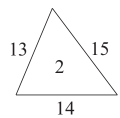

Figure 7t

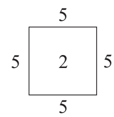

Figure 8t

Obtained are 168 and 50 solid *hastas*.

Example for the procedure of timber-sawing.

What is the fruit (*phala*) for ⟨sawing a plank⟩, whose length and width are respectively ten and three, along six routes? /E22cd/

Setting-down:

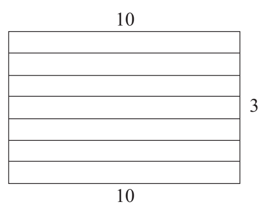

Figure 9t

Routes 6. The fruit (*phala*) of this is 180 solid *hastas*. Thus the procedure of brick-piling and timber-sawing.

Rule for the procedure of heaped-up grain (*rāśikā*).

In the case of heaped-up grain, one-ninth of the circumference ⟨of the base of the grain heaped up in the shape of a cone⟩ is multiplied by the square of one-sixth ⟨of the same in order to obtain the volume of the grain⟩. /25ab/

Example.

In the case of a heaped-up grain whose circumference is thirty-six, what shall be the volume in *bāhus* (= *hastas*)? /E23ab/

Setting-down:

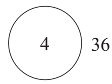

Figure 10t

Obtained is 144 solid *hastas*, which measure the same in Magadha's Khārī.[104]
Thus the procedure of heaped-up grain.

Rule for the procedure for shadow (*chāyā*).

> When half the gnomon (*śaṅku*) is divided by the ⟨shadow⟩ increased by the gnomon, the elapsed or remaining portion of the daylight (*dina*) will be ⟨obtained⟩. /25cd/

Example.

> The shadow measures 'Jaina sages' (24) when the gnomon measures eight *aṅgulas*. What is the remaining or elapsed portion of the daylight at that time? /E23cd/

Setting-down: gnomon 8 *aṅgulas*, shadow 24 *aṅgulas*. Obtained is the elapsed or remaining portion of the daylight $\boxed{\begin{matrix}1\\8\end{matrix}}$ Thus the computation of ⟨the elapsed or remaining portion of⟩ the daylight.

Rule for the knowledge of the shadow.

> The gnomon, multiplied by the distance between the feet of the lamp and of the gnomon and divided by the height of the top of the lamp less the man (i.e., gnomon), shall be the shadow. /F9 = L 234/

Example.

> If the ground (distance) between the gnomon and the lamp measures three *hastas* and the height of the lamp three and a half *karas* (= *hastas*), then how long will be the shadow of that gnomon which measures 'suns' (12) *aṅgulas*? Tell quickly. /FE7 = L 235/

[104] L 7 reads: 'A twelve-edge solid (i.e., cube) whose width, length and thickness measure one *hasta* each is called a solid *hasta*. The measure of one solid *hasta* used for grain is "Magadha's Khārikā" stipulated in the treatises.'

Setting-down:

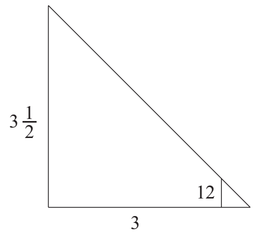

Figure 11t

The ⟨length in⟩ *aṅgulas* of the shadow obtained is 12. Thus the knowledge of the shadow.

Rule for the knowledge of the height of the lamp (*dīpa-auccya*).

> When the gnomon is divided by the shadow, multiplied by the distance between the feet of the man (gnomon) and of the lamp, and increased by the man, the height of the lamp will indeed be ⟨obtained⟩. /F10 = L 236/

Example.

> If the ground (distance) between the lamp and the gnomon measures three *hastas* and the shadow is equal to sixteen *aṅgulas* ⟨in length⟩, how much will be the height of the lamp? ⟨Also,⟩ at that time from this ⟨height⟩,[105] the distance between the lamp and the gnomon should be told to me. /FE8 = L 237/

Setting-down: gnomon ⟨in *aṅgulas*⟩ 12, shadow in *aṅgulas* 16, distance between the gnomon and the lamp in *hastas* 3. The height of the lamp obtained is $\begin{array}{|c|}11\\4\\\hline\end{array}$ *hastas*.

Thus the height of the lamp.

Thus the procedure for shadow.

⟨Concluding remark.⟩

> May you not fall in the cunning snares of the works of critics. Read again and again ⟨this⟩ textbook on mathematics composed by Śrīdhara, which is delightful to a man of taste. /26/

> Up to the mountain of gods (i.e., Himalaya) in the North and the Malaya mountain in the South and between the Eastern and Western Seas, there is no mathematician other than Śrīdhara. /F11/

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105 Instead of '⟨Also,⟩ at that time from this ⟨height⟩,' L/ASS reads 'Tell quickly,' and L/VIS 'Likewise, from these two (i.e., the shadow and the height of the lamp).'

Thus, the *Mathematics in Twenty-Five* ⟨*Verses*⟩ composed by the revered teacher Śrīdhara is completed.

3. COMMENTARY

The references to the L in this section are not exhaustive. For the parallel verses in the L see Table 2 above.

S1: Homage
Just like the first verse of the Tr, this verse also claims that the author is Śrīdhara and that the work is an epitome of his larger work. The god to whom homage is paid, Śiva, is also common to the two works as well as to the PG.

S2: Weight, system 1.
In the following tables for the weights and measures, the conversion ratios printed in italic face indicate that they have actually been given in the texts; others have been obtained by calculation. The unit names are spelled out in the first column but in the top row they are abbreviated to fit into the narrow space.

	gu	mā	ka	pa
guñjā	1			
māṣa	*5*	1		
karṣa*	80	*16*	1	
pala	320	64	*4*	1

*One *karṣa* of gold is *suvarṇa.*

Exactly the same table occurs in Tr Pb5 = PG 10 and in L 4.

S3ab: Weight, system 2.

	gu	va	ga
guñjā	1		
valla	*3*	1	
gadyāṇaka	48	*16*	1

The weight units *valla* and *gadyāṇaka* do not occur in the Tr and PG. This table is part of the table of L 3:

	ya	gu	va	dha	gha	ga
yava	1					
guñjā	*2*	1				
valla	6	*3*	1			
dharaṇa	48	24	*8*	1		
ghaṭaka	84	42	*14*	$1\frac{3}{4}$	1	
gadyāṇaka	96	48	16	*2*	$1\frac{1}{7}$	1

(The units of system 2 are italicized.)

S3cd–S4: Volume.

Def: Measuring cup for one *kuṭapa* = $3\times3\times1\frac{1}{2}$ *aṅgulas* = $13\frac{1}{2}$ cubic *aṅgulas*.

	ku	pra	ā	dro	khā
kuṭapa	1				
prastha	*4*	1			
āḍhaka	16	*4*	1		
droṇa	64	16	*4*	1	
khāri	1024	256	64	*16*	1

Exactly the same table, except for the definition of the *kuṭapa* measure in S3cd, occurs in Tr Pb6 = PG 11 and in L 8. For Sūrya's quotation of S3cd see Hayashi 1995, 246-7.
According to L 7–8,

1024 *kuḍavas* = 1 *khārī* = 1 cubic *hasta*,

where *hasta* is the same as *kara*, that is, 24 *aṅgulas*. Hence follows exactly the same value of *kuḍava* as above,

1 *kuḍava* = $13\frac{1}{2}$ cubic *aṅgulas*.

Srinivasan 1979, 71, says without documentation that Bhāskarācārya's *kuḍava* is defined by 'a vessel … in the shape of a cube measuring $3\frac{1}{2}$ *aṅgulas* in every dimension.' This *kuḍava* is equal to $42\frac{7}{8}$ cubic *aṅgulas* and about three times larger than the above *kuḍava* inferred from L 7–8. I cannot find the mention of 'a cube measuring $3\frac{1}{2}$ *aṅgulas* in every dimension' in any of Bhāskara's works.

S5: Money.

	va	kā	pa	dra	ni
varāṭaka	1				
kākiṇī	*20*	1			
paṇa	80	*4*	1		
dramma	1280	64	*16*	1	
niṣka	20480	1024	256	*16*	1

Exactly the same table occurs in L 2. A similar table, without *niṣka*, occurs also in Tr Pb4 = PG 9. It is noteworthy that the name *dramma* never occurs in the Tr and PG, which employ *purāṇa* in its instead.

	va	kā	pa	pu
varāṭaka	1			
kākiṇī	*20*	1		
paṇa	80	*4*	1	
purāṇa	1280	64	*16*	1

S6–S7a: Length.

	ya	a	ka	da	kro	yo
yava	1					
aṅgula	*3*	1				
kara	72	*24*	1			
daṇḍa	288	96	*4*	1		
krośa	576000	192000	8000	*2000*	1	
yojana	2304000	768000	32000	8000	*4*	1

The same table, without *yava*, occurs in Tr Pb7 = PG 12 and, with 8 *yavas* = 1 *aṅgula*, in L 5–6a.

S7bcd: Area.

Def: Measuring rod of one *vaṃśa* ('bamboo') = 10 or 8 *hastas* (or *karas*).

1 *nivartana* = area of the square of 20 *vaṃśas* × 20 *vaṃśas*.

Neither this nor any other unit of area occurs in the Tr and PG. It is common in mathematical works of India that the units of length such as *hasta* are employed also for the area. The volume, on the other hand, is sometimes expressed by the term *ghana-hasta* ('solid *hasta*') but more often simply by *hasta*.

The unit *nivartana* occurs nowhere else in the GP.

L 6bcd gives the same definitions of *vaṃśa* and *nivartana* as above, although it does not refer to the alternative conversion ratio, 8 *hastas*.

Tr Pb8 = PG 13 give a table of time units but the GP, like the L, does not.

S8–S9: Decimal number and place names.
The names as well as the number (18) of the decimal names and places of the GP are the same as those of Tr Pb2–3 = PG 7–8 and L 10–11 except for the substitution of several synonyms: *abja* (Tr, PG, L) for *padma* (lotus), *mahāsaroja* (Tr, PG) for *mahāpadma* (big lotus), and *saritāṃ pati* (Tr, PG) and *jaladhi* (L) for *saritpati* (lord of rivers or sea).

1ab: Sum and difference.
The first quarter of the verse (GP 1a) prescribes the sum and the difference of integers by means of the decimal place-value notation.
The 'sum' (*saṃkalita*) for which the Tr (1–2) and the PG (14–15) prescribe rules is not the ordinary sum of integers but that of the first n terms of the natural series, and the 'difference' (*vyavakalita*) in them (Tr 3–4 = PG 16–17) is that of the two finite natural series.
The second quarter (GP 1b) gives rules for the addition/subtraction of zero to/from a number and the addition of a number to zero,

$$a \pm 0 = a, \quad 0 + a = a.$$

Subtraction of a number from zero is probably not intended here. These rules are no doubt meant for the sum and difference of digits at each decimal place.
Tr 8ab = PG 21ab treat the sum and difference with zero. It is natural that the Tr and PG give the rules for zero when they prescribes algorithms for multiplication, since their rules for the 'sum' and 'difference' are not directly concerned with the decimal notation.
For multiplication etc. of zero see GP 9ab below.

E1ab: Examples for sum and difference.
The procedure for each example may be reconstructed as follows.

$$\begin{array}{rr} & 6\,0 \\ & 9 \\ & 1\,2 \\ \hline \langle\text{Sum}\rangle & 8\,1 \end{array} \qquad \begin{array}{rr} & \\ & 1\,0\,0 \\ & 8\,1 \\ \hline \langle\text{Diff.}\rangle & 1\,9 \end{array}$$

With regard to the first example for the sum of integers $(7+8+9+16+93+60+76+50)$ given in GT 14, the commentator Siṃhatilaka (14th century) describes the actual procedure for the summation as follows.

पट्टके भूमौ वा पूर्वं सप्त तदधो ऽष्टावित्यादिक्रमेणाधः पञ्चाशद्यावल्लिखिताङ्कश्रेणिः पूर्वव्याख्यातक्रमोत्क्रमाभ्यां मिलिता।[106] सप्तमध्ये क्षिप्ता अष्टौ जाताः पञ्चदश १५।[107] एतन्मध्ये क्षिप्ता नव जाताश्चतुर्विंशतिः २४।इत्यादिक्रमेण। तथाधोऽङ्कात्।[108] षट्सप्ततिसत्कषण्मध्ये क्षिप्तास्त्रयो जाता नव ९।[109] एतन्मध्ये षट्क्षेपे जाताः पञ्चदश १५।पञ्चदश इत्याद्युत्क्रमेण मिलिताः …।[110] (GT, pp. 3–4)

On a writing board (*paṭṭaka*) or on the ground (*bhūmi*) is written down the series of digits: seven at first, eight below it, and so on up to fifty, and summed up in the direct or inverse order as explained above. The eight added to the seven becomes fifteen 15; the nine added to it becomes twenty-four 24, etc. In this way, ⟨the summation is made⟩ in the direct order. Similarly, ⟨the summation may be made⟩ from the lowest digit. The three ⟨of the ninety-three⟩ added to the six existing in the seventy-six becomes nine 9; when the six ⟨of the sixteen⟩ is added to it, fifteen 15 is produced; to the fifteen, etc. In this way, they are summed up in the inverse order.

In this description, the addition to zero, $0 + 6 = 6$, and the addition of zero, $6 + 0 = 6$, are not mentioned probably because they are trivial. It should also be noted that the expression 'direct or inverse order' (*krama-utkrama*) is concerned with the order of the numbers and not with the order of the digits composing each number, that is, the order of the decimal places. Thus the procedure may be reconstructed as follows.

$$\begin{array}{ccc} & & 7 \\ & & 8 \\ & & 9 \\ & 1 & 6 \\ & 9 & 3 \\ & 6 & 0 \\ & 7 & 6 \\ & 5 & 0 \\ \cline{2-3} & 3 & 9 \\ \cline{1-2} 3 & 1 & \end{array}$$

Direct order: Sum of the units' place $= 7 + 8 + 9 + 6 + 3 + 0 + 6 + 0 = 39$. Sum of the tens' place $= 1 + 9 + 6 + 7 + 5 + 3 = 31$. Total $= 319$.

Inverse order: Sum of the units' place $= 0 + 6 + 0 + 3 + 6 + 9 + 8 + 7 = 39$. Sum of the tens' place $= 3 + 5 + 7 + 6 + 9 + 1 = 31$. Total $= 319$.

106 ऽष्टावित्यादि] ऽष्टौ इत्यादि GT; लिखिताङ्क] लिखिताऽङ्क GT; पूर्व] पूर्वं GT.

