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ALBANESE AND PICARD 1-MOTIVES

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Abstract. — Let X be an n -dimensional algebraic variety over a field of characteristic zero. We describe algebraically defined Deligne 1-motives $\text{Alb}^+(X)$, $\text{Alb}^-(X)$, $\text{Pic}^+(X)$ and $\text{Pic}^-(X)$ which generalize the classical Albanese and Picard varieties of a smooth projective variety. We compute Hodge, ℓ -adic and De Rham realizations proving Deligne’s conjecture for H^{2n-1} , H_{2n-1} , H^1 and H_1 .

We investigate functoriality, universality, homotopical invariance and invariance under formation of projective bundles. We compare our cohomological and homological 1-motives for normal schemes. For proper schemes, we obtain an Abel-Jacobi map from the (Levine-Weibel) Chow group of zero cycles to our cohomological Albanese 1-motive which is the universal regular homomorphism to semi-abelian varieties. By using this universal property we get “motivic” Gysin maps for projective local complete intersection morphisms.

Résumé (1-motifs d’Albanese et de Picard). — Soit X une variété algébrique de dimension n sur un corps de caractéristique 0. Nous décrivons les 1-motifs de Deligne $\text{Alb}^+(X)$, $\text{Alb}^-(X)$, $\text{Pic}^+(X)$ et $\text{Pic}^-(X)$ définis algébriquement, qui généralisent les variétés d’Albanese et de Picard classiques d’une variété projective lisse. Nous calculons les réalisations de Hodge, ℓ -adique et de De Rham, montrant ainsi la conjecture de Deligne pour H^{2n-1} , H_{2n-1} , H^1 et H_1 .

Nous étudions la fonctorialité, l’universalité, l’invariance par homotopie et l’invariance par formation de fibrés projectifs. Nous comparons nos 1-motifs homologiques et cohomologiques pour les schémas normaux. Pour des schémas propres, nous obtenons une application d’Abel-Jacobi du groupe de (Levine-Weibel) Chow des zéro-cycles vers notre 1-motif cohomologique d’Albanese, qui est l’homomorphisme universel régulier vers les variétés semi-abéliennes. En utilisant cette propriété universelle, nous obtenons des applications de Gysin « motiviques » pour les morphismes projectifs localement intersection complète.

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