

SÉMINAIRES ET CONGRÈS 15

**PARTIAL DIFFERENTIAL EQUATIONS
AND APPLICATIONS**

PROCEEDINGS OF THE CIMPA SCHOOL HELD IN LANZHOU

(2004)

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Abstract. — This volume contains expanded versions of lecture notes of CIMPA's school held in Lanzhou, July 2004. These texts offer a detailed survey, including the most recent advances, of some topics in analysis of partial differential equations arising from physics, mechanics and geometry such as: Korteweg-de Vries equation, harmonic maps, Birkhoff normal form and KAM theorem for infinite dimensional dynamical systems, vorticity of Euler equation, semi-classical analysis of Schrödinger and Dirac equations, and limiting situations of semilinear elliptic equations. They are mainly aimed at students and young researchers interested in these subjects.

Résumé (Equations aux dérivées partielles et applications. Actes de l'école du CIMPA, Lanzhou, 2004)

Ce volume comprend des versions élargies des notes de cours de l'école du CIMPA à Lanzhou, juillet 2004. Ces textes donnent un survol, y compris les progrès les plus récents, sur certains thèmes en analyse des équations aux dérivées partielles d'origine physique, mécanique ou géométrique tels que : l'équation de Korteweg-de Vries, les applications harmoniques, la forme normale de Birkhoff and le théorème de KAM pour des systèmes dynamiques de dimension infinie, le tourbillon de l'équation d'Euler, l'analyse semi-classique des équations de Schrödinger et de Dirac, et des situations limites des équations elliptiques semi-linéaires. La plupart des textes pourraient être lus par des étudiants ou des chercheurs débutants qui s'intéressent à ces sujets.

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