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A TORSION JACQUET-LANGLANDS CORRESPONDENCE

Frank CALEGARI & Akshay VENKATESH

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par Frank CALEGARI & Akshay VENKATESH

Abstract. — We prove a numerical form of a Jacquet-Langlands correspondence for torsion classes on arithmetic hyperbolic 3-manifolds.

***Résumé.* (Une correspondance de Jacquet-Langlands torsion)** — Nous établissons une forme numérique d'une correspondance de Jacquet-Langlands pour les classes de torsion sur des variétés hyperboliques arithmétiques de dimension 3.

CONTENTS

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The mathematical debt this book owes to the work of Cheeger [**24**] and Müller [**68**] should be clear: the main result about comparison of torsion homology makes essential use of their theorem.

Some of the original ideas of this book were conceived during the conference "Explicit Methods in Number Theory" in Oberwolfach during the summer of 2007. Various parts of the manuscript were written while the authors were resident at the following institutions: New York University, Stanford University, Northwestern University, the Institute for Advanced Study, the University of Chicago, the University of Melbourne, the University of Sydney, and the Brothers K coffee shop; we thank all of these institutions for their hospitality.

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