LEONARD DICKSON'S

HISTORY OF THE THEORY OF NUMBERS:

AN HISTORICAL STUDY WITH

MATHEMATICAL IMPLICATIONS

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ABSTRACT. — In 1911, the research mathematician Leonard Dickson embarked on a *historical* study of the theory of numbers which culminated in the publication of his three-volume *History of the Theory of Numbers*. This paper discusses the genesis of this work, the historiographic style revealed therein, and the mathematical contributions which arose out of it.

RÉSUMÉ. — *HISTORY OF THE THEORY OF NUMBERS* DE LEONARD DICKSON: ÉTUDE HISTORIQUE AVEC DES PROLONGEMENTS MATHÉMATIQUES. — En 1911, le mathématicien Leonard Dickson s'est lancé dans une étude *historique* de la théorie des nombres, qui a culminé avec la publication de son *History of the Theory of Numbers* en trois volumes. Notre étude examine la genèse de ce travail, l'approche historiographique qui la sous-tend et les contributions mathématiques qui en découlent.

In 1911, only a decade into what would become a forty-year-long career in the mathematics department at the University of Chicago, Leonard Dickson had a résumé which solidly identified him as a distinguished mathematician. He had, for example, authored roughly 150 mathematical papers (primarily in group theory at this point) and three books, served as editor of the *American Mathematical Monthly* from 1902 to 1908 and recently assumed this post for the *Transactions of the American Mathematical Society*, and passed swiftly through the ranks from assistant to associate to full professor at one of the premiere mathematics institutions in this country. Yet, in 1911, he threw what seems a rather twisted turn into his professional plans by pursuing a *historical* project which would

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interrupt his pure mathematical research for the better part of nine years. That investigation? The title of this paper gives it away: he embarked on a study of the history of the theory of numbers. This almost ninety-year-old decision raises (raised?) a flurry of questions. In this paper, we focus on the genesis of this work, the historiographic style revealed therein by Dickson, and one of the surprising mathematical contributions which arose out of it. An understanding of the origins of this work, however, begins with an understanding of its author, the then thirty-seven-year-old Leonard Dickson.

Born in Independence Iowa in 1874, Dickson spent his boyhood in Cleburne, Texas and ultimately attended the University of Texas for his undergraduate and master's education.[1] With his master's degree in hand and two years of teaching experience under his belt, Dickson chose the strong Eliakim Hastings Moore, Oskar Bolza, and Heinrich Maschke triumvirate at the young University of Chicago over the up-and-coming Harvard with William Fogg Osgood and Maxime Bôcher as the place to pursue his doctorate. Dickson's mathematical career would ultimately hinge on this decision [Fenster 1997, pp. 9–13].

At the time, Chicago, with its sights set on emulating the German tradition of scholarship, stood in marked contrast to most American institutions. Specifically, Moore, Bolza and Maschke formed the core of the original far-sighted Chicago Mathematics Department which promoted *both* research and teaching and which emphasized in its graduate program the training of future productive researchers [Parshall & Rowe 1994, pp. 363–426], [Fenster 1997, pp. 10–11].

While Dickson pursued a Ph.D. at the young Chicago from 1894 to 1896, the then group-theoretically minded Moore inspired him to write a thesis on (what we would call) permutation groups [Dickson 1897]. Although group theory would remain among Dickson's research interests throughout his career, he would add finite field theory, invariant theory, the theory of algebras and number theory to his repertoire of research interests. Dickson reflected Chicago's influence—particularly that of Moore—in more ways than in his research interests, however. The

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[1] [Bell 1938] and [Albert 1955] serve as the standard sources for biographical information on Dickson. This author also consulted [University of Texas 1899, 1914] and [Parshall 1991].

department's sustained commitment to research, high standards for pub-
lication, and their vision for the American (as opposed to New England)
mathematical community came to permeate Dickson's mathematical per-
sona in these formative years. In the spring of 1900, the Chicago Mathe-
matics Department invited Dickson to join them as an assistant professor.
From this position, Dickson made significant contributions to the consol-
idation and growth of the algebraic tradition in America [Fenster 1997,
p. 21]. Specifically, Dickson spent forty years (all but the first two) of
his professional career on the faculty at Chicago where he directed 67
Ph.D. students, wrote more than 300 publications, served as editor of the
Monthly and the *Transactions*, and guided the American Mathematical
Society as its President from 1916–1918.

   And, yet, this mathematical workhorse, who played billiards and bridge
by day and did mathematics from 8: 30 p.m. to 1: 30 a.m. every night
[Albers & Alexanderson 1991, p. 377], spent nearly a decade of his career
writing a three-volume, 1500-page history of the theory of numbers.
The lurking question is: why? Why did Dickson interrupt his own pure
investigations of mathematics to write a history of the theory of numbers?

   Dickson's most celebrated student, A. Adrian Albert, has suggested
that Dickson wrote the book to become more acquainted with number
theory. More precisely, Albert wrote, "*Dickson always said that mathe-
matics is the queen of sciences, and that the theory of numbers is the
queen of mathematics. He also stated that he had always wished to work
in the theory of numbers and that he wrote his monumental* History of the
Theory of Numbers *so that he could know all of the work which had been
done in the subject*" [Albert 1955, p. 333].