107 क्षिप्ता अष्टौ] क्षिप्ताऽष्टौ GT.

108 तथाधोऽङ्कात्।] तथा अधोऽङ्कात् GT.

109 सत्कषण्मध्ये] सत्कद्षट्मध्ये GT.

110 पञ्चदश इत्याद्य्] पञ्चदशेत्याद्य् GT.

1cd: Multiplication.
This verse prescribes for the multiplication of integers by means of the decimal place-value notation. This method seems to be identical with the one called *kapāṭa-sandhi* ('door-junction') in the Tr and PG (*kavāṭa-* in PG). Tr 5–6ab = PG 18–19ab read:

> विन्यस्याधो गुण्यं कवाटसन्धिक्रमेण गुणराशेः ।
> गुणयेद्विलोमगत्यानुलोममार्गेण वा क्रमशः ॥ Tr 5 = PG 18 ॥
> उत्सार्योत्सार्य ततः कवाटसन्धिर्भवेदिदं करणम् ॥ Tr E80 । Tr 6ab = PG 19ab ।

> Having placed the multiplicand below the multiplier as in the junction of two doors, multiply successively in the inverse or direct order, ॥ moving ⟨the multiplier⟩ each time. This process is known as *kavâṭa-sandhi* ("the door-junction method"). (tr. by K. S. Shukla, '॥' added by me)

According to this, the multiplication by the *kapāṭa-sandhi* was made 'in the inverse or direct order' (*vilomagatyānulomamārgeṇa vā*), but the GP refers to the 'inverse order' only. See the phrase 'the digits of the multiplicand, beginning from the last ⟨place⟩' (*guṇyasyāntyādikān aṅkān*). The decimal places are numbered from the units' place and therefore the 'last place' is the highest place.

E1cd: Examples for multiplication.

$$127 \times 24 = 3048, \qquad 16132 \times 123 = 1984236.$$

The working process of the first example by means of the *kapāṭa-sandhi* in the inverse order may be reconstructed as follows. The product at each step is written down in the row of the multiplicand. These calculations were, it should be noted, made on a writing board (a sort of laptop blackboard) with a piece of chalk or on the ground with a stick and therefore it was easy to erase or rewrite previously written digits. Here, the digits newly written down at each step are printed in italic face.

Multiplier: Multiplicand:	*2 4* *1 2 7*	$2 \times 4 = 8$:	2 4 2 8 *8* 7
$1 \times 2 = 2$:	2 4 *2* 1 2 7	Move '24':	*2 4* 2 8 8 7
$1 \times 4 = 4$:	2 4 2 *4* 2 7	$7 \times 2 = 14$, $288 + 14 = 302$:	2 4 *3 0 2* 7
Move '24':	*2 4* 2 4 2 7	$7 \times 4 = 28$, $20 + 28 = 48$:	2 4 3 0 *4 8*
$2 \times 2 = 4$, $4 + 4 = 8$:	2 4 2 *8* 2 7 ↗		

According to the anonymous commentary on the PG (p. 13), the *kapāṭa-sandhi* in the direct order is carried out as follows.

Multiplier:			*2*	*4*
Multiplicand:	*1*	*2*	*7*	
$7 \times 4 = 28$:			2	4
	1	2	7	*8*
			2	
$7 \times 2 = 14$:			2	4
$2 + 14 = 16$:	1	2	*6*	8
		1		
Move '24':		*2*	*4*	
	1	2	6	8
		1		
$2 \times 4 = 8$:		2	4	
$16 + 8 = 24$:	1	2	*4*	8
		2		
$2 \times 2 = 4$:		2	4	
$2 + 4 = 6$:	1	*6*	4	8
Move '24':	*2*	*4*		
	1	6	4	8
$1 \times 4 = 4$:	2	4		
$6 + 4 = 10$:	1	*0*	4	8
	1			
$1 \times 2 = 2$:	2	4		
$1 + 2 = 3$:	*3*	0	4	8

The above examples show that in the inverse order the calculation can be carried out in only two rows but that in the direct order a third supplementary row is necessary. This seems to be the reason the anonymous commentator remarks (PG, p. 14): 'The *kavāṭa-sandhi* is easy to do in the inverse order. Therefore it has been mentioned first ⟨in the verse⟩.' (*vilomagatyā kavāṭasandhiḥ sukara eva iti sa eva pūrvamuddiṣṭaḥ*).

The anonymous author of a commentary on the Tr preserved in A2 tells us another variation of the *kapāṭa-sandhi*, which keeps all the intermediate results of multiplication at each decimal place in supplementary rows as well as in the row of the multiplicand. His comment on the second example (896 × 37) of Tr E3 reads as follows:

षण्णवत्यधिकाष्टशतानां[111] न्यासः

३	७		
	८	९	६

[112] उत्सार्योत्सार्यगुणनाज्जातं

			३	७
२	४	६	३	२
	५	७	८	
	⟨२⟩	६	४	
		१		

विनष्टं येन गुणितं। अंकयोजना⟨1⟩ लब्धं | ३३१५२ |

(Tr, A2, fol. 5a)

111 अधिकाष्टशतानां] अधिकाष्टौ शतानां A2.

112

३	७		
	८	९	६

]

३	७	
८	९	६

A2.

Setting-down of ⟨the multiplicand⟩ eight hundred and ninety-six ⟨together with the multiplier, thirty-seven⟩ $\begin{array}{|cccc|} 3 & 7 & & \\ & 8 & 9 & 6 \\ \hline \end{array}$ By the multiplication with repeated shift ⟨of the multiplier⟩ produced is

$$\begin{array}{ccccc|} & & & 3 & 7 \\ \multicolumn{1}{|c}{2} & 4 & 6 & 3 & 2 \\ \cline{1-1}\cline{5-5} & \multicolumn{1}{|c}{5} & 7 & \multicolumn{1}{c|}{8} & \\ & \multicolumn{1}{|c}{2} & 6 & \multicolumn{1}{c|}{4} & \\ \cline{2-2}\cline{4-4} & & \multicolumn{1}{|c|}{1} & & \\ \cline{3-3} \end{array}$$

⟨That⟩ by which ⟨the multiplicand is⟩ multiplied (i.e., the multiplier 37) is erased. What is obtained from the summation of the digits is $\begin{array}{|c|} 33152 \\ \hline \end{array}$

The steps of the procedure intended here may be reconstructed as follows.

Multiplier: $\mathit{3}\ \mathit{7}$
Multiplicand: $\ \ \mathit{8}\ \mathit{9}\ \mathit{6}$

$8 \times 3 = 24:$
$$\begin{array}{ccccc} & 3 & 7 & & \\ \mathit{2} & \mathit{4} & 8 & 9 & 6 \end{array}$$

$8 \times 7 = 56:$
$$\begin{array}{ccccc} & 3 & 7 & & \\ 2 & 4 & \mathit{6} & 9 & 6 \\ & \mathit{5} & & & \end{array}$$

Move '37':
$$\begin{array}{ccccc} & & \mathit{3} & \mathit{7} & \\ 2 & 4 & 6 & 9 & 6 \\ & 5 & & & \end{array}$$

$9 \times 3 = 27:$
$$\begin{array}{ccccc} & & 3 & 7 & \\ 2 & 4 & 6 & 9 & 6 \\ & 5 & \mathit{7} & & \\ & \mathit{2} & & & \end{array}$$

$9 \times 7 = 63:$
$$\begin{array}{ccccc} & & 3 & 7 & \\ 2 & 4 & 6 & \mathit{3} & 6 \\ & 5 & 7 & & \\ & 2 & \mathit{6} & & \nearrow \end{array}$$

Move '37':
$$\begin{array}{ccccc} & & & \mathit{3} & \mathit{7} \\ 2 & 4 & 6 & 3 & 6 \\ & 5 & 7 & & \\ & 2 & 6 & & \end{array}$$

$6 \times 3 = 18:$
$$\begin{array}{ccccc} & & & 3 & 7 \\ 2 & 4 & 6 & 3 & 6 \\ & 5 & 7 & \mathit{8} & \\ & 2 & 6 & & \\ & & \mathit{1} & & \end{array}$$

$6 \times 7 = 42:$
$$\begin{array}{ccccc} & & & 3 & 7 \\ 2 & 4 & 6 & 3 & \mathit{2} \\ & 5 & 7 & 8 & \\ & 2 & 6 & \mathit{4} & \\ & & 1 & & \end{array}$$

Delete '37':
$$\begin{array}{ccccc} 2 & 4 & 6 & 3 & 2 \\ & 5 & 7 & 8 & \\ & 2 & 6 & 4 & \\ & & 1 & & \end{array}$$

Add up: $\mathit{3}\ \mathit{3}\ \mathit{1}\ \mathit{5}\ 2$

Tr E3 = PG E3: $1296 \times 21 = 27216$, $896 \times 37 = 33152$, $8065 \times 60 = 483900$.
Tr E4: $987654321 \times 753 = 743703703713$, $6543 \times 8702 = 56937186$.

2ab: Division.

This verse prescribes for the division of integers by means of the decimal place-value notation. The verse only refers to the stepwise subtraction of the product of the divisor and an appropriate number (partial quotient) from the dividend.

The same rule for division is given in Tr 9 = PG 22, which recommends that the common factors in the dividend and in the divisor be cancelled beforehand.

Example (the reverse of E1cd).

$$3048 \div 24 = 127, \qquad 1984236 \div 123 = 16132.$$

According to the anonymous commentary on the PG, the working process of the first example may be reconstructed as follows. The quotient obtained at each step is written down above the dividend.

$$\begin{array}{lcccc}
\text{Dividend:} & \mathit{3} & \mathit{0} & \mathit{4} & \mathit{8} \\
\text{Divisor:} & \mathit{2} & \mathit{4} & & \\
\\
\text{Quotient:} & & \mathit{1} & & \\
3-2\times 1 = 1: & \mathit{1} & 0 & 4 & 8 \\
 & 2 & 4 & & \\
\\
 & & 1 & & \\
10-4\times 1 = 6: & & \mathit{6} & 4 & 8 \\
 & 2 & 4 & & \\
\\
 & & 1 & & \\
\text{Move `24':} & & 6 & 4 & 8 \\
 & & \mathit{2} & \mathit{4} & \\
\\
 & & 1 & \mathit{2} & \\
6-2\times 2 = 2: & & \mathit{2} & 4 & 8 \\
 & & 2 & 4 & \nearrow
\end{array}$$

$$\begin{array}{lccc}
 & 1 & 2 & \\
24-4\times 2 = 16: & \mathit{1} & \mathit{6} & 8 \\
 & 2 & 4 & \\
\\
 & 1 & 2 & \\
\text{Move `24':} & 1 & 6 & 8 \\
 & & \mathit{2} & \mathit{4} \\
\\
 & 1 & 2 & \mathit{7} \\
16-2\times 7 = 2: & & \mathit{2} & 8 \\
 & & 2 & 4 \\
\\
 & 1 & 2 & 7 \\
28-4\times 7 = 0: & & & \mathit{0} \\
 & & 2 & 4
\end{array}$$

2cd: Square and cube.

This verse reduces the square and the cube of an integer to the multiplication respectively of two and three equal numbers, while the Tr and PG teach several methods for each, including the same method as well as one designed specifically for each of the square and the cube. See Tr 10–11 = PG 23–24 for square and Tr 14–15 = PG 27–28 for cube.

E2ab: Examples for square and cube.

$$9^2 = 81, \quad 123^2 = 15129, \quad 9^3 = 729, \quad 123^3 = 1860867.$$

The ms. also contains a table of the squares and cubes of the natural numbers from one to nine but it seems to be a later interpolation.
Tr E5 = PG E4 (square): $1^2 = 1$, $2^2 = 4, \ldots$ $9^2 = 81$, $25^2 = 625$, $36^2 = 1296$, $63^2 = 3969$, $432^2 = 186624$, $7802^2 = 60871204$.
Tr E6 =PG E5 (cube): $1^3 = 1$, $9^3 = 729$, $15^3 = 3375$, $256^3 = 16777216$, $203^3 = 8365427$.
3–4ab: Square-root.
This verse prescribes an algorithm for obtaining the square-root by means of the decimal place-value notation.
Tr 12–13 = PG 25–26 prescribe the same algorithm.

Example (the reverse of the first two examples of E2ab).

$$\sqrt{81} = 9, \qquad \sqrt{15129} = 123.$$

The working process of the latter may be reconstructed as follows (o = odd place).

	o o o	Root × 2
	1 5 1 2 9	
$1 - 1^2 = 0$:	5 1 2 9	*2*
$5 - (2 \cdot 1) \cdot 2 = 1$:	*1* 1 2 9	2
$11 - 2^2 = 7$:	*7* 2 9	2 *4*
$72 - 24 \cdot 3 = 0$:	9	2 4 *6*
$9 - 3^2 = 0$:	*0*	2 4 6

Here also, the digits newly written down at each step are printed in bold face. Needless to say, the actual calculation on a laptop blackboard or on the ground was made in a single row by rewriting the original digits '15129' step by step and by adding the digits for the root successively.

4cd–6ab: Cube-root.
This verse prescribes an algorithm for obtaining the cube-root by means of the decimal place-value notation.
Tr 16–18 = PG 29–31 prescribe the same algorithm.

Example (the reverse of the last two examples of E2ab).

$$\sqrt[3]{729} = 9, \qquad \sqrt[3]{1860867} = 123.$$

The working process of the latter may be reconstructed as follows (c = cube place).

	c			c			c	Root
	1	*8*	*6*	*0*	*8*	*6*	*7*	
$1 - 1^3 = 0$:		8	6	0	8	6	7	*1*
$8 - (3 \cdot 1^2) \cdot 2 = 2$:		*2*	6	0	8	6	7	1 *2*
$26 - 3 \cdot 1 \cdot 2^2 = 14$:		*1*	*4*	0	8	6	7	1 2
$140 - 2^3 = 132$:		1	*3*	*2*	8	6	7	1 2
$1328 - (3 \cdot 12^2) \cdot 3 = 32$:				*3*	*2*	6	7	1 2 *3*
$326 - 3 \cdot 12 \cdot 3^2 = 2$:						*2*	7	1 2 3
$27 - 3^3 = 0$:							*0*	1 2 3

6cd–7ab: Part class and the sum and difference of fractions.

$$\begin{array}{|c|c|} a & \pm c \\ b & d \\ \hline \end{array} \rightarrow \begin{array}{|c|c|} ad & \pm bc \\ bd & bd \\ \hline \end{array} \rightarrow \begin{array}{|c|} ad \pm bc \\ bd \\ \hline \end{array} \quad : \quad \frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$$

The part class (*bhāga-jāti*) is the reduction of fractions having different denominators to fractions having the same denominator (the operation indicated by the first arrow) and the sum and difference are made of those having the same denominator (the operation indicated by the second arrow).

