Mémoires
de la SOCIÉTÉ MATHÉMATIQUE DE FRANCE

Numéro 118
Nouvelle série

TOPOLOGICAL PROPERTIES OF RAUZY FRACTALS

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2009

SOCIÉTÉ MATHÉMATIQUE DE FRANCE
Publié avec le concours du Centre National de la Recherche Scientifique

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P.O. Box 6248
Providence RI 02940
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Tarifs

Vente au numéro : 28 € ($ 42)
Abonnement Europe : 247 €, hors Europe : 281 € ($ 424)
Des conditions spéciales sont accordées aux membres de la SMF.

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ISSN 0249-633-X

ISBN 978-2-85629-290-7

Directeur de la publication : Bernard HELFFER

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2000 *Mathematics Subject Classification.* – 28A80, 11A63, 54F65.

Key words and phrases. – Rauzy fractal, tiling, beta-numeration, connectivity, homeomorphy to a disk, fundamental group.

Both authors are supported by the "Amadée" grant FR-13-2008.
The first author is supported by projects ANR-06-JCJC-0073 and BLAN07-1-184548 granted by the French National Research Agency.
The second author is supported by project S9610 granted by the Austrian Science Foundation (FWF). This project is part of the FWF national research network S96 "Analytic combinatorics and probabilistic number theory".
The drawings of the graphs are done with help of the software yFiles.

TOPOLOGICAL PROPERTIES OF RAUZY FRACTALS

Anne Siegel, Jörg M. Thuswaldner

Abstract. – Substitutions are combinatorial objects (one replaces a letter by a word) which produce sequences by iteration. They occur in many mathematical fields, roughly as soon as a repetitive process appears. In the present monograph we deal with topological and geometric properties of substitutions, in particular, we study properties of the *Rauzy fractals* associated to substitutions.

To be more precise, let σ be a substitution over the finite alphabet $\mathcal{A}$. We assume that the incidence matrix of σ is primitive and that its dominant eigenvalue is a unit Pisot number (*i.e.*, an algebraic integer greater than one whose norm is equal to one and all of whose Galois conjugates are of modulus strictly smaller than one). It is well-known that one can attach to σ a set $\mathcal{T}$ which is called *central tile* or *Rauzy fractal* of σ. Such a central tile is a compact set that is the closure of its interior and decomposes in a natural way in $n = |\mathcal{A}|$ subtiles $\mathcal{T}(1), \ldots, \mathcal{T}(n)$. The central tile as well as its subtiles are graph directed self-affine sets that often have fractal boundary.

Pisot substitutions and central tiles are of high relevance in several branches of mathematics like tiling theory, spectral theory, Diophantine approximation, the construction of discrete planes and quasicrystals as well as in connection with numeration like generalized continued fractions and radix representations. The questions coming up in all these domains can often be reformulated in terms of questions related to the topology and the geometry of the underlying central tile.

After a thorough survey of important properties of unit Pisot substitutions and their associated Rauzy fractals the present monograph is devoted to the investigation of a variety of topological properties of $\mathcal{T}$ and its subtiles. Our approach is an algorithmic one. In particular, we dwell upon the question whether $\mathcal{T}$ and its subtiles induce a tiling, calculate the Hausdorff dimension of their boundary, give criteria for their connectivity and homeomorphy to a closed disk and derive properties of their fundamental group.

The basic tools for our criteria are several classes of graphs built from the description of the tiles $\mathcal{T}(i)$ $(1 \leq i \leq n)$ as the solution of a graph directed iterated function system and from the structure of the tilings induced by these tiles. These graphs are of interest in their own right. For instance, they can be used to construct the

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