   Dickson's developing research interests substantiate this claim. Of the
200 papers he wrote prior to 1923, the year he published the third
(and final) volume of his *History of the Theory of Numbers* (hereinafter
History), only ten considered number-theoretic topics.[2] In 1927, however,
his pure mathematical researches began to focus on additive number
theory, on the ideal Waring theorem, in particular. In a long series of
papers, he and his students provided an almost complete verification of the
theorem which loosely states that every positive integer is a sum of I
integral n-th powers for sufficiently large I. Moreover, Dickson guided

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[2] [Albert 1955, pp. 334–345] contains a bibliography of Dickson's work.

twenty-nine of his last thirty-two doctoral students in number-theoretic dissertations [University of Chicago 1931, 1938, 1941]. These twenty-nine students, along with Dickson's contributions to the ideal Waring theorem and three number theory texts he published in 1929, 1930, and 1939 [Dickson 1929, 1930, 1939] seem to indicate that if he intended for his historical study to acquaint him with the subject so that he could work in the field himself, he had certainly accomplished what he set out to do.

In some sense, given the time period under discussion, this connection between the history of mathematics and pure mathematical results comes as no surprise. The early decades of this century, in fact, represented a "*golden age for the history of mathematics*" since "*the historians of mathematics were professional mathematicians working in good mathematics departments*" [Gray 1998, p. 54].[3] Still, however, some members of the mathematical community viewed those who wrote *about* mathematics, in contrast to those who "*d[id] something*" in mathematics (*i.e.* "*proved new theorems*" or "*added to mathematics*"), as "*second-rate minds*" [Hardy 1940, p. 61]. With a solid reputation as a "*powerful*" [Mac Lane 1992] and prolific research mathematician [Fenster & Parshall 1994, pp. 185–186], Dickson apparently had no qualms about devoting time to the history of mathematics for more than a third of his career.

He may, however, have had other reasons for undertaking this historical work. In his initial letter to the Carnegie Institution seeking interest in the project, for example, Dickson outlined that "[*i*]*t would seem desirable to have undertaken in this country something of the kind done by the British Association, the Deutsche Mathematiker-Vereinigung, etc., in the preparation by specialists of note of extensive Reports each covering an important branch of science... I have already given a solid year's work to such an expository Report on the theory of numbers (integral and algebraic),...*" [Dickson 1911]. Thus the British and German mathematical Report[s], and, in particular, the lack of similar offerings in America,

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[3] [Merzbach 1989, p. 642] also documents that "*with one notable exception* [David Eugene Smith]*,"* the historians of mathematics in America before World War I came from "*those trained in mathematics and allied fields rather than from historians."* From post-World War I to 1930, the American historians of mathematics had strong ties with—and leadership roles within—the American Mathematical Society [*Ibid.*, p. 650].

may have encouraged Dickson to write his own compendium on the subject of number theory. In the case of graduate training, it was not at all unusual for the American mathematicians to look to the Europeans for ideas [Parshall & Rowe 1994]. The initiative Dickson outlined in his letter to Woodward, however, required not only an acquaintance with the European literature but also an awareness of a perceived void in American publications. Moreover, the opening sentence of his letter seems to suggest that Dickson wanted to raise American mathematics to the European standard in this particular realm.[4]

But Dickson himself gave another—more altruistic—reason for writing what grew into this three volume *History*. In the preface to the second volume, Dickson asserts that he embarked on this historical study because "*it fitted with my conviction that every person should aim to perform at some time in his life some serious useful work for which it is highly improbable that there will be any reward whatever other than his satisfaction therefrom*" [Dickson 1920, p. xxi]. Robert Carmichael extinguished any doubts of Dickson's sincerity in his review of this second volume for the *Monthly*. Carmichael, a number theorist who not only wrote the review of Dickson's *History* for the *Monthly*, but also read the proof sheets for the entire second and third volumes, described Dickson's motivation in similar terms. As Carmichael expressed it, "[i]t *is refreshing and inspiring to find a man, when he pauses at a breathing place in the excellent performance of a great task, willing to set forth in a quiet way the fact that he has been moved by the highest and most unselfish ideal of duty*" [Carmichael 1921, p. 78]. In the end, though, as we will see, whether motivated by a desire to acquaint himself with number theory, to publish an American report on the theory of numbers, or to fulfill this "*highest and unselfish ideal of duty,*" this historical initiative led Dickson to one of his most celebrated mathematical contributions.

First, however, let's take a closer look at Dickson's *History* itself. Dickson's view of the role of the historian dictated how he both prepared and wrote his book. As he saw it, "[w]*hat is generally wanted* [in a historical study] *is a full and correct statement of the facts, not an*

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[4] Throughout his career, Dickson remained avidly committed to establishing standards of excellence for and in the community of American mathematicians. See [Fenster, forthcoming].