The GP treats the part class in this verse together with the sum and difference of fractions probably because the only purpose of the former is the latter, while the Tr and PG treat it separately in the section for the 'homogenization of fraction' (*kalā-savaṇana* in Tr and *-savarṇa* in PG), that is, reduction of a composite fraction to a simple one. According to the Tr and PG, it consists of six types of reduction: (1) part class (Tr 23ab, PG 36), (2) multi-part class (Tr 23cd = PG 38ab), (3) part-part class (Tr 25ab = PG 38cd), (4) part-increase class (Tr 24 = PG 39), (5) part-decrease class (PG 40)[113], and (6) part-mother class (Tr 25 = PG 42).

Tr 19 = PG 32 treat the sum and difference of fractions. It is also noted there that the denominator of an integer (*acchedana*, lit. 'one that has no denominator') should be unity.

Note that in the following example not the product but the least common multiple of all denominators is taken to be the common denominator.

113 The Tr does not have this verse in its proper place although half a verse identical with PG 40ab is cited in the solution of Tr E7.

E2cd: Examples for the part class and the sum and difference of fractions.

$$\begin{array}{|c|c|c|}1&1&1\\2&3&4\\\hline\end{array} \rightarrow \begin{array}{|c|c|c|}6&4&3\\12&12&12\\\hline\end{array} \rightarrow \begin{array}{|c|}13\\12\\\hline\end{array}$$

$$\begin{array}{|c|c|}3&\overset{\circ}{13}\\1&12\\\hline\end{array} \rightarrow \begin{array}{|c|c|}36&\overset{\circ}{13}\\12&12\\\hline\end{array} \rightarrow \begin{array}{|c|}23\\12\\\hline\end{array}$$

Tr E7 = PG E6 (sum): $\frac{1}{4}+\frac{1}{3}+\frac{1}{6}+\frac{1}{12}=\frac{5}{6}$, $2\frac{1}{2}+(3-\frac{1}{4})+6=\frac{45}{4}$.
Tr E8 = PG E8 (difference): $1-\frac{1}{2}-\frac{1}{3}-\frac{1}{6}=0$, $5-(3-\frac{1}{2})-2\frac{1}{3}=\frac{1}{6}$.
Tr E13 = PG E14 (part class): $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\frac{6}{7}+\frac{7}{8}+\frac{8}{9}=7\frac{305}{2520}$.

7cd: Multi-part class and multiplication of fractions.

$$\begin{array}{|c|c|c|c|}a_1&a_2&\cdots&a_n\\b_1&b_2&\cdots&b_n\\\hline\end{array} \rightarrow \begin{array}{|c|}a_1a_2\cdots a_n\\b_1b_2\cdots b_n\\\hline\end{array} \quad : \quad \prod_{i=1}^{n}\frac{a_i}{b_i}=\frac{\prod_{i=1}^{n}a_i}{\prod_{i=1}^{n}b_i}$$

The 'multi-part class' (*prabhāga-jāti*) is expressed as $\frac{a_1}{b_1}$'s $\frac{a_2}{b_2}$'s … $\frac{a_n}{b_n}$ while 'multiplication'(*guṇana*) as $\frac{a_1}{b_1}$ multiplied by $\frac{a_2}{b_2}$ multiplied by … multiplied by $\frac{a_n}{b_n}$.
In this half verse the GP treats the multi-part class together with the multiplication of fractions since the arithmetical procedure is the same in both, while the Tr and PG treat it separately in the section for the 'homogenization of fractions.' See under GP 6cd–7ab above.
Tr 20ab = PG 33ab treat the multiplication of fractions.

E3ab: Examples for the multi-part class and the multiplication of fractions.

$$\text{Multi-part class:}\quad \begin{array}{|c|c|c|}1&1&1\\4&2&3\\\hline\end{array} \rightarrow \begin{array}{|c|}1\\24\\\hline\end{array}$$

$$\text{Multiplication:}\quad \begin{array}{|c|c|}1&1\\3&2\\\hline\end{array} \rightarrow \begin{array}{|c|}1\\6\\\hline\end{array}$$

Tr E9 (multiplication): $3\frac{1}{2}\times 1\frac{1}{4}=\frac{35}{8}$, $\frac{1}{2}\times\frac{1}{4}=\frac{1}{8}$.
PG E10 (multiplication): $2\frac{1}{2}\times 1\frac{1}{2}=3\frac{3}{4}$, $60\frac{1}{3}\times\frac{5}{2}=150\frac{5}{6}$.
Tr E14 (multi-part class): $1\times\frac{1}{2}+(1\times\frac{1}{2})\times\frac{1}{3}+\{(1\times\frac{1}{2})\times\frac{1}{3}\}\times\frac{1}{5}$ of *kākiṇī* $=14$ *varāṭakas.*
Tr E15 (multi-part class): $2\frac{1}{2}\times\frac{1}{4}+3\times\frac{1}{16}+\frac{1}{8}\times\frac{3}{10}=\frac{68}{80}$.
PG E15 (multi-part class): $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{4}+\frac{1}{3}\times\frac{1}{5}\times\frac{1}{6}\times\frac{1}{10}+2\frac{1}{2}\times\frac{1}{6}\times\frac{1}{7}=\frac{3103}{25200}$.

8ab: Division of fractions.

$$\begin{array}{|c|c|} a & c \\ b & d \\ \hline \end{array} \rightarrow \begin{array}{|c|c|} a & d \\ b & c \\ \hline \end{array} \rightarrow \begin{array}{|c|} ad \\ bc \\ \hline \end{array} \quad : \quad \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}.$$

Tr 20cd = PG 33cd give the same rule for division.

Example (the reverse of the second example of E3ab).

$$\begin{array}{|c|c|} 1 & 1 \\ 6 & 3 \\ \hline \end{array} \rightarrow \begin{array}{|c|c|} 1 & 3 \\ 6 & 1 \\ \hline \end{array} \rightarrow \begin{array}{|c|} 1 \\ 2 \\ \hline \end{array}$$

Tr E10 = PG E11: $6\frac{1}{4} \div 2\frac{1}{2} = 2\frac{1}{2}$, $60\frac{1}{4} \div 3\frac{1}{2} = 17\frac{3}{14}$.

8cd: Square etc. of fractions.

$$\begin{array}{|c|} a \\ b \\ \hline \end{array} \rightarrow \text{square} \begin{array}{|c|} a^2 \\ b^2 \\ \hline \end{array} \quad : \quad \left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2},$$

$$\begin{array}{|c|} a \\ b \\ \hline \end{array} \rightarrow \text{cube} \begin{array}{|c|} a^3 \\ b^3 \\ \hline \end{array} \quad : \quad \left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3},$$

$$\begin{array}{|c|} a \\ b \\ \hline \end{array} \rightarrow \text{square-root} \begin{array}{|c|} \sqrt{a} \\ \sqrt{b} \\ \hline \end{array} \quad : \quad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}},$$

$$\begin{array}{|c|} a \\ b \\ \hline \end{array} \rightarrow \text{cube-root} \begin{array}{|c|} \sqrt[3]{a} \\ \sqrt[3]{b} \\ \hline \end{array} \quad : \quad \sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}.$$

Tr 21–22 = PG 34–35 give the same rules in the following order: square, square-root, cube, cube-root.

E3cd: Examples for the square etc. of fractions.

$$\begin{array}{|c|} 3 \\ 1 \\ 2 \\ \hline \end{array} \rightarrow \begin{array}{|c|} 7 \\ 2 \\ \hline \end{array} \rightarrow \text{square} \begin{array}{|c|} 49 \\ 4 \\ \hline \end{array}, \qquad \begin{array}{|c|} 3 \\ 1 \\ 2 \\ \hline \end{array} \rightarrow \begin{array}{|c|} 7 \\ 2 \\ \hline \end{array} \rightarrow \text{cube} \begin{array}{|c|} 343 \\ 8 \\ \hline \end{array}$$

$$\begin{array}{|c|} 49 \\ 4 \\ \hline \end{array} \rightarrow \text{square-root} \begin{array}{|c|} 7 \\ 2 \\ \hline \end{array}, \qquad \begin{array}{|c|} 343 \\ 8 \\ \hline \end{array} \rightarrow \text{cube-root} \begin{array}{|c|} 7 \\ 2 \\ \hline \end{array}$$

Tr E11 = PG E12 (square): $(2\frac{1}{2})^2 = 6\frac{1}{4}$, $(5\frac{1}{4})^2 = 27\frac{9}{16}$, $(\frac{1}{2})^2 = \frac{1}{4}$, $(\frac{1}{3})^2 = \frac{1}{9}$.

Tr E12 = PG E13 (cube): $(7\frac{1}{2})^3 = 421\frac{7}{8}$, $(15\frac{1}{4})^3 = 3546\frac{37}{64}$, $(\frac{1}{4})^3 = \frac{1}{64}$, $(\frac{1}{3})^3 = \frac{1}{27}$.

In the Tr and PG also, the examples for the square-root and cube-root are the reverse cases of the examples for the square and cube.

9ab: Zero class.

$$a \times 0 = 0 \times a = 0, \quad 0^2 = 0, \quad 0^3 = 0, \quad \text{etc.}$$

Here, 'etc.' seems to imply 'square-root' and 'cube-root'. Divisions of zero and by zero are not mentioned here probably because they are not necessary in the arithmetical computation by means of the decimal notation.
Tr 8cd = PG 21cd give similar rules for zero.
For the sum and difference with zero see GP 1ab above.

9cd–10ab: Part-increase/decrease class.

$$\text{Part of unity:}\quad \begin{array}{|r|} a \\ \pm b \\ c \\ \hline \end{array} \rightarrow \begin{array}{|c|} ac \pm b \\ c \\ \hline \end{array} \quad : \quad a \pm \frac{b}{c} = \frac{ac \pm b}{c}$$

$$\text{Part of oneself:}\quad \begin{array}{|r|} a \\ b \\ \pm c \\ d \\ \hline \end{array} \rightarrow \begin{array}{|c|} a(d \pm c) \\ bd \\ \hline \end{array} \quad : \quad \frac{a}{b} \pm \frac{a}{b} \cdot \frac{c}{d} = \frac{a(d \pm c)}{bd}$$

Tr 24 = PG 39 give the rules for the part-increase class and PG 40 for the part-decrease class. The Tr edited by Dvivedī does not have a verse corresponding to PG 40 but cites in the prose commentary on Tr E7 a verse which consists of PG 39ab (= Tr 24ab) and PG 40ab.

E4ab: Examples for the part-increase/decrease class.

$$\begin{array}{|c|} 3 \\ 1 \\ 3 \\ \hline \end{array} \rightarrow \begin{array}{|c|} 3 \times 3 + 1 \\ 3 \\ \hline \end{array} \rightarrow \begin{array}{|c|} 10 \\ 3 \\ \hline \end{array} \quad : \quad 3 + \frac{1}{3} = \frac{10}{3}$$

$$\begin{array}{|c|} 3 \\ \overset{\circ}{1} \\ 3 \\ \hline \end{array} \rightarrow \begin{array}{|c|} 3 \times 3 - 1 \\ 3 \\ \hline \end{array} \rightarrow \begin{array}{|c|} 8 \\ 3 \\ \hline \end{array} \quad : \quad 3 - \frac{1}{3} = \frac{8}{3}$$

$$\begin{array}{|c|} 3 \\ 1 \\ 1 \\ 3 \\ \hline \end{array} \rightarrow \begin{array}{|c|} 3 \times (3 + 1) \\ 3 \\ \hline \end{array} \rightarrow \begin{array}{|c|} 4 \\ 1 \\ \hline \end{array} \quad : \quad \frac{3}{1} + \frac{3}{1} \cdot \frac{1}{3} = \frac{4}{1}$$

$$\begin{array}{|c|} 3 \\ 1 \\ \overset{\circ}{1} \\ 3 \\ \hline \end{array} \rightarrow \begin{array}{|c|} 3 \times (3 - 1) \\ 3 \\ \hline \end{array} \rightarrow \begin{array}{|c|} 2 \\ 1 \\ \hline \end{array} \quad : \quad \frac{3}{1} - \frac{3}{1} \cdot \frac{1}{3} = \frac{2}{1}$$

For the expression 'made into the same color' (*savarṇite*) that occurs in the prose part, see 6cd–7ab above.

Tr E16 = PG E18 (part of unity): $1\frac{1}{2} + 5\frac{1}{4} + 8\frac{1}{3} = \frac{3}{2} + \frac{21}{4} + \frac{25}{3} = 15\frac{1}{12}$.

Tr E17 = PG E19 (part of oneself): $3\frac{1}{2} + (3\frac{1}{2}) \cdot \frac{1}{4} + \{(3\frac{1}{2}) \cdot \frac{1}{4}\} \cdot \frac{1}{6} + \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{3} + (\frac{1}{2} \cdot \frac{1}{3}) \cdot \frac{1}{4} = \frac{245}{48} + \frac{5}{6} = 5\frac{15}{16}$.

10cd–11ab: Inverse-problem class.

$$x \pm a = b \;\rightarrow\; x = b \mp a, \quad x \times a = b \;\rightarrow\; x = b \div a,$$
$$x \div a = b \;\rightarrow\; x = b \times a, \quad x^2 = a \;\rightarrow\; x = \sqrt{a},$$
$$\sqrt{x} = a \;\rightarrow\; x = a^2, \quad x \pm \frac{a}{b} \cdot x = c \;\rightarrow\; x = \frac{c}{1 \pm a/b}.$$

The Tr edited by Dvivedī does not treat this topic. PG 78 gives the same rules except the last. It is noteworthy that the L (48–49) gives the same rules including the last one.

E4cd: Example for the inverse-problem class.
Statements of the problem:

$$x_1 - 2 = x_2, \quad x_2 \times 3 = x_3, \quad x_3^2 = x_4, \quad x_4 + \frac{1}{3}x_4 = x_5, \quad x_5 \div 3 = 16.$$

Solution (reconstructed): Apply the rule of the inverse operations inversely (10cd–11ab).

$$x_5 = 16 \times 3 = 48, \quad x_4 = \frac{48}{1 + 1/3} = 36, \quad x_3 = \sqrt{36} = 6,$$
$$x_2 = 6 \div 3 = 2, \quad x_1 = 2 + 2 = 4.$$

PG E102: $x_1 \times \frac{5}{2} = x_2$, $x_2 \div 3 = x_3$, $x_3^2 = x_4$, $x_4 + 9 = x_5$, $\sqrt{x_5} = x_6$, $x_6 - 1 = 4$. $x_1 = 4\frac{4}{5}$.

