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FIVE LIVES OF A *GEOMETRIA SUBTERRANEA* (1708-1785). AUTHORSHIP AND KNOWLEDGE CIRCULATION IN PRACTICAL MATHEMATICS

Thomas Morel

Abstract. — In 1708, the subterranean geometer August Beyer (1677-1753) wrote a manuscript entitled *Geometria subterranea,* detailing the instruments and operations of underground surveying, of which several handwritten copies still exist. A modified version of this practical geometry was published by its author in 1749 and a second edition was printed in 1785, well after Beyer's death, by a mathematics professor of the Freiberg mining academy, J.F. Lempe (1757-1801). Analysing successive versions of this text shows the evolution of the discipline in the 18th century.

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T. Morel, Univ. Artois, EA 2462, Laboratoire de Mathématiques de Lens (LML), F-62300 Lens, France.

Courrier électronique : thomas.morel@espe-lnf.fr

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In addition, several questions about subterranean geometry have a general interest for the history of practical mathematics. The concept of authorship, in both senses of paternity of a text and of moral authority, proves to be ambigous, and in last resort unfit to understand the evolution and circulation of this kind of useful knowledge. Moreover, the growing institutionalization of engineering training in the 18th century could be thought to imply a swift progress in the mathematization of actual practices. The example of Beyer's *Geometria subterranea* shows that the short-term influence of technical schools was sometimes mixed, while highlighting other circulation realms for practical geometry.

RÉSUMÉ (Cinq vies d'une *Geometria subterranea* (1708-1785). Autorité et circulation des connaissances en mathématiques pratiques.)

En 1708, le géomètre souterrain Auguste Beyer (1677-1753) rédige un manuscrit intitulé *Geometria subterranea* dans lequel il décrit les instruments et opérations de l'arpentage minier. Plusieurs copies manuscrites de ce texte ont été conservées. Une version modifiée de ce texte fut publiée par son auteur en 1749, tandis qu'une seconde édition réalisée par J.F. Lempe (1757-1801), professeur de mathématiques à l'Académie des mines de Freiberg, parut en 1785, c'est-à-dire bien après la mort de Beyer. Les versions successives de ce texte témoignent de l'évolution de la discipline au cours du XVIII^{e} siècle.

Plusieurs questions relatives à cette discipline ont en outre un intérêt plus général pour l'histoire des mathématiques pratiques. Le concept d'autorité, au double sens de paternité d'un texte et d'influence morale, se revèle ambigu et *in fine* inadapté pour comprendre l'évolution et la circulation de ce type de savoirs utiles. On pourrait de plus penser que l'institutionnalisation croissante de la formation des ingénieurs au XVIII^{e} siècle implique une mathématisation croissante des pratiques effectives. L'exemple de la *Geometria subterranea* d'Auguste Beyer montre que le succès des écoles techniques fût, dans un premier temps, relativement mitigé, tout en mettant en évidence des dynamiques alternatives de circulation des savoirs géométriques pratiques.

1. INTRODUCTION

This paper describes the evolution of subterranean geometry in the 18th century, a discipline then belonging to the mathematical sciences. We focus on one particular work, a *Geometria subterranea* written around 1708 by the mining official August Beyer (1677-1753). At least five versions of this text have been preserved, three of them manuscripts from the first half of the century, as well as two printed editions respectively published in 1749 and 1785. This dense material allows for a minute analysis of its content and a close comparison of successive rewritings of the original text. We can thus track the evolution of underground surveying over the course of a century.

The fact that a manuscript written in 1708 would still be in use three generations later might be interpreted as a relative stagnation. On the contrary, this article shows that despite an apparent stability, numerous evolutions concerned both the mathematical content and the social structure of this discipline. When Beyer wrote the first version of his manuscript, this topic was neither taught in schools nor at universities while manuscripts were transmitted from master to pupils in a guild-like fashion [Sennewald 2002]. When the last version of this work appeared in 1785, subterranean geometry was a well-identified discipline and a public object of science—at least in the German-speaking world. Mining academies had been created, and a dozen textbooks were available while new instruments or methods were regularly discussed in technical journals. [1]

A similar trend could be observed in many other fields of practical mathematics: the 18th century saw a significant increase of the ambitions, if not of the achievements, of mechanical and mathematical sciences. [2] New institutions were created for military and civil engineering, from the *École royale des ponts et chaussées* (1747) and the *École royale du génie de Mézières* (1748) in France to the *Bergakademien* in Saxony (1765) and several other German states [Taton 1964 ; Guagnini 2004].

In that context, we will also use Beyer's text as a case study to ask more general questions about the elaboration and diffusion of knowledge in practical mathematical sciences. Practical geometry, and its subdiscipline subterranean geometry, were especially important and widespread in mining regions in order to direct extracting operations, draw maps and settle property limits underground. Following its numerous metamorphoses over the course of the 18th century helps understand its evolution and reveals a general pattern, with analytical methods increasingly replacing the graphical and piecemeal approaches of the previous century.

