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GYROKINETIC SIMULATIONS OF MAGNETIC FUSION PLASMAS

by

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Abstract. – This lecture presents the gyrokinetic framework and details the various numerical schemes used in nonlinear simulations to compute turbulent transport in magnetic fusion plasmas. The basic features of tokamak magnetic configuration and of fusion plasmas are recalled. Fundamental elements of the gyrokinetic theory are carefully introduced, including the derivation of velocity drifts and of the quasi-neutrality. From the numerical point of view, the main focus is put on the three existing classes of numerical methods, namely Particle-In-Cell, Eulerian and semi-Lagrangian. Their properties are discussed, and their strengths and weaknesses are exhaustively reviewed.

***Résumé* (Simulations gyrocinétiques des plasmas de fusion magnétique).** – Ce cours présente l'approche gyrocinétique et les différents schémas numériques utilisés dans les simulations pour calculer le transport turbulent dans les plasmas de fusion par confinement magnétique. Les caractéristiques essentielles de la configuration magnétique des tokamaks et des plasmas de fusion sont rappelées. Les élements fondamentaux de la théorie gyrocinétique sont introduits, dont en particulier les équations des trajectoires et l'électro-neutralité. Du point de vue numérique, les trois classes de méthodes numériques existantes sont présentées en détail, à savoir Particle-In-Cell, eulerienne et semi-lagrangienne. Leurs propriétés sont discutées, et leurs forces et faiblesses respectives sont passées en revue.

1. Introduction

In magnetic fusion devices, the power gain strongly increases with the energy confinement time. As a matter of fact, the quality of the plasma energy confinement largely determines the size and therefore the cost of a fusion reactor. This confinement time turns out to be mainly governed by the plasma turbulence which develops in such devices – of relative magnitude of a few percents in the hot core – and the associated transport. Understanding its origin and properties in view of its possible

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control is one of the critical issues in fusion science [**37**]. Such a quest belongs to the more general framework of general studies on turbulence, which is considered as one of the most difficult problems in physics, because of its inherent nonlinear character with many degrees of freedom. Plasma turbulence is rich of at least three additional properties with respect to neutral fluids: (*i*) Multiple fluids (an electron fluid and other ion fluids) with considerably different mobility are coupled through electromagnetic fields and weak collisional interactions. (*ii*) Strong confinement magnetic fields provide highly anisotropic turbulent structures. (*iii*) The inhomogeneities in density, temperature, and magnetic fields place the plasma naturally out of thermodynamical equilibrium, and tend to excite several micro-instabilities over a wide spectral range. In particular, the toroidal magnetic configuration strongly affects the linear properties of micro-instabilities as well as nonlinearly evolved turbulent structures. Finally, because of the weak collisionality in the hot core of fusion plasmas, which is often said to be collisionless (for thermal particles, the mean free path is expected to be of the order of several kilometers in ITER), kinetic resonances as well as significant deviations from the Maxwellian of the distribution function of the various species and particle orbit effects cannot be ignored. Because of this almost collisionless character, conventional fluid models becomes insufficient, and kinetic descriptions of the plasma in phase space are required. Kinetic models are drastically more demanding in terms of numerical resources than fluid models. In such first-principle descriptions of plasmas, the six dimensional evolution equation for the distribution function – Vlasov or Fokker-Planck equations – is solved for each specie, coupled to the self-consistent equations for the electromagnetic fields, namely Maxwell's equations. These coupled equations are non-linear. No general analytic solution of these equations exists, such that numerical simulations are necessary. Fortunately, as far as turbulent fluctuations are concerned, they develop at much lower typical frequencies than the high frequency cyclotron motion. Therefore, this 6D problem can be restricted to a 5D one by incorporating part of this small scale temporal behavior into the larger scales temporal dynamics of both the distribution function and the fields. The useful part of the distribution function then evolves in a five dimensional phase space generated by four slow variables and an adiabatic invariant. This model is known as the *gyrokinetic* model. In a first approximation, the gyrokinetic distribution function can be thought of as deduced from its six-dimensional kinetic analogue by an average procedure over the fast-varying gyrophase angle. This model consensually provides today's deepest insight on plasma behavior. But even with this dimensional reduction, the task is not easy at all. Solving 5D gyrokinetic equations for each specie reveals extremely challenging. First-principle gyrokinetic codes, which have been developed for this stage, make an intensive use of massively parallel supercomputers and require state-of-the-art high performance computing (HPC). They have greatly benefited from the drastic increase of both hardware capabilities (at present, Petaflop calculators are able to process 10^{15} floating point operations per second) and of new computational techniques based on fast solvers, massive parallelization protocols, etc. These numerical tools have already played an important role in clarifying a number of pending issues in

turbulent transport. Enlightening exhaustive reviews on the subject can be found in references [**142, 132, 78, 57**], listed in chronological order.

The development of such gyrokinetic codes would not be possible without a strong collaboration between physicists, mathematicians and now computer scientists. The numerical schemes, developed for Vlasov equations in 2 to 6 dimensional phase spaces, have evolved all along the last twenty-five years, in direct link with the evolution of HPC resources. Historically, particle in cell methods (PIC) [**16**] have been most popular, and represent widely adopted approaches to numerical simulations of kinetic plasmas. They used to be considered as the most efficient tool to describe plasma dynamics, essentially because they are capable of describing many physical phenomena in the full dimensional case, at relatively small computational costs. However, it is well known that the numerical noise inherent to PIC methods constitutes a strong limiting factor to accurately describe the distribution function in phase space on long time scales. Moreover, the numerical noise only slowly decreases, like $1/\sqrt{N}$, when the number N of particles is increased. To remedy this problem, alternative methods have also been developed. Eulerian methods, in which the Vlasov equation is discretized on a mesh of phase space, are one of those. Among them, the Fourier transform uses fast Fourier transform of the distribution function in phase space. Other methods like the Fourier-Hermite or Hermite transform methods exploit the fact that the Hermite basis can offer some advantage when modeling distribution functions with Gaussian-shaped profiles. However, due to wave-particle resonances which play an important role in the collisionless or weakly collisional regime, fine-scale structures inherently develop in velocity space. This requires the spectral expansion of a large number of Hermite modes to achieve high accuracy. Some of these algorithms use artificial damping to smooth out these fine-scale structures [**124**]. The drawback is that the collisionless nature of the system is lost, leading to distortions of the distribution function [**123**]. Another drawback of such methods is that they are only valid for periodic boundary conditions [**86, 87**]. Consequently, for non-periodic boundary conditions, Gibbs oscillations form at the boundary of the grid and lead to spurious oscillations which can propagate to the entire distribution function. A finite element method has also been proposed [**159, 158**]. Although it is well suited for complex boundary conditions which may arise in many practical applications, the numerical resolution can become cumbersome when dealing with the Vlasov equation in high dimension. Another scheme for the Vlasov equation is the flux corrected transport (FCT) [**18, 17**], or more recently the flux balance method (FBM) [**52**]: the basic idea is to compute the average of the Vlasov equation solution in each cell of the phase space grid by a conservative method. One of the common flaws of these algorithms is the non-preservation of the positivity, which reveals problematic for long simulation runs since numerical oscillations develop. Attempts to overcome this problem of the positive and flux conservative (PFC) methods have been tried [**54**]. Finally, the semi-Lagrangian method aims at taking advantages of both Lagrangian and Eulerian numerical schemes. It consists of computing