11cd: Optional-number class.
Type of problem: a linear equation in a single unknown number whose statements can be reduced to the form ax without a constant.
Rule: Assume the unknown quantity to be unity, calculate according to the statements of the problem, and divide the 'visible quantity' (given constant) by the result of the calculation.

This is a special case of the 'optional-number operation' (*iṣṭa-karman*) or the so-called *regula falsi*. Given the relationship $ax = b$, calculate the left-hand side of the equation for any optional number (say p): $ap = q$. Then, x is obtained by $x = bp/q$. In the present rule, the 'optional' number (p) is always assumed to be unity and therefore it is not necessary to multiply the visible quantity (b) by the optional number.
The Tr and PG do not treat this topic. L 51 gives a general rule for the optional-number operation.

It is noteworthy that the GP gives four examples for this rule in exactly the same manner as the L; that is, in both works, the first example (GP E5ab, L 52) is designed for the optional-number class itself but the other three are originally designed respectively for the visible-number class (GP E5cd, L 53), the remainder class (GP E6ab, L 54), and the difference class (GP E6cd, L 55). Interestingly, the problems of L 53 and 54 are solved in the prose commentary by assuming the optional number to be unity just as in the GP. Moreover, in the prose part of L 54 exactly the same three kinds of solutions as in the GP are mentioned. See GP E6ab below.

Immediately before the example of the remainder class (E6ab), a verse (F1) which prescribes an algorithm specifically designed for that class is given. But it is identical with the verse of GT 67 of Śrīpati and is not used in the solution of E6ab. This proves that the verse (F1) is a later interpolation.

E5ab: Example of the optional-number class; purely numerical.

Statements of the problem:

$$x_1 \times 3 = x_2, \quad x_2 \div 5 = x_3, \quad x_3 + \frac{1}{5} \cdot x_1 = x_4, \quad x_4 \div 4 = 2.$$

Solution (reconstructed): Assume $x_1 = 1$ and calculate according to the statements.

$$1 \times 3 = 3, \quad 3 \div 5 = \frac{3}{5}, \quad \frac{3}{5} + \frac{1}{5} \cdot 1 = \frac{4}{5}, \quad \frac{4}{5} \div 4 = \frac{1}{5}.$$

Hence follows:

$$x_1 = 2 \div \frac{1}{5} = 10.$$

L 52 (number): $x_1 \times 5 = x_2$, $x_2 - \frac{1}{3} \cdot x_2 = x_3$, $x_3 \div 10 = x_4$, $x_4 + \frac{1}{3} \cdot x_1 + \frac{1}{2} \cdot x_1 + \frac{1}{4} \cdot x_1 = 70 - 2$. $x_1 = 48$.

E5cd: Example of the visible-number class; parrots.

Statement of the problem:

$$x - \frac{x}{6} - \frac{x}{3} = 6.$$

Solution (reconstructed): Assume $x = 1$ and calculate according to the statements.

$$1 - \frac{1}{6} - \frac{1}{3} = \frac{1}{2}.$$

Hence follows

$$x = 6 \div \frac{1}{2} = 12.$$

Tr E23 (pillar): $x - \frac{x}{2} - \frac{x}{6} - \frac{x}{12} = 2$ *hastas*. $x = 8$ *hastas*.

Tr E24 (pillar): $x - \frac{x}{2} - \frac{x}{9} - \frac{x}{8} - \frac{x}{6} = 49$ *aṅgulas*. $x = 21$ *hastas*.
Tr E25 (pillar): $x - \frac{x}{2} - \frac{x}{12} - \frac{x}{6} = 1\frac{1}{2}$ *hastas*. $x = 6$ *hastas*.
Tr E26 (pearls): $x - \frac{x}{3} - \frac{x}{5} - \frac{x}{6} - \frac{x}{10} = 6$. $x = 30$.
Tr E27 (elephants): $x - (\frac{x}{2} + \frac{x}{2} \cdot \frac{1}{3}) - (\frac{x}{6} + \frac{x}{6} \cdot \frac{1}{7}) - (\frac{x}{8} + \frac{x}{8} \cdot \frac{1}{9}) = 4$. $x = 1008$.
Tr E28 (wasps): $x - \frac{x}{6} - \frac{x}{3} - \frac{x}{4} - \frac{x}{5} - \frac{x}{30} = 1$. $x = 60$.
Tr E29 (number): $x - (\frac{x}{4} + \frac{x}{4} \cdot \frac{1}{4}) - (\frac{x}{3} + \frac{x}{3} \cdot \frac{1}{3}) - (\frac{x}{5} + \frac{x}{5} \cdot \frac{1}{5}) = 11$. $x = 3600$.
PG E96 (pillar): $x - \frac{x}{4} - \frac{x}{3} - \frac{x}{6} = 3$ *hastas*. $x = 12$ *hastas*.
L 53 (lotus flowers): $x - \frac{x}{3} - \frac{x}{5} - \frac{x}{6} - \frac{x}{4} = 6$. $x = 120$.

Tr 27cd and PG 74ab prescribe an algorithm each for this type of problems which Śrīdhara calls *stambha-uddeśaka* ('pillar-problem'). The L treats the above problem (L 53) as an example for the optional-number operation.

F1: Remainder class (Śrīpati's verse, GT 67, p. 44, line 11, is cited).
Type of problem:

$$x_1 - \frac{a_1}{b_1} \cdot x_1 = x_2, \quad x_2 - \frac{a_2}{b_2} \cdot x_2 = x_3, \quad \ldots, \quad x_n - \frac{a_n}{b_n} \cdot x_n = c.$$

Rule:

$$x_1 = c \div \left\{ \left(\prod_{i=1}^{n} (b_i - a_i) \right) \div \left(\prod_{i=1}^{n} b_i \right) \right\}.$$

This rule is used neither in the next example (E6ab) which is designed for the remainder class nor in any other examples of the GP. See above under 11cd.

E6ab: Example of the remainder class; parrots.
Statements of the problem:

$$x_1 - \frac{1}{6}x_1 = x_2, \quad x_2 - \frac{1}{2}x_2 = x_3, \quad x_3 - \frac{1}{3}x_3 = 10.$$

Solution 1 (reconstructed) by the rule of the optional-number class (11cd): Assume $x_1 = 1$ and calculate according to the statements.

$$1 - \frac{1}{6} \cdot 1 = \frac{5}{6}, \quad \frac{5}{6} - \frac{1}{2} \cdot \frac{5}{6} = \frac{5}{12}, \quad \frac{5}{12} - \frac{1}{3} \cdot \frac{5}{12} = \frac{5}{18}.$$

Hence follows

$$x_1 = 10 \div \frac{5}{18} = 36.$$

Solution 2 by the rule of the optional-number class (11cd) with the help of the rule of the part-decrease class (10ab): Assume $x_1 = 1/1$ and apply the rule of the part-decrease class.

$$\begin{array}{|c|} 1 \\ 1 \\ \overset{\circ}{1} \\ 6 \\ \overset{\circ}{1} \\ 2 \\ \overset{\circ}{1} \\ 3 \\ \hline \end{array} \rightarrow \begin{array}{|c|} 5 \\ 6 \\ \overset{\circ}{1} \\ 2 \\ \overset{\circ}{1} \\ 3 \\ \hline \end{array} \rightarrow \begin{array}{|c|} 5 \\ 12 \\ \overset{\circ}{1} \\ 3 \\ \hline \end{array} \rightarrow \begin{array}{|c|} 10 \\ 36 \\ \hline \end{array} \rightarrow \begin{array}{|c|} 5 \\ 18 \\ \hline \end{array}$$

The rest is the same as above.
Solution 3 (reconstructed) by the rule of the inverse operations (10ab–11cd):

$$x_3 = \frac{10}{1-1/3} = 15, \quad x_2 = \frac{15}{1-1/2} = 30, \quad x_1 = \frac{30}{1-1/6} = 36.$$

F1, which is placed immediately before this example, would solve the same problem as follows, but this solution is not even mentioned in the text.

$$x_1 = 10 \div \{(6-1)(2-1)(3-1) \div (6 \cdot 2 \cdot 3)\} = 36.$$

PG E97 (number): $x_1 - \frac{1}{2}x_1 = x_2$, $x_2 - \frac{2}{3}x_2 = x_3$, $x_3 - \frac{3}{4}x_3 = x_4$, $x_4 - \frac{4}{5}x_4 = 3$. $x_1 = 360$.
L 54 (money): $x_1 - \frac{1}{2}x_1 = x_2$, $x_2 - \frac{2}{9}x_2 = x_3$, $x_3 - \frac{1}{4}x_3 = x_4$, $x_4 - \frac{6}{10}x_4 = 63$ *niṣkas*. $x_1 = 540$ *niṣkas*.

The Tr does not treat this type of problems. The PG solves the above problem (E97) by the same rule as that for the pillar problems (see under GP E5ab above) after rewriting the statements of the problem. The L treats the above problem (L 54) as an example for the optional-number operation.

E6cd: Example of the difference class; cows.
Statement of the problem:

$$x - \left(\frac{x}{2} - \frac{x}{6}\right) = 6.$$

Solution (reconstructed): Calculate the difference of the two fractions,

$$x - \frac{x}{3} = 6,$$

and to this, as in E6ab, apply either the optional-number class or the optional-number class with the part-decrease class or the inverse-operation class. The answer is 9.

PG E98 (cows): $x - \frac{x}{2} - \frac{x}{4} - (\frac{x}{2} - \frac{x}{4}) \times 2 \div 5 = 3$. $x = 20$.
L 55 (black bees): $x - \frac{x}{5} - \frac{x}{3} - (\frac{x}{3} - \frac{x}{5}) \times 3 = 1$. $x = 15$.

The Tr does not treat this type of problems but PG 74cd prescribes an algorithm for the above problem (E98). The L treats the above problem (L 55) as an example for the optional-number operation.

F2: Root class.
Type of problem:

$$x \pm a\sqrt{x} = b.$$

Rule:

$$x = \left(\frac{\sqrt{4b + a^2} \mp a}{2} \right)^2 .$$

The Tr does not treat this problem but PG 75 gives the same formula for the negative sign. L 65 prescribes a formula equivalent to the above for both signs.

FE1: Example for the root class; purely numerical.
Statements of the problem:

$$x_1 - \sqrt{x_1} = x_2, \quad x_2 - \frac{1}{6}x_2 = x_3, \quad x_3 + \sqrt{x_3} = 30.$$

Solution (reconstructed): By the root class,

$$x_3 = \left(\frac{\sqrt{4 \times 30 + 1^2} - 1}{2} \right)^2 = 25.$$

Either by the optional-number class or by the optional-number class with the part-decrease class or by the remainder class or by the inverse-operation class, $x_2 = 30$. Again by the root class, $x_1 = 36$.

PG E99 (number): $x_1 - \sqrt{x_1} = x_2$, $x_2 - \frac{1}{6}x_2 = x_3$, $x_3 - \sqrt{x_3} = x_4$, $x_4 - \frac{1}{5}x_4 = x_5$, $x_5 - 2\sqrt{x_5} = 8$. $x_1 = 36$.
L 67 (swans): $x - \frac{7}{2}\sqrt{x} = 2$. $x = 16$.
L 68 (number): $x + 9\sqrt{x} = 1200 + 40$. $x = 961$.

F3: Seen-root-part class.
Type of problem:

$$x \pm \frac{a}{b} \cdot x \pm c\sqrt{x} = d.$$

Rule: Rewrite the problem as

$$x \pm \frac{c}{1 \pm a/b} \cdot \sqrt{x} = \frac{d}{1 \pm a/b},$$

and to this apply the root class (F2).

The Tr does not treat this problem but PG 76 gives an algorithm obtained by applying the root class to the above equation. L 66, like F3, advises to apply the root class after rewriting the statement of the problem.

FE2: Example for the seen-root-part class; purely numerical.
Statement of the problem:

$$x + \frac{1}{27}x + \frac{1}{3}x + \frac{1}{9}x + \sqrt{x} = 129.$$

Solution (reconstructed): The three fractions are added up,

$$x + \frac{13}{27}x + \sqrt{x} = 129,$$

and to this the seen-root-part class is applied.

$$x = \left(\frac{\sqrt{4 \cdot \left(129 \div \frac{40}{27}\right) + \left(1 \div \frac{40}{27}\right)^2} - 1 \div \frac{40}{27}}{2} \right)^2 = 81.$$

PG E100 (monkeys): $x - (\frac{x}{3} + \frac{x}{3} \cdot \frac{1}{3}) - \sqrt{x} = 2$: $x = 9$.
L 69 (swans): $x - 10\sqrt{x} - \frac{1}{8}x = 2 \times 3$. $x = 144$.
L 70 (arrows): $x - 4\sqrt{x} - \frac{x}{2} = 6 + 3 + 1$. $x = 100$.
L 71 (wasps): $x - \sqrt{x/2} - \frac{8}{9}x = 2$. $x = 72$.
L 72 (number): $x + 18\sqrt{x} + \frac{x}{3} = 1200$. $x = 576$.

F4: Tower class.
Type of problem: Let n be the number of the floors of a tower and x_i the number of persons on the i-th floor at the beginning. From the i-th floor persons equal in number to an a-th part of the number of persons on the $(i-1)$-th floor simultaneously move to the $(i-1)$-th floor with the result that the number of persons on every floor becomes equal (y). Naturally, x_i, y and a are integers.

$$\begin{aligned}
x_n \qquad\qquad\qquad - \frac{x_{n-1}}{a} &= y,\\
x_{n-1} + \frac{x_{n-1}}{a} - \frac{x_{n-2}}{a} &= y,\\
&\vdots\\
x_2 + \frac{x_2}{a} - \frac{x_1}{a} &= y,\\
x_1 + \frac{x_1}{a} \qquad\qquad &= y.
\end{aligned}$$

Rule: Let us assume that a solution for y (say b) has already been obtained. The verse prescribes an algorithm for obtaining x_i, which may be paraphrased as follows (cf. the table below). (1) Put b n-times vertically. (2) Divide the i-th term from the bottom by $(a+1)^i$ (except the topmost term). (3) Multiply each term by a. (4) Starting from the bottom, add each term to the immediately above. (5) Divide the topmost term by a (this is not stated in the verse). The results are solutions for x_i. In this rule, a is called the 'desired ⟨number⟩' (*iṣṭa*) and b 'like digits' (*sadṛśa-aṅka*).