The importance of the institutionalization of engineering schools in continental Europe in spurring the mathematization of various technical activities has often been underlined. [3] Without denying the long-term

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1 For a brief overview of the state of subterranean geometry at the end of the 18th century and especially the creation of the mining academy of Freiberg, see [Morel 2013, p. 164-190].

2 About mathematization in the 18th century, see [Lowood 1990] for a summary of the debate and several interesting case studies. About the limits and slow progress of this process see [Vérin 1993, p. 243-333, p. 357], [Belhoste et al. 1990].

3 Institutional history and the history of teaching mathematical and mechanical sciences have produced important works such as [Taton 1964] or [Belhoste 1998]. See [Schubring 2003] for an overwiew about the institutional history of mathematics.

influence of these institutions, their immediate impact on contemporary technicians has to be reassessed and put in perspective. As early as 1782, the professor of mathematics at the mining academy of Freiberg claimed that academic teaching had within a few years deeply improved the practice of subterranean geometry:

> And so subterranean geometry stayed, for those who had to perform it, in the usual craft usage, until this most valuable institution, the mining academy that was built here in 1765, gave to everyone who had the capacity and desire of thinking, through the learning of mathematics and other auxiliary sciences, the opportunity not only [to master] the principles of subterranean geometry but also its complete scope, and could convince himself not only of the basics of subterranean geometry, but also of its whole range, and in how many kinds of cases it may be applied usefully to mining, and in what kind of tighter connection it stands with mathematics. [4]

This assertion, full of emphasis and rhetorical elaboration, sums up almost perfectly the challenges that historians of practical mathematics face about the 18th century. New institutions systematically blamed the artisan character and lack of theory they considered to be inherent to previous methods. They rejected the use of manuscripts and advocated an open circulation of knowledge. They also pretended to have instantly improved actual practices, as if heavy scientific books presenting intricate methods could both convince practitioners and solve every concrete problem at once, without any downside. To balance these obviously one-sided reports, statements from practitioners about the early history of these institutions are generally scarce and equally biased.

To give a more nuanced view of the development of practical mathematics, we need to understand how the practitioners themselves were working, reflecting on and improving their methods in the early eighteenth century. The question of authorship turns out to be a major, and evolving, issue. How were knowledge and know-how produced by underground surveyors? What did authorship mean in circles where methods were constantly transmitted and improved? Mathematical practitioners

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4 [Lempe 1782, p. 10–11], introduction by J.F.W. Charpentier: "Es blieb also die Markscheidekunst immer noch bey denen, die sie ausüben sollten, in der gewöhnlichen handwerksmäßigen Behandlung, bis durch die preißwürdigsten Anstalten, der im Jahr 1765 hier errichteten Bergwerksakademie die Gelegenheit allgemein wurde, wodurch sich ein jeder, der Fähigkeiten und Lust zum Denken hatte, durch Erlernung mathematischer und anderer Hülfswissenschaften, nicht nur von den Gründen der Markscheidekunst, sondern auch von ihrem ganzen Umfange, und auf wie mancherley Fälle sie beym Bergbau brauchbar anzuwenden ist, und in was für genauer Verbindung sie mit der Mathematik steht, selbst überzeugen konnte." All translations, unless otherwise stated, are from the author of the present article.

had to reconcile two fundamental sets of values: while they had to ensure practicability and to cope with very specific problems, they also looked for tested methods that could easily be reproduced and systems of representation that would ensure an easy and unambiguous communication with their fellow surveyors.

To address these questions, we present a material and intellectual biography of August Beyer's *Geometria subterranea.* [5] We will first introduce Beyer's work in the context of the early 18th century technical world of the mines, describing the scope and methods of subterranean geometry. We then focus on the manuscripts, studying the development and structure of Beyer's first *Geometria subterranea*, its diffusion and the various copies that were made. The influence of printing, the genesis of the first published version and the differences with the original manuscript have then to be analyzed. Beyer's decision reflected an evolution of the discipline, while its adaptation to a new readership had a direct influence on both its presentation and its content. We finally study the last edition, published some thirty years after Beyer's death and twenty years after the creation of mining academies. Analyzed as an academic textbook, this *Geometria subterranea* reveals precious information about the specific dynamics of mathematical practices as well as about the inherent difficulties associated with the institutionalization of engineering training.

2. AUGUST BEYER (1677-1753), MATHEMATICAL PRACTITIONER AND MINING EXPERT

2.1. *Subterranean geometry in the German mining states*

In 1708, a mining official named August Beyer (1677-1753) started writing a manuscript describing the use of geometry in the silver mines of the Ore Mountains (*Erzgebirge*), in the Electorate of Saxony. Beyer was a mining expert and wore the official title of subterranean geometer (*Markscheider*). He was therefore a mathematical practitioner of a rare kind, combining his geometrical knowledge with technical skills to play an important legal

[5] Recent works in the history of early modern mathematics have seriously attempted to write biographies of scientific or technical works. [Métin 2016] studies numerous manuscripts written and compiled by military engineers of the sixteenth and seventeenth centuries, while [Joffredo 2017] presents a meticulous analysis of the genesis and reception of Gabriel Cramer's *Introduction à l'analyse des lignes courbes algébriques* (1750). See the bibliographies of both theses for further references.