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Panoramas et Synthèses

Numéro 39/40 2013

SOCIÉTÉ MATHÉMATIQUE DE FRANCE
Publié avec le concours du Centre national de la recherche scientifique

Panoramas & Synthèses
39-40, 2013, p. 179–238

MHD SIMULATIONS FOR FUSION APPLICATIONS

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Abstract. – Nuclear fusion holds forth the promise of being a clean and safe solution to meet the world's energy demand in the foreseeable future without producing long-lived radioactive waste or weapons-grade material. The most mature configuration for magnetically confining a fusion plasma is the tokamak; a current carrying toroidal plasma characterized by strong externally produced magnetic fields. The temperatures, densities, and current will diffuse across the magnetic field lines at some rate, determining the confinement properties of the tokamak. The tokamak can also develop global instabilities if the current and/or pressure exceed certain instability thresholds. This set of lectures is aimed at describing analytical formulations and associated numerical methods for quantitatively describing both the slow (diffusive) motion associated with transport and the faster (wavelike) motion associated with instabilities. The former uses slow time scale ordering to remove the wavelike motion, and a time-dependent field-aligned coordinate transformation to isolate the cross-field transport from the faster transport along the magnetic field lines. The latter uses a combination of high-order finite elements, a particular representation of the magnetic and velocity vector fields, and an implicit time advance algorithm with desirable properties.

Résumé **(Simulations magnétohydrodynamiques pour des applications de fusion)**

La fusion nucléaire promet une énergie propre et sécuritaire capable de soutenir la demande mondiale en énergie dans les années futures sans la production de déchets radioactifs de longue durée ou de matières fissiles. La configuration la plus avancée pour le confinement magnétique d'un plasma à haute température est le tokamak, lequel est caractérisé par un plasma toroïdal conduisant un courant électrique et soumis à de forts champs magnétiques externes. Les températures, densités et courants diffusent à travers les lignes de champ magnétique à un taux qui détermine les propriétés de confinement du tokamak. Des instabilités globales peuvent cependant se développer à l'intérieur du tokamak lorsque le courant et/ou la pression dépassent un certain seuil. Cette série de cours à pour but de décrire les formulations analytiques, ainsi que leurs méthodes numériques correspondantes, pour le mouvement lent de diffusion associé au transport ainsi que le mouvement ondulatoire rapide associé aux instabilités. La première formulation utilise une échelle de temps longs afin d'éliminer

Key words and phrases. – Fusion, magnetohydrodynamics, tokamak, computational methods.

This work was supported by USDOE Contract No. DE-AC02-76CH03073.

le mouvement ondulatoire rapide, ainsi qu'une transformation de coordonnées alignée avec le champ magnétique et évoluant temporellement afin d'isoler le transport lent perpendiculaire aux lignes de champ du transport rapide le long des lignes de champ. La seconde formulation utilise une combinaison d'éléments finis d'ordre élevé, une représentation judicieuse des champs vectoriels de vélocité et de champ magnétique, ainsi qu'un algorithme implicite avancé ayant les propriétés désirées pour l'intégration temporelle.

1. Tokamak Fusion Basics and the MHD Equations

1.1. Tokamak Fusion Basics. – A strong case can be made for the development of fusion energy [**18**]. Worldwide demand for energy continues to increase due to both population increases and economic development. Most of this population growth and new energy demand is in urban areas, which implies the need for large centralized power generation. By many estimates, worldwide oil and gas production is near or past its peak, which implies the need for an alternative source. The only possibilities are: coal, nuclear fission, or nuclear fusion.

There is increasing evidence that release of greenhouse gasses is causing global climate change. This comes both from historical data and from detailed climate projections. This makes nuclear energy (both fission and fusion) preferable to fossil (coal). Nuclear fusion has three advantages over nuclear fission that could become critical: (i) It has inherent safety, so there is no possibility of a meltdown accident; (ii) There are no weapons proliferation considerations; and (iii) The waste disposal problems are greatly alleviated.

Controlled fusion uses isotopes of Hydrogen. A Deuterium (D) nuclei that collides with a Tritium (T) nuclei can fuse to produce a Helium nuclei and a neutron, and 17.5 MeV of energy. The neutron can be captured by a Lithium nuclei to produce another Helium nuclei and a Tritium nuclei. Because the process breeds its own Tritium and both Deuterium and Lithium are naturally abundant, there is essentially an unlimited supply of fuel.

In order to create the conditions for D and T to fuse, you must create a mixture of DT nuclei (or ions) and their associated electrons, called a plasma, and heat it to high temperature and high pressure. You need about 5 atmosphere pressure at a temperature of 10 keV ($100,000,000^0$ degrees K). A plasma at that temperature can never come into contact with material walls so we must confine it with magnetic fields.