(1)	(2)	(3)	(4) and (5)	
b	b	ab	$b+\frac{b}{(a+1)}+\frac{b}{(a+1)^2}+\cdots+\frac{b}{(a+1)^{n-1}}$	$=x_n$
b	$\frac{b}{(a+1)^{n-1}}$	$\frac{ab}{(a+1)^{n-1}}$	$\frac{ab}{(a+1)}+\frac{ab}{(a+1)^2}+\cdots+\frac{ab}{(a+1)^{n-1}}$	$=x_{n-1}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
b	$\frac{b}{(a+1)^2}$	$\frac{ab}{(a+1)^2}$	$\frac{ab}{(a+1)}+\frac{ab}{(a+1)^2}$	$=x_2$
b	$\frac{b}{(a+1)}$	$\frac{ab}{(a+1)}$	$\frac{ab}{(a+1)}$	$=x_1$

This algorithm can be obtained by solving the above mentioned n equations for x_i inversely:

$$\begin{aligned}
x_1 &= \frac{ay}{a+1}\\
x_2 &= \frac{ay}{a+1}+\frac{x_1}{a+1}=\frac{ay}{a+1}+\frac{ay}{(a+1)^2}\\
&\vdots\\
x_{n-1} &= \frac{ay}{a+1}+\frac{x_{n-2}}{a+1}=\frac{ay}{a+1}+\frac{ay}{(a+1)^2}+\cdots+\frac{ay}{(a+1)^{n-1}}\\
x_n &= y+\frac{x_{n-1}}{a}=y+\frac{y}{a+1}+\frac{y}{(a+1)^2}+\cdots+\frac{y}{(a+1)^{n-1}}
\end{aligned}$$

From this it is also obvious that y must be a multiple of $(a+1)^{n-1}$ in order that every x_i is an integer, although this is not stated in the verse.

The same rule (F4) together with the same example (FE3) and the answer in a column are found on the obverse of the first folio of one of the Baroda manuscripts (B2) of the *Triśatikā*. For B2 see under Śrīdhara-*Triśatikā* in the Bibliography.

It was customary for a scribe to begin his copying a Sanskrit work on the reverse side of the first folio (1b), leaving its obverse (1a) blank. In the present case also, the *Triśatikā* begins on fol. 1b and the stuff on 1a is written by a hand different from that of the main work. The text of the main work is separated from the left and the right margins by a double vertical line on each side but the text on 1a is not. It is therefore very likely that someone, possibly the previous owner of the manuscript (Yati Hemacandra Jī, Baroda) or a related person, copied this rule and the example from some other source just as a casual memo.

Another set of a rule and an example for the tower class is found in a Patan manuscript (H2) of the *Triśatikā*. For its text and translation see the Appendix.
The rule, which is totally different from F4, may be paraphrased as follows (cf. the chart below). (1) Divide any optional number (*iṣṭa*, say b) by the multiplier (a) plus one. (2) Put the quotient below and multiply it by the multiplier. (3) Also put the same quotient above and add the desired number (b) to it. (4) To the sum repeatedly apply the same calculations, that is, (i) division by $(a+1)$ [indicated by a right arrow in the chart], (ii) multiplication by a [a right arrow down] and (iii) addition of b [a right arrow up]. (5) Reduce the fractions obtained and b into those having equal denominators and delete the denominators. The results are the solutions for x_i and y. In this rule, a is called the 'multiplier' (*guṇa*) and b the 'optional ⟨number⟩' (*iṣṭa*).

$$
\begin{array}{llll}
 & & & \frac{b}{(a+1)^{n-1}} + \cdots + \frac{b}{(a+1)} + b = x_n \\
 & & \nearrow & \\
 & \frac{b}{(a+1)} + b \to \cdots & & \\
 & \nearrow & \searrow & \\
b \to \frac{b}{a+1} & & & \frac{ab}{(a+1)^{n-1}} + \cdots + \frac{ab}{(a+1)} = x_{n-1} \\
 & \searrow & & \\
 & \frac{ab}{(a+1)} = x_1 & &
\end{array}
$$

Step (5) is necessary because here the results $(x_1, x_2, \ldots, x_n)$ of Step (4) are not integers unless the optionally chosen number b is a multiple of $(a+1)^{n-1}$.

Among the manuscripts of the *Triśatikā* available to me (see the Bibliography), only H2 contains this rule. Moreover, H2 is affected with interpolations in places. It is very likely that this rule is also an interpolation.

FE3: Example for the tower class; a tower having seven floors.
Statements of the problem:

$$
\begin{aligned}
x_7 \phantom{{}+x_6} - x_6 &= y,\\
x_6 + x_6 - x_5 &= y,\\
x_5 + x_5 - x_4 &= y,
\end{aligned}
$$

$$\begin{aligned} &\vdots \\ x_2 + x_2 - x_1 &= y, \\ x_1 + x_1 \qquad &= y. \end{aligned}$$

Solution (reconstructed): Since $n = 7$ and $a = 1$, let $y = (1+1)^{7-1} = 64$. Then,

(1)	(2)	(3)	(4) and (5)	
64	64	$64 \times 1 = 64$	$(63 + 64)/1 = 127$	$= x_7$
64	$64/2^6 = 1$	$1 \times 1 = 1$	$62 + 1 = 63$	$= x_6$
64	$64/2^5 = 2$	$2 \times 1 = 2$	$60 + 2 = 62$	$= x_5$
64	$64/2^4 = 4$	$4 \times 1 = 4$	$56 + 4 = 60$	$= x_4$
64	$64/2^3 = 8$	$8 \times 1 = 8$	$48 + 8 = 56$	$= x_3$
64	$64/2^2 = 16$	$16 \times 1 = 16$	$32 + 16 = 48$	$= x_2$
64	$64/2 = 32$	$32 \times 1 = 32$	32	$= x_1$

Example in the Patan Ms. HJJM 10728 (see the Appendix).

$$\begin{aligned} x_5 \qquad\quad - \frac{x_4}{2} &= y, \\ x_4 + \frac{x_4}{2} - \frac{x_3}{2} &= y, \\ x_3 + \frac{x_3}{2} - \frac{x_2}{2} &= y, \\ x_2 + \frac{x_2}{2} - \frac{x_1}{2} &= y, \\ x_1 + \frac{x_1}{2} \qquad\quad &= y. \end{aligned}$$

Solution (reconstructed): $n = 5$ and $a = 2$. Let, for example, $b = 1$ and apply the given algorithm.

$$\begin{tikzcd}
&&&&&&&& \frac{121}{81} = x'_5 \\
&&&&&& \frac{40}{27} \arrow[r] & \frac{40}{81} \arrow[ur] \arrow[dr] & \\
&&&& \frac{13}{9} \arrow[r] & \frac{13}{27} \arrow[ur] \arrow[dr] & & & \frac{80}{81} = x'_4 \\
&& \frac{4}{3} \arrow[r] & \frac{4}{9} \arrow[ur] \arrow[dr] & & & \frac{26}{27} = \frac{78}{81} = x'_3 & & \\
b = 1 \arrow[r] & \frac{1}{3} \arrow[ur] \arrow[dr] & & & \frac{8}{9} = \frac{72}{81} = x'_2 & & & & \\
& & \frac{2}{3} = \frac{54}{81} = x'_1 & & & & & &
\end{tikzcd}$$

Delete the denominators 81's and a set of solutions is obtained: $\{x_i\} = \{81x_i'\} = \{54, 72, 78, 80, 121\}$ and $y = 81b = 81$.

12-3ab: $(2n+1)$-quantity operation.
When the quantity (y) of a certain category (for example, price) is proportionate to the quantity (x_i) of each of n categories (for example, length, width, weight, number, etc.), and if $y = b$ when $x_i = a_i$ for all i's, then

$$y = \frac{b\prod_{i=1}^{n} x_i}{\prod_{i=1}^{n} a_i}.$$

The computation is carried out as follows.
(1) The 'standard' (*pramāṇa*, a_i) and the 'requisite' (*icchā*, x_i) of each category are put side by side and at the bottom of the columns the 'standard-fruit' (*pramāṇa-phala*, b) and the 'requisite-fruit' (*icchā-phala*, y) are also put.

$$\begin{array}{|cc|} a_1 & x_1 \\ a_2 & x_2 \\ \vdots & \vdots \\ a_n & x_n \\ b & y \\ \hline \end{array}$$

(2) The two 'fruits', b and y, are exchanged, where the unknown y is expressed either by a vacant place or by a *bindu* (dot) that indicates a vacant place. (3) The denominators of fractions, if any, are mutually moved to the opposite side. (4) The product of the terms of 'the side with more numerators' (the side of x_i in this case) is divided by the product of the terms of 'the side with fewer numerators' (the side of a_i in this case).
This rule includes the three-quantity operation:

$$\begin{array}{|cc|} a_1 & x_1 \\ b & y \\ \hline \end{array} \quad \rightarrow \quad \begin{array}{|cc|} a_1 & x_1 \\ y & b \\ \hline \end{array} \quad \rightarrow \quad y = \frac{bx_1}{a_1},$$

but the *nyāsa* ('setting-down') for the accompanying example (E7ab) arranges the three terms horizontally in the usual manner:

$$\begin{array}{|c|c|c|} a_1 & b & x_1 \\ \hline \end{array} \quad \rightarrow \quad y = \frac{bx_1}{a_1}.$$

In the case of the inverse three-quantity operation, the two 'fruits' are not exchanged but the rule about the division is applied here also.

$$\begin{array}{|cc|} a_1 & x_1 \\ b & y \\ \hline \end{array} \quad \rightarrow \quad y = \frac{a_1 b}{x_1}.$$

This is usually expressed as follows.

$$\boxed{a_1 \,|\, b \,|\, x_1} \quad \rightarrow \quad y = \frac{a_1 b}{x_1}.$$

As the example for the inverse three-quantity operation (E9) is lost, it is impossible to know which style its *nyāsa* has employed.

Tr 29 = PG 43 prescribe for the three-quantity operation, Tr 30 = PG 44cd for the inverse three-quantity operation, and Tr 31 = PG 45 for the five-quantity operations etc. PG 44ab treats the 'going forward and backward' (*gati-nivṛtti*) type of problems, stipulating an advance calculation of the daily increase of the distance, property, etc.
L 73 prescribes for the direct and inverse three-quantity operation and L 82 for the five-quantity operation etc.

It is noteworthy that the present rule includes all the $(2n + 1)$-quantity operations beginning with the three-quantity operation and even the inverse three-quantity operation. This means that not only the five-quantity operation etc. but also the three-quantity operation and the inverse three-quantity operation employ the vertical two-column arrangement of the given $(2n + 1)$ terms. On the other hand, most Sanskrit texts including the Tr, PG and L employ the horizontal arrangement for the direct and inverse three-quantity operations and the vertical arrangement for the rest. The only other text that prescribes the vertical two-column arrangement for the three-quantity operation also is the BSS (12.11cd–12) which is known to be the first that gave the rules in such a way that we can infer the arrangement of the given terms. But the BSS (12.10–11ab) also allows the horizontal arrangement for the direct and inverse three-quantity operations and this arrangement became more popular for the three-quantity operations than the vertical one in later India. Interestingly, al-Bīrūnī (11th century) refers to the vertical two-column arrangement for the three-quantity operation. For a lucid exposition of the history of the three-quantity operation see Sarma 2002.

E7ab: Example for the three-quantity operation; mango fruit.
According to the rule of GP 12-3ab, the solution should be as follows.

$$\begin{array}{r|cc|} \textit{paṇas} & \frac{13}{2} & \frac{16}{1} \\ \text{mango} & 100 & \cdot \\ \hline \end{array} \quad \rightarrow \quad \begin{array}{|cc|} 13 & 16 \\ 1 & 2 \\ \cdot & 100 \\ \hline \end{array} \quad \rightarrow \quad \frac{16 \times 2 \times 100}{13 \times 1} = 246\frac{2}{13} \text{ pieces.}$$

But the *nyāsa* of this example put the three terms as usual in three cells arranged horizontally, which implies the following computation.

$$\begin{array}{|c|c|c|}\hline 6 & 100 & 16 \\ \frac{1}{2} & & \\ \hline\end{array} \quad \rightarrow \quad \frac{100\times 16}{6\frac{1}{2}} = 100\times 16\times\frac{2}{13} = 246\frac{2}{13}.$$

Hereafter I express a three-quantity operation $\boxed{a|b|x}$ as (a, b, x) and the two categories of quantity as 'weight-money' etc.

Tr E30 = PG E25 (sandal wood, weight-money): (1 *pala* 1 *karṣa*, $10\frac{1}{2}$ *paṇas*, 9 *palas* 1 *karṣa*) = 4 *purāṇas* 13 *paṇas* 2 *kākiṇīs* 16 *varāṭakas*.
Tr E31 = PG E26 (black pepper, money-weight): ($1\frac{1}{4}$ *paṇas*, $1\frac{1}{3}$ *palas*, $10-\frac{1}{3}$ *paṇas*) = 10 *palas* 1 *karṣa* 3 *māṣas* $4\frac{5}{9}$ *guñjās*.
Tr E32 (grain, money-volume): ($100\frac{1}{3}$ coins,[114] $60\frac{1}{2}$ *khārīs*, 1 coin) = 9 *droṇas* 2 *āḍhakas* 2 *prasthas* $1\frac{139}{301}$ *kuḍavas*.
Tr E33 = PG E27[115] (grain, volume-money): ($1\frac{1}{2}$ *droṇas* 3 *kuḍavas*, 8 coins, 1 *khārī* 1 *droṇa*) = $87\frac{91}{99}$ coins.
Tr E34 = PG E29 (gold, weight-money): (1 *suvarṇa*, $70\frac{1}{3}$ coins, $1-\frac{1}{10}$ *māṣa*) = $3\frac{153}{160}$ coins.
Tr E35 = PG E30 (a man's walk, length-time): (8 *yojanas*, $\frac{1}{3}$ day, 100 *yojanas*) = 8 months $26\frac{2}{3}$ days.[116]
Tr E36 = PG E31 (a worm's crawl, length-time): ($\frac{1}{8}$ *aṅgula*, $\frac{1}{4}$ day, 10 *yojanas*) = 33600 years.[117]
Tr E37 (commission of surety, money-money): 'When the commission of surety is six percent separately, what will it be for one thousand coins inclusively?' (106 coins, 6 coins, 1000 coins) = $56\frac{32}{53}$ coins.
Tr E38 = PG E34 (pearls and necklaces, number-number and number-number).[118] Tr: (1 necklace, 8 pearls, 20 necklaces) = 160 pearls, (6 pearls, 1 necklace, 160 pearls) = $26\frac{2}{3}$ necklaces. PG: (8 pearls, 20 necklaces, 6 pearls) = $26\frac{2}{3}$ necklaces.

114 Śrīdhara often uses the word *rūpa* for prices of commodities in his examples in the Tr and PG. Following K. S. Shukla (p. 24 of his translation of the PG), I regard it as a synonym of *rūpaka* meaning a coin of any denomination.

115 The verbal expression of PG E27cd is different from that of Tr E33cd. Here, the sign of equation means the numerical equality.

116 30 days = 1 month.

117 360 days = 1 year.

118 The Tr takes up this problem twice: here under the three-quantity operation and in Tr E40 under the inverse three-quantity operation. Here, it is solved by means of two successive three-quantity operations. On the other hand, the PG treats it only as an example for the inverse three-quantity operation.

Tr E39 = PG E36 (gold, weight-purity and purity-weight).[119] Tr: (1 *suvarṇa*, 16 *varṇas*, 168 *suvarṇas*) = 2688 *varṇas*, (11 *varṇas*, 1 *suvarṇa*, 2688 *varṇas*) = 244 *suvarṇas* 5 *māṣas* $4\frac{1}{11}$ *guñjās*. PG: (16 *varṇas*, 168 *suvarṇas*, 11 *varṇas*) = 244 *suvarṇas* 5 *māṣas* $4\frac{1}{11}$ *raktikās*.
PG E28 (grain, money-volume):[120] ($100\frac{1}{3}$ coins, $60\frac{1}{2}$ *khārīs*, $\frac{1}{4}$ coin) = 2 *droṇas* 1 *āḍhaka* $2\frac{178}{301}$ *prasthas*.
PG E32 (an elephant's walk, length-time): ($\frac{1}{2}(1+\frac{1}{4})(1-\frac{1}{3})(1+\frac{1}{2})$ *yojana* per $\{6 \cdot \frac{1}{5} \cdot \frac{1}{9} \cdot \frac{1}{3} \cdot (1+\frac{1}{4})\}$ day$-2(1-\frac{1}{3})$ *yojana* per $1\frac{1}{2}$ days, 1 day, 100 *yojanas*) = $\frac{243}{373}$ day.[121]
PG E33 (a man's earning, money-time): ($7\frac{1}{2}$ coins per $1\frac{1}{3}$ days$-\frac{1}{2}$ coin per day, 1 day, 100 coins) = $19\frac{21}{41}$ days.
L 74 (saffron, money-weight): ($\frac{3}{7}$ *niṣka*, $2\frac{1}{2}$ *palas*, 9 *niṣkas*) = 52 *palas* 2 *karṣas*.
L 75 (camphor, weight-money): (63 *palas*, 104 *niṣkas*, $12\frac{1}{4}$ *palas*) = 20 *niṣkas* 3 *drammas* 8 *paṇas* 3 *kākiṇīs* $11\frac{1}{9}$ *varāṭakas*.
L 76 (brown rice, money-volume): (2 *drammas*, $1\frac{1}{8}$ *khārīs*, 70 *paṇas*) = 2 *khārīs* 7 *droṇas* 1 *āḍhaka* 2 *prasthas*.

E7cd: Example for the five-quantity operation; interest.

$$\begin{array}{r|cc|} \text{months} & \frac{3}{2} & \frac{12}{1} \\ \text{capital} & 100 & 60 \\ \text{interest} & 5 & \cdot \\ \hline \end{array} \rightarrow \begin{array}{|cc|} 3 & 12 \\ 1 & 2 \\ 100 & 60 \\ \cdot & 5 \\ \hline \end{array} \rightarrow \frac{12 \times 2 \times 60 \times 5}{3 \times 1 \times 100} = 24.$$

Hereafter, in the references to other texts, I express the two vertical columns of a five-quantity operation etc. horizontally as $(a_1, a_2, \ldots, a_n, b; x_1, x_2, \ldots, x_n)$ just for convenience.

Tr E44 = PG E39 (interest, time-money-money): (1 month, 100 coins, 5 coins; 1 *year*, 60 coins) = 36 coins.
Tr E45 = PG E40 (interest, time-money-money): ($\frac{1}{3}$ month, $100\frac{1}{2}$ coins, $1\frac{1}{2}$ coins; $7\frac{1}{2}$ months, $17-\frac{1}{4}$ coins) = $5\frac{1}{8}$ coins.
Tr E46 = PG E41 (gold, purity-weight-money): (16 *varṇas*, 1 *suvarṇa*, 60 coins; 10 *varṇas*, 63 *suvarṇas*) = $2362\frac{1}{2}$ coins.
Tr E47 (gold, purity-weight-money): (16 *varṇas*, 1 *suvarṇa*, 73 coins; 11 *varṇas*, $1\frac{1}{2}$ *suvarṇas*) = $75\frac{9}{32}$ coins.

119 The PG treats this problem under the inverse three-quantity operation.
120 Close to Tr E32 above.
121 This and the next examples are problems called 'going forward and backward'.

Tr E48 = PG E42 (gold, purity-weight-money): (16 *varṇas,* $\frac{1}{2}$ *suvarṇa* -1 *guñjā,* $20\frac{1}{2}$ coins; $11\frac{1}{2}$ *varṇas,* 3 *guñjās*) = $1\frac{111}{832}$ coins.
Tr E49 = PG E43 (wages for carriers, volume-length-money): (8 *droṇas,* 1 *yojana,* 6 *paṇas*; 1 *khārī* 1 *droṇa,* 3 *yojanas*) = 2 *purāṇas* 6 *paṇas* 1 *kākiṇī.*
Tr E50 = PG E44 (wages for laborers, number-time-money): (3 persons, 2 days, 5 coins; 8 persons, 9 days) = 60 coins.
L 83 (interest, time-money-money): (1 month, 100, 5; 1 year, 16) = $9\frac{3}{5}$ (the monetary unit not given).
L 84 (interest, time-money-money): ($1\frac{1}{3}$ months, 100, $5\frac{1}{5}$; $3\frac{1}{5}$ months, $62\frac{1}{2}$) = $7\frac{4}{5}$ (the monetary unit not given).

E8: Example for the seven-quantity operation; cloth.

$$\begin{array}{r|cc|} \text{pieces} & 1 & 2 \\ \text{length} & 10 & 12 \\ \text{width} & 3 & 3 \\ \textit{drammas} & 8 & \cdot \end{array} \quad \rightarrow \quad \begin{array}{|cc|} 1 & 2 \\ 10 & 12 \\ 3 & 3 \\ \cdot & 8 \end{array} \quad \rightarrow \quad \frac{2 \times 12 \times 3 \times 8}{1 \times 10 \times 3} = 19\frac{1}{5} \ \textit{drammas.}$$

Tr E51 = PG E45 (blanket, length-length-number-money): (2, 8, 1, 10 coins; 3, 9, 2) = $33\frac{3}{4}$ coins (the unit of length not given).
L 85 (cloth, length-length-number-money): (3 *hastas,* 8 *hastas,* 8, 100 *niṣkas*; $\frac{1}{2}$ *hasta,* $3\frac{1}{2}$ *hastas,* 1) = 0 *niṣka,* 14 *drammas* 9 *paṇas* 1 *kākiṇī* $6\frac{1}{3}$ *varāṭakas.*

E9: Example for the inverse three-quantity operation.
Most part of this example is lost but, judging from the three words that have survived, this is concerned with exchange of a piece of gold whose purity is eight *varṇas* and another piece equivalent to it. As has been pointed out in L 78, the inverse three-quantity operation is applicable to the weight-purity relationship of gold as well as to the age-money relationship of living beings and to the unit-quantity relationship of grain. The L gives one example each for them. The Tr and PG give one or two examples for each of them as well as one for the area-number relationship of cloth. I cite only those for gold.

Tr E42 = PG E38 (gold, weight-purity): ($12\frac{1}{2}$ *varṇas,* $100\frac{1}{2}$ *suvarṇas,* $10\frac{1}{4}$ *varṇas*) = 122 *suvarṇas* 8 *māṣas* $4\frac{36}{41}$ *raktikās.*[122]

122 Dvivedī's text of the Tr reads 12 and **रपणाः** respectively for 122 and **माषाः**. The former must be a misprint. The word *rapaṇa* cannot be attested in any other Sanskrit works. Dvivedī glosses the latter as follow (p. 19): **षोडशरपणैः सुवर्णः।पञ्चरक्तिकाभिरेको रपण इति गणिततः सिध्यति।** But all the manuscripts of the Tr I consulted read **माष** and not **रपण**. Dvivedī does not specify the manuscript he used for his edition.

L 80 (gold, purity-weight): (10 *varṇas*, 1 *gadyāṇaka*, 15 *varṇas*) = $\frac{2}{3}$ *gadyāṇaka*.

13cd–23ab: Lost. Cf. the parallel verses in the Tr, PG, and L indicated in Table 2 above.

E10–E19: Lost. Cf. the parallel verses in the Tr, PG, and L indicated in Table 2 above.

E20: Example of an inequiperpendicular inequilateral quadrilateral.
Most part of this example is lost except a geometrical figure and a short sentence attached to it. The figure shows a quadrilateral with two diagonals and two perpendiculars, though they are distorted in the manuscript. It also contains the numerals for the lengths of the four sides (25, 39, 60, 52) except the base (60) and of the two perpendiculars ($\frac{189}{5}$, $\frac{224}{5}$) though one of the two 2's is missing. Therefore, the quadrilateral is determined uniquely: it is cyclic and its diagonals are orthogonal to each other, their lengths being 56 and 63. The prose sentence after the figure states that its area is 1764.
This area can be obtained either from the four sides by means of Brahmagupta's formula, i.e., the extended Heron's formula,

$$A = \sqrt{(88-25)(88-39)(88-60)(88-52)} = 1764,$$

where $(25+39+60+52)/2 = 88$, as it is a cyclic quadrilateral, or from the two diagonals (56, 63),

$$A = \frac{56 \times 63}{2} = 1764,$$

as they are orthogonal to each other. However, the first method is more plausible than the second because Tr E80 asks for the area of exactly the same quadrilateral with the answer 1764 in its prose part,[123] and Tr 43

123 Dvivedī's text of the Tr reads 1784 but this '8' must be a misprint. All the manuscripts of the Tr I consulted read 1764. The verse of Tr E80 reads as follows in his text:

> पञ्चकृतिर्यस्य मुखं षष्टिर्भूमिर्भुजौ त्रयोदशकरौ।
> त्रिचतुर्गुणौ यथाक्रममृजुनि सखे तस्य किं गणितम्॥ Tr E80॥

The word ऋजुनि ('in straight') of the fourth पाद does not make sense in this context. ऋजुनि सखे should be emended to अनृजुमुखे ('when it is non-straight-face') as in manuscripts A2 and L3, although other manuscripts are also corrupt at this point. See the next footnote. With this emendation, the above verse means:

> What is the calculated ⟨area⟩ of that ⟨quadrilateral⟩ whose face (top) is the square of five, base sixty, and arms (two flank sides) thirteen *karas* multiplied respectively by three and four, when it is non-straight-face?/Tr E80/

immediately before it prescribes Brahmagupta's formula (see below). It would then be natural to think that the lost folio contained a verse that gave the same formula immediately before GP E20. The same formula is stated in PG 117 also but the uniquely extant manuscript of the PG breaks off after PG 118 that prescribes a formula for approximate square-roots. Thus, no example for the formula is preserved in the extant PG.
In the PG, Śrīdhara classifies trilaterals into three, that is, equilateral, isosceles and scalene, and quadrilaterals into five, that is, equilateral (square), elongated (rectangular), equi-bilateral, equi-trilateral and in-equilateral.

आयतसमचतुरश्रे द्वित्रिसमभुजे विषमचतुरश्रम् ।
समविऽमद्विसमभुजत्र्यश्राण्यथ वृत्तचापे च॥PG 110॥
क्षेत्राणि दशैतानि हि फलमेषां साधयेत्स्वकरणेन।PG 111ab॥

Elongated and equilateral quadrilaterals, equi-bilateral and equi-trilateral quadrilaterals, inequilateral quadrilateral, equilateral, in-equilateral and equi-bilateral trilaterals, and also circle and bow: ॥ these are the ten ⟨basic⟩ figures, whose areas one can obtain by means of the calculation appropriate for each.

He further classifies quadrilaterals into equiperpendicular (*samalamba*) and inequiperpendicular (*asamalamba*) and gives one area formula for each category. Let a be the base, b and c the flank sides, and d the top of a quadrilateral, and h the height when it is equiperpendicular. Śrīdhara prescribes

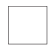
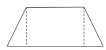
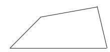
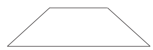
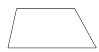
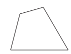
$$A = \frac{a+d}{2} \cdot h \quad \text{(PG 115)}$$

for equiperpendicular figures and

$$A = \sqrt{(s-a)(s-b)(s-c)(s-d)} \quad \text{(PG 117)},$$

where $s = \frac{a+b+c+d}{2}$, for inequiperpendicular figures. Both formulas are also meant for the three kinds of trilaterals where $d = 0$.

Table 3: Śrīdhara's Classification of Quadrilaterals

	Equiperpendicular	Inequiperpendicular	in PG
	Straight-face	Non-straight-face	in Tr
Equilateral		–	
Elongated		–	
Equi-bilateral			
Equi-trilateral			
Inequilateral			

In the Tr, Śrīdhara treats trilaterals and the first two categories of quadrilaterals separately from other equiperpendicular figures. Thus he prescribes

$$A = ab \quad \text{(Tr 42ab)}$$

for equilateral and elongated quadrilaterals,

$$A = \frac{a+d}{2} \cdot h \quad \text{(Tr 42cd)}$$

for 'other ⟨straight-face⟩ quadrilaterals',[124]

$$A = \sqrt{(s-a)(s-b)(s-c)(s-d)} \quad \text{(Tr 43abc)},$$

for non-straight-face quadrilaterals[125] and trilaterals, and lastly

$$A = \frac{a}{2} \cdot h \quad \text{(Tr 43d)}$$

as an alternative formula for trilaterals.