To a first approximation, both the electrons and DT ions exhibit spiral orbits around magnetic field lines as indicated in Fig. 1. They describe circular orbits perpendicular to the field and free-stream in the direction of the field lines. In a *tokamak* [**56**], the magnetic field is bent into a torus so that the free-streaming motion does not lead to loss of particles. The tokamak has large electromagnets to produce

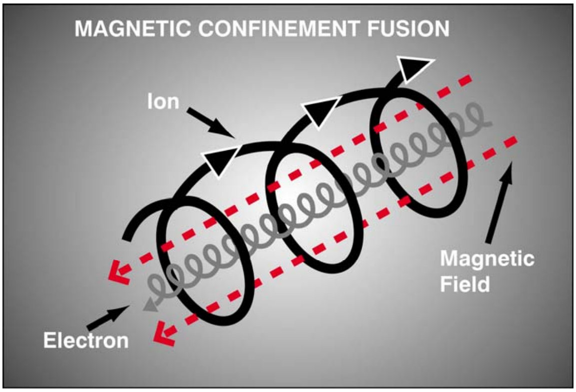

FIGURE 1. Charged particles have helical orbits in a magnetic field; they describe circular orbits perpendicular to the field and free-stream in the direction of the field.

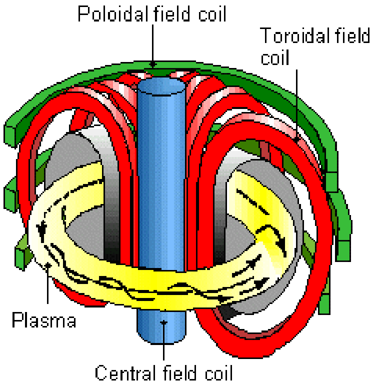

FIGURE 2. The tokamak creates toroidal magnetic field to confine the plasma and induces a toroidal plasma current to heat and confine the plasma.

the confining magnetic fields and to induce electrical current into the plasma. This current both heats the plasma and provides an essential "twist" to the magnetic field in the plasma so that the electrons and ions do not drift out of the confinement region. The toroidal-field coils produce the strongest magnetic field. These fields are directed in the toroidal direction (the long way around the torus). The poloidal-field coils produce weaker fields in the same plane as the fields produced by the plasma currents. These are orthogonal to the toroidal field, and serve to shape the plasma cross section. The magnetic field from the central field coil changes in time such as to induce current into the plasma through transformer action. The coil arrangement in a standard tokamak is shown in Fig. 2.

The doughnut-shaped tokamak plasma can develop global instabilities if the current it carries is too large or poorly distributed, or if the pressure is too high. This is the motivation for developing a set of partial differential equations that describe the dynamics of the plasma in the tokamak. The mathematical description of a plasma we consider here is called *extended magnetohydrodynamics*, or just simply *magnetohydrodynamics* (MHD). This treats the plasma as a conducting fluid that interacts with the magnetic fields produced by the electrical currents flowing in the plasma and from external sources. A tokamak is characterized by having a very strong externally imposed toroidal magnetic field (going the long way around the torus) so that the plasma pressure, p is much less than the magnetic pressure, $B^2/2\mu_0$ (SI Units). This is referred to as *low-*β, where $\beta \equiv 2\mu_0 p/B^2$. Special numerical methods are required to obtain accurate numerical solutions of low-β plasmas [**35**].

The organization of these lecture notes is as follows. In Sec. 1.2 we present the most general form for the MHD equations and some simpler approximate forms that are often used. These equations contain phenomena occuring on multiple timescales associated with (1) wave motion, (2) magnetic reconnection, and (3) transport of the magnetic field, energy, and particles. The fastest timescales that are present in the equations come from the presence of wave solutions. We refer to these waves as "ideal MHD waves" since they are present in the ideal form of the equations when all dissipative terms have been set to zero. The remainder of this first lecture discusses properties of those waves.

In Lecture 2, we discuss techniques for solving the equations of Sec. 1.2 over timescales that are very long compared to the ideal MHD wave transit times. This involves making an approximation that essentially removes the waves from the system. This is a very good approximation for describing slowly varying phenomena in a plasma configuration that remains stable as it evolves in time. These equations thus contain only the transport timescales, and are useful for describing the evolution of the current, pressure, temperature, and density profiles of a stable plasma over timescales comparable to the length of a discharge.

In order to solve the full set of MHD equations in which the wave phenomena is not removed, an implicit time advance utilizing high-order finite elements is described. As background, we introduce the finite element method in Lecture 3 and introduce a particular type of finite element that enforces continuity of the solution and the first derivative across element boundaries. These are referred to as C^1 elements [**51**]. By using C^1 elements and applying the Galerkin method, systems of equations with up to 4^{th} order spatial derivatives can be solved. Much of the introductory material presented in Lectures 1-3 and more can also be found in a recent textbook published about the time these lectures were presented [**35**].

Finally, in Lecture 4, we describe a highly accurate implicit time advance algorithm for magnetized plasma. A particular representation of the magnetic and velocity vector fields is chosen that preserves the divergence free condition on the magnetic field and allows accurate description of nearly divergence-free velocity fields that do not compress the toroidal magnetic field. These representations involve derivatives of scalar