Of course, Brahmagupta's formula stated in PG 117 and Tr 43abc gives the correct area only when the figure is circumscribed, but nowhere in the PG and Tr it is mentioned, although Śrīdhara's extant two examples, the quadrilateral (25, 39, 52, 60) in Tr E80 and naturally the trilateral (13, 14, 15) in Tr E81, are cyclic.

124 In the Tr, 'equiperpendicular' quadrilaterals are called 'straight-face'. This expression does not occur here in Tr 42 itself but does occur in Tr E77 (*ṛju-mukha*) and in Tr 49 (*ṛju-vadana*). This nomenclature has not so far been found in any other Sanskrit mathematical works. The 'straight-face' seems to signify that the 'face' (i.e., the top) of a trapezium (including square and rectangle) lies 'straightly' in the same direction as the base, that is, in parallel to the base. It would then be quite natural to expect the expression 'non-straight-face' (*anṛju-mukha*) for 'inequiperpendicular' quadrilaterals. This expression does not occur in the Dvivedī's edition of the Tr but does occur in Tr E80 in two of the manuscripts I have consulted. See the preceding footnote.

125 See the preceding footnote.

Brahmagupta himself is ambiguous about the condition of the formula. He prescribes two formulas in one verse, one for a 'gross' area (*sthūla-phala*) and the other for 'accurate' one (*sūkṣma-*).

> स्थूलफलं त्रिचतुर्भुजबाहुप्रतिबाहुयोगदलघातः।
> भुजयोगार्धचतुष्टयभुजोनघातात्पदं सूक्ष्मम्॥ BSS 12.21॥
>
> The product of the halves of the sums of the sides and countersides of a tri- and quadri-lateral is a gross area. The square-root of the product of fourfold of half the sum of the sides, each decreased by a side, is the accurate.

The accurate one is Brahmagupta's formula while the gross one is the so-called 'surveyor's rule,' that is, in the same notation as above,

$$A = \frac{a+d}{2} \times \frac{b+c}{2}.$$

The BSS has no example for these formulas but the commentator Pṛthūdakasvāmī (10th century) illustrates them by three trilaterals and five quadrilaterals (fols. 65b–67a), probably following the above-mentioned Śrīdhara's classification of tri- and quadri-laterals.

(1) 12, 12, 12: gross 72, accurate $\sqrt{3888}$;
(2) 10, 13, 13: gross 65, accurate 60;
(3) 14, 13, 15: gross 98, accurate 84;
(4) 10, 10, 10, 10: gross 100, accurate 100;
(5) 5, 12, 5, 12: gross 60, accurate 60;
(6) 14, 13, 13, 4: gross 117, accurate 108;
(7) 39, 25, 25, 25: gross 800, accurate 768;
(8) 60, 52, 39, 25: gross $1933\frac{3}{4}$, accurate 1764.

Pṛthūdaka uses the same quadrilateral (8) for explaining Brahmagupta's rules for the diagonals of an 'inequilateral' (*viṣama*) quadrilateral (BSS 12.28), which is also required to be cyclic, and for the segments of the diagonals and perpendiculars in a needle figure (*sūcī-kṣetra,* BSS 12.32), which is constructed on a quadrilateral, which is not required to be cyclic, but he does not mention anywhere in his commentary that the quadrilateral he uses is cyclic.
However, it is probable that Brahmagupta gave the formula for cyclic quadrilaterals as has been argued several times. For one of the recent studies in this direction see Kichenassamy 2010. In that case, as Kichenassamy points out, it follows that most successors, including Śrīdhara, Bhāskara

II, and even the commentator Pṛthūdaka, misunderstood the condition of Brahmagupta's formula. Bhāskara II says:

द्विपञ्चाशन्मितव्येकचत्वारिंशन्मितौ भुजौ।
मुखं तु पञ्चविंशत्या तुल्यं षष्ट्या मही किल॥L 186॥
अतुल्यलम्बकं क्षेत्रमिदं पूर्वैरुदाहृतम्।
षट्पञ्चाशत्त्रिषष्टिश्च नियते कर्णयोर्मिती।
कर्णौ तत्रापरौ ब्रूहि समलम्बं च तच्छ्रुती॥L 187॥

The two arms (flank sides) measure fifty-two and thirty-nine respectively, and the face and the base are said to be equal to twenty-five and sixty respectively.॥ This is the figure given by our predecessors as an illustration of the inequiperpendicular figure. Fifty-six and sixty-three are the lengths of its ears (diagonals) ⟨given by them as⟩ determinate. ⟨But, even if the four sides are given, its diagonals are indeterminate and the figure is not necessarily inequiperpendicular.⟩ Tell ⟨me⟩ then other pair of ears and, ⟨when it is equiperpendicular⟩, the equal perpendicular and the ears.

In the prose passage that follows, assuming one of the diagonals to be 32, he obtains another diagonal, $\sqrt{621} + \sqrt{2700}$, and then, assuming the figure to be equiperpendicular, he calculates the perpendicular, $\sqrt{38016/25}$, and the diagonals, $\sqrt{5049}$ and $\sqrt{2176}$. And then he concludes that prose passage with a criticism of his predecessors beginning with Brahmagupta:

एवं चतुरस्रे तेष्वेव बाहुष्वन्यौ कर्णौ बहुधा भवतः।एवमनियतत्वे ऽपि नियतावेव कर्णावानीतौ ब्रह्मगुप्ताद्यैः।तदानयनं यथा। (End of the prose passage after L 187)

In this way, when the ⟨four⟩ arms of a quadrilateral are the same as those ⟨given by our predecessors⟩, a number of paris of ears exist. Thus, in spite of the state of being indeterminate, a pair of ears were calculated as if ⟨they were⟩ determinate by Brahmagupta and others. That calculation is as follows. (Here he cites BSS 12.28.)

23cd: Circumference and area of a circle.

$$C = \frac{22}{7}d, \quad A = \frac{22}{7}\left(\frac{d}{2}\right)^2.$$

The Tr uses $\sqrt{10}/1$ for the ratio of the circumference to the diameter (π):

$$C = \sqrt{10d^2}, \quad A = \sqrt{10\left(\frac{d}{2}\right)^4} \qquad \text{(Tr 45)}.$$

This is the only approximation to that ratio (π) preserved in the extant works of Śrīdhara if we except 22/7 for the time being. He uses the same approximation in his formulas for the area of a circle segment (Tr 47), for the volume of a truncated cone (Tr 54), and for the volume of a sphere (Tr 56):

$$A = \sqrt{\left(\frac{a+h}{2}\cdot h\right)^2 \times 10 \div 9} \qquad \text{(Tr 47)};$$

$$V = \frac{\sqrt{\left\{d_1^2 + (d_1+d_2)^2 + d_2^2\right\}^2 \times 10} \times h}{24} \qquad \text{(Tr 54)};$$

$$V = \frac{d^3}{2} + \frac{d^3}{2} \times \frac{1}{18} \qquad \text{(Tr 56)}.$$

In the last formula, 19/6 is used as an approximation to $\sqrt{10}$:

$$\sqrt{10} \approx 3 + \frac{1}{2\times 3} = \frac{19}{6}.$$

However, it has been logically inferred from Rāghavabhaṭṭa's statements in his commentary on the *Śāradātilaka* that one of his lost works, namely *Bṛhatpāṭī*, employed $\sqrt{10}/1$ for practical calculation and 22/7 for accurate one. See Hayashi 1995, 244. Therefore, Śrīdhara was the first in India who used 22/7 for that ratio. Bhāskara II employed 22/7 and 3927/1250 (L 199).

E21ab: Example for the circumference and area of a circle.
Given $d = 10$. Answer: $C = 31\frac{3}{7}$, $A = 78\frac{4}{7}$.

Tr E85: Given $d = 10$. Answer: $C = \sqrt{1000}$, $A = \sqrt{6250}$.
For the calculation of these square-roots, Tr 46 (= PG 118) gives the approximation formula,

$$\sqrt{K} = \frac{\sqrt{Ka^2}}{a} \approx \frac{[\sqrt{Ka^2}]}{a},$$

where a is an optional large integer and $[\sqrt{Ka^2}]$ is the integer part of $\sqrt{Ka^2}$, which is calculated by means of the algorithm based on the decimal notation, for which see GP 3–4ab above.
Thus the prose part after Tr 46 gives approximations to the above square-roots by taking $a = 100$, that is, $C \approx 31\frac{31}{50}$, $A \approx 79\frac{1}{20}$.

F5: Area of a segment of a circle.

$$A = \frac{(a+h)h}{2} + \frac{(a+h)h}{2} \cdot \frac{1}{20},$$

where a = chord, h = arrow (height of the segment).

The verse of F5 is exactly the same as the one Gaṇeśa ascribes to his father Keśava while commenting on L 213 that gives an example for Bhāskara I's formula for the arc anonymously cited in L 212. The L does not give any formula for the area of a circle segment.
For Śrīdhara's own formula for a circle segment given in Tr 47, see above under GP 23cd. For an analysis of the formula of F5, see Hayashi 1995, 242–45.

FE4: Example for the area of a circle segment.
Given $a = 13$, $h = 3$. Answer: $A = 25\frac{1}{5}$.
The diameter of the circle intended here turns out to be $17\frac{1}{12}$.

The problem given here is numerically the same as that of Tr E86, but the expression is different.

> चापाकृतिनि क्षेत्रे त्रिहस्तबाणे त्रयोदशकरज्ये ।
> किं भवति फलं विद्वन् गणयित्वा कथय यदि वेत्सि ॥ Tr E86 ॥

> In a bow-like figure whose arrow is three *hastas* and chord thirteen *karas* (= *hastas*), what is the area? O learned! Calculate and tell ⟨the answer to me⟩, if you know.

The prose part after the verse gives the answer $\sqrt{640} \approx 25\frac{29}{100}$. For the approximation of the square-root, see above under GP E21ab.

F6: Arrow, chord and diameter of a circle (L 204).

$$h = \frac{d - \sqrt{(d-a)(d+a)}}{2}, \quad a = 2\sqrt{h(d-h)}, \quad d = \left(\frac{a}{2}\right)^2 \div h + h.$$

The Tr does not have these formulas.

FE5: Example for the arrow, chord and diameter (L 205).
1. Given $d = 10$, $a = 6$. Answer: $h = 1$.
2. Given $d = 10$, $h = 1$. Answer: $a = 6$.
3. Given $a = 6$, $h = 1$. Answer: $d = 10$.

F7: Definition of 'arm' (*bāhu*) and 'edge' (*koṭi*) (L 135).
One of the two sides orthogonal to each other of a right-angled triangle is called 'arm' (*bāhu*) and the other 'edge' (*koṭi*).

The state of being orthogonal is expressed as 'in the direction competing with it' (*tatspardhinyāṃ diśi*). In the L, a right-angled triangle is called 'noble trilateral' (*jātya-tryasra*).

The Tr does not give any definition of these terms.

F8: Sides of right-angled triangles (L 136).

$$c = \sqrt{a^2 + b^2}, \qquad b = \sqrt{c^2 - a^2}, \qquad a = \sqrt{c^2 - b^2}.$$

Tr 51, which is the last verse of the chapter on plane figures (*kṣetra-vyavahāra*), gives these formulas. The verse occupies the last part of a procedure for obtaining the segments (*avadhā*, lit. 'place down') of the base of a trapezium from its four sides. The procedure goes as follows.

– Tr 49: Transform the given trapezium[126] (a, b, c, d) to the trilateral $(a-d, b, c)$ by removing the rectangle delimited by the perpendiculars drawn from the two vertices.

– Tr 50ab: Calculate the area A of the trilateral by Tr 43 and then the perpendicular or height by $h = 2A/(a-d)$.

– Tr 50cd: Regard (x_1, h, b) and (x_2, h, c), where x_1 and x_2 are the segments of the base divided by the perpendicular, as right trilaterals (x_1 and x_2 = arms, h = edge, b and c = ears) and calculate the arms by the next formulas.

– Tr 51: the same three formulas as above.

It should however be noted that Tr 51 can also be read independently.

भुजकोट्योः कृतिहीनात्पृथक्पृथक्कर्णवर्गतो मूले ।[127]
कोटिभुजौ तत्कृत्योर्युतितो मूलं प्रजायते कर्णः ॥ Tr 51 ॥

> The square-root from the square of the ear decreased by the square of the arm or of the edge, is respectively the edge or the arm. The square-root from the sum of the squares of them (i.e. the arm and edge) becomes the ear.

FE6: Examples for the sides of right-angled triangles (L 137).
1. Given $a = 3$, $b = 4$. Answer: $c = 5$.
2. Given $c = 5$, $a = 3$. Answer: $b = 4$.
3. Given $b = 4$, $c = 5$. Answer: $a = 3$.

126 The trapezium is here called 'straight-face quadrilateral' (**ऋजुवदन - चतुर्बाबु**). See the previous footnote.

127 हीनात्] होनात् Dvivedī's text.

24ab: Volume of an excavation.

$$V = Ah,$$

where A is the area of a horizontal cross-section of the excavation and h its depth.

Tr 52–53: $V = \bar{a_1} \times \bar{a_2} \times \bar{a_3}$, where $\bar{a_i}$ is the mean length, mean width and mean depth, that is, $\bar{a_i} = \left(\sum_{j=1}^{n_i} a_{ij}\right) / n_i$.

E21cd: Examples for the volume of an excavation.
1. Given $A = 84$, $h = 3$. Answer: $V = 252$.
2. Given $A = 25$, $h = 3$. Answer: $V = 75$.
The example referred to by the word 'previous' is lost but it is known from the figures given in the manuscript that the horizontal cross-section of the first example is the Heron's integral triangle (13, 14, 15; $A = 84$) and the second a square of side 5 $(A = 25)$.
In the given figure of the Heron's triangle, the side of 14 is taken to be its base. This is because the height is also an integer (12) only in that case; the height is $12\frac{12}{13}$ for the base 13 and $11\frac{1}{5}$ for the base 15.

Tr E87 (a lotus pool): Given $a_1 = 16$, $a_2 = 5$, $a_{3j} = 2, 3, 4$ *hastas.* Answer: $V = 240$ ⟨cubic *hastas*⟩.
Tr E88 (an excavation): Given $a_1 = 12$, $a_{2j} = 3, 4, 5$, $a_3 = 8$ *hastas.* Answer: $V = 384$ ⟨cubic *hastas*⟩.
Tr E89 (an excavation): Given $a_1 = 16\frac{1}{2}$, $a_2 = 10\frac{1}{4}$, $a_3 = 8$ *hastas.* Answer: $V = 1353$ ⟨cubic⟩ *hastas.*
Tr E90 (an excavation): Given $a_1 = a_2 = a_3 = 16$ *hastas.* Answer: $V = 4096$ ⟨cubic *hastas*⟩.

24cd: Brick-piling and timber-sawing.
1. The 'fruit' (*phala*), that is, the total volume of the piled bricks is

$$V = Ah,$$

where A is the area of a horizontal cross-section of the piled bricks and h its height.
2. The 'fruit' (*phala*), that is, the total area of the surfaces cut out of a plank by means of a saw is

$$S = Am,$$

where A is the area of a cut surface and m the number of cutting lines called *mārga* (road or route).

These rules seem to have been meant for evaluating respectively the work of piling bricks, which is proportionate to the weight of the piled bricks and therefore to the total volume of the bricks (cf. the example in Tr E97–98 under E22ab below), and the work of sawing, which is proportionate to the total surface area cut by the saw (cf. Tr E99–E100 under E22cd below).

Tr 58 gives, in addition to the same rule as above for the volume of the piled bricks, a rule for the number of the bricks:

$$N = V \div v,$$

where v is the volume of one brick.
Tr 59 gives a formula for vertical cutting (*ūrdhva-ccheda*),

$$S = abm \div (2 \times 12)^2 \quad \langle\text{square}\rangle \ \textit{hastas},$$

where a and b are the length and the thickness of the timber measured in *aṅgulas*.
Tr 60 gives a formula for horizontal cutting (*tiryak-cheda*),

$$S = Am \div (6 \times 4)^2 \quad \langle\text{square}\rangle \ \textit{hastas},$$

where A is the area in *aṅgulas* of a cut surface.

E22ab: Examples for brick-piling.
1. Given $A = 84$, $h = 2$. Answer: $V = 168$.
2. Given $A = 25$, $h = 2$. Answer: $V = 50$.
For the area of the cross-section in each problem, see under E21cd above.

Tr E95–95 (a rectangular altar): Given the altar's length = 6, width = 3, height = $\frac{1}{2}$ *hastas*; a brick's length = 1 *hasta*, width = $\frac{1}{2}$ *hasta*, height = 6 *aṅgulas*. Answer: $V = 9$ solid *hastas*, $v = \frac{1}{8}$ solid *hasta*, $N = 72$.
Tr E97–98 (wages for digging a pit): Given the pit's length = 10, width = 2, height = 4 *hastas*; the rate of wages = 1 coin (*rūpa*) per a pit of $3\frac{1}{2} \times 2 \times 3\frac{1}{2}$ solid *hastas*. Answer: $V = 80$ solid *hastas*, $v = 24\frac{1}{2}$ solid *hastas*, $N = 3\frac{13}{49}$ coins.

E22cd: Example for timber-sawing.
Given $A = 10 \times 3$ ⟨square *hastas*⟩, $m = 6$. Answer: $S = 180$ ⟨square⟩ *hastas*.
The unit of S is, in the manuscript, expressed as 'solid *hastas*' (*ghana-hastāḥ*), that is, the cubic *hasta*, but it should be the 'square' *hastas*, which must have been expressed simply as 'hastāḥ'.
The figure in the manuscript shows a rectangle divided by 'five' equidistant straight lines parallel to the longer sides. If this is not a mistake of 'six' lines, the six 'strips' made by the five parallel lines and by the two longer sides

of the rectangle probably represent the six cut surfaces rearranged on the same plane.

It may be pointed out here that the piece of timber intended here is unrealistically thick. According to Pherū (GSK 3.92), a piece of timber whose width (i.e. thickness) is more than one *gaja* (which is estimated to be $63 \sim 72$ cm) cannot be cut by a saw (the ordinary *hasta* is equated to about 18 inches or 45.7 cm). See SaKHYa 2009, 157 (and 99 for *gaja*).

Tr E99 (vertical cutting of timber of acacia catechu): Given $a = 12$, $b = \frac{1}{4}$ *hasta*, $m = 5$. Answer: $S = 15$ ⟨square⟩ *hastas*.

Tr E100 (wages for the sawing in E99): By the three-quantity operation, (sawing of 100×1 ⟨square⟩ *hastas* of acacia catechu, 6 coins, 15 ⟨square⟩ *hastas*) = $\frac{9}{10}$ coin.

Tr E101 (horizontal cutting of timber of acacia catechu): Given the timber's diameter = 1 *hasta*, $m = 10$. Answer: $A = \sqrt{10 \times 12^4} = \sqrt{207360} \approx 455\frac{9}{25}$ ⟨square *aṅgulas*⟩, $S = 455\frac{9}{25} \times 10 \div (6 \times 4)^2 = 7\frac{163}{180}$ ⟨square⟩ *hastas*. Wages calculated by the three-quantity operation, $(100, 6, 7\frac{163}{180}) = \frac{1423}{3000}$ coin.

25ab: Volume of heaped-up grain.

$$V = \left(\frac{C}{6}\right)^2 \cdot \frac{C}{9},$$

where C is the circumference of the circular base of a cone-like heap of grain.

Tr 61–62 gives the formula, $V = \left(\frac{C}{6}\right)^2 \cdot h$, where h is the height. In the attached example (Tr E102), h equal to $C/9$ is given. According to BSS 12.50, L 227, GSK 3.97, and PV A29–31, $h = C/9$ or $C/10$ or $C/11$ according to whether the grain is fine or coarse.

E23ab: Example.

Given $C = 36$ *hastas*. Answer: $V = 144$ solid *hastas*.

Tr E102: Given $C = 36$, $h = 4$. Answer: $V = 144$ solid *hastas*.

25cd: Time from shadow.

The elapsed or remaining portion of the daylight is

$$t = \frac{g/2}{s+g} \text{ daylight},$$

where g is the length of the gnomon and s the length of the shadow.

Tr 65 gives the same relationship of the time and the shadow length in the form, $t = \frac{g}{2(s+g)}$, and also in its reverse form, $s = \frac{g/2}{t} - g$.[128]
For equivalent and different formulas for shadow-time relationship given in Sanskrit mathematical works, see GSK pp. 160–62.

E23cd: Example.
Given $g = 8$ *aṅgulas*, $s = 24$ *aṅgulas*. Answer: $t = \frac{1}{8}$ of the daylight.

Tr E106: 1. Given $g = 12$ *aṅgulas*, $s = 3g$ *aṅgulas* in the West. Answer: $t = \frac{1}{8}$ of the daylight elapsed. 2. Given $g = 8$ *aṅgulas*, $s = 3g$ *aṅgulas* in the West. Answer: $t = \frac{1}{8}$ of the daylight elapsed.
Tr E107: 1. Given $g = 12$ *aṅgulas*, $t = \frac{1}{8}$ daylight elapsed or remaining. Answer: $s = 36$ *aṅgulas* in the West or East. 2. Given $g = 8$ *aṅgulas*, $t = \frac{1}{8}$ daylight elapsed or remaining. Answer: $s = 24$ *aṅgulas* in the West or East.

F9: Shadow (L 234).

$$s = \frac{dg}{h - g},$$

where d is the distance between the feet of the lamp and of the gnomon and h the height of the lamp.

FE7: Example (L 235).
Given $g = 12$ *aṅgulas*, $d = 3$ *hastas*, $h = 3\frac{1}{2}$ *hastas*. Answer: $s = \frac{1}{2}$ *hasta* = 12 *aṅgulas*.

F10: Height of the lamp (L 236).

$$h = \frac{dg}{s} + g.$$

FE8: Example (L 237).
Given $d = 3$ *hastas*, $s = 16$ *aṅgulas*, $\langle g = 12$ *aṅgulas*$\rangle$. Answer: $h = 66$ *aṅgulas* = $\frac{11}{4}$ *hastas*.

26: Concluding remark (merits of the GP).
The first half of this verse reminds one of the concluding verse of the last chapter called 'Net of Digits' of the L, where Bhāskara dealt with combinatorics of digits (numerical figures) which is a topic he invented.

न गुणो न हरो न कृतिर्न घनः पृष्टस्तथापि दुष्टानां
गर्वितगणकबटूनां स्यात्पातोऽवश्यमङ्कपाशे ऽस्मिन् ॥ L 271 ॥

128 Read दिनगतशेषोद्धृते instead of दिनगतशेषे हृते of Tr 65d in Dvivedī's text.

> Although neither multiplier, nor divisor, nor square, nor cube is asked ⟨in this chapter⟩, second-rate haughty mathematicians would inevitably fall in this net of digits.

The phrase ‘Read again and again’ (*paṭha paṭha*) of the second half occurs also in the concluding verse of the BG.

> **गणक भणितिरम्यं बाललीलावगम्यं**
> **सकलगणितसारं सोपपत्तिप्रकारम्।**
> **इति बहुगुणयुक्तं सर्वदोषैर्विमुक्तं**
> **पठ पठ मतिवृद्ध्यै लघ्विदं प्रौढिसिद्ध्यै** ॥ BG 102 ॥ [129]

> This book is pleasant for reciting (reading) [or elegant in style], is easy to understand even for young people, is the essence of the whole mathematics, and is accompanied by proof methods. In short, this book is full of merits and free from faults. Read again and again, o calculator, this small ⟨book⟩ in order to enhance your intelligence and to obtain maturity.

F11: Another concluding remark (praise of Śrīdhara).
This verse has been found attached to most (but not all) of the extant manuscripts of the *Triśatikā* of the same author. See, for example, A1, fol. 25b, H1, fol. 22a, H2, fol. 11b, L1, fol. 23a, L2, fol. 23b, L3, fol. 10a, and T1, fols. 28b-29a. In P1, which is incomplete, the commentator cites the verse while commenting on the first verse of the Tr in which the name Śrīdhara occurs. B1, which is complete, does not have it.

129 The last compound reads ‘प्रौढसिद्ध्यै’ in my edition (2009) of the BG but it should be ‘प्रौढि - ’; ‘प्रौढ MGTP] प्रोढ A’ in the corresponding footnote should also be corrected to ‘प्रौढि MGT] प्रौढ P, प्रोढ A.’

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APPENDIX: MĀḌAJĀTI IN HJJM 10728

Included here are a rule and an example of the tower class found in the Patan manuscript HJJM 10728 (= H2) of the *Triśatikā*. For my interpretation, see under F4 and FE3 in the Commentary.

Text (H2, fol. 5)

माडजातौ करणसूत्रमार्या॥[1]

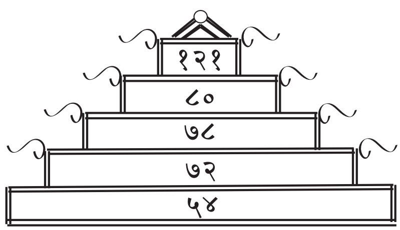
इष्टं सैकगुणाप्तं तदधः संस्थाप्य गुणगुणं सेष्टम्॥[2]
तदुपर्यावृत्कृत्वा तुल्यहराणां छिदां लोपः॥[3]

उदा॥

आरूढा नृपयोषितः कतिपया मल्लाहवं वीक्षितं
चित्रालङ्कृतयः सुनीतिचतुरास्ताः पञ्चभूपुष्पके।[4]
सार्धं भङ्गभियावतीर्णमपि ता जाताः समाः स्युः कथम्
वेत्सि त्वं यदि माडजातिकुतुकं मित्र प्रवक्षाशु तत्⟨॥⟩[5]

न्यासः॥पञ्चभूमिके॥ ५॥अधोवर्तनांश[6] ‖ १ २ ‖ जाताः पञ्चभूमिके समारूढा⟨ः⟩।[7]

पृथक्पृथङ्नार्यादर्शनम्।[8] न्यास⟨ः⟩।



⟨क्षेत्र १२⟩[9]

जाता समा संख्या॥ ८१⟨॥⟩[10] एवं माडजातिः समाप्ता॥[11]

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1 माड] मांड H2; आर्या] आर्याः H2.

2 संस्थाप्य] संस्थाप्ये H2 (‘e’ crossed out).

3 आवृत्कृत्वा] आवत्॥कृत्वा H2.

4 योषितः] योषितेः॥H2 (‘e’ crossed out); कृतयः] कृतयः।H2 (‘।’ crossed out); ताः] ताः॥ H2; भू] भूमि H2.

5 जाताः समाः स्युः कथम्] जाता स्युः।कथं H2.

6 अधोवर्तनांश] अधोवत्ताणांश H2.

7 जाताः] जाताः॥H2.

8 पृथङ्नार्या] पृथक् : नार्या H2.

9 This figure, as usual in Sanskrit manuscripts, occupies not its proper place (after the second ‘न्यासः’) but a space (of 13 *akṣaras* × 5 lines of writing) at the top left corner of the page (fol. 5b).

10 संख्या] त्रिय H2.

11 माडजातिः समाप्ता] मांडजातिसमाप्तः H2.

Translation

A computational rule in the Āryā meter for the tower class.

> An optional number is divided by the multiplier increased by one. Put that (quotient) multiplied by the multiplier below and, on the other hand, put that (same quotient) increased by the optional number above and repeat ⟨the same calculation⟩. Delete the denominators of ⟨all the fractions obtained, after they are reduced to those that⟩ have equal denominators.

Ex.

> A certain number of royal ladies, elegant, lovely and adorned with various ornaments, mounted on the stands (*puṣpaka*) built on five levels (*bhū*) ⟨for⟩ watching a wrestling sport. ⟨The number of the ladies on each level was⟩ increased by half ⟨of itself⟩ and also ⟨simultaneously decreased by half of the number of the ladies on the next level, both due to their⟩ descending, for fear of collapse ⟨of the stands⟩. ⟨In consequence⟩, they became the same ⟨in number in all levels⟩. How? If you know the curious tower class, o friend, tell ⟨me⟩ that quickly.

Setting-down: on the ⟨stands having⟩ five levels, 5; part of those who are on the ⟨next⟩ lower level, $\begin{array}{||c||}1\\2\\\hline\end{array}$ ⟨The number of women⟩ who mounted on ⟨the stands having⟩ five levels is produced ⟨by calculation⟩. Showing the ⟨number of⟩ women at each level, setting-down:

[Figure 12t (see kṣetra 12 in the Text above)]

The number ⟨of ladies on each level, after moving⟩, becomes equal: 81. Thus is completed ⟨the section on⟩ the tower class.

Acknowledgments

I wish to express my sincere thanks to S. R. Sarma san for his valuable comments and suggestions on the draft of this paper, which greatly helped me improve it in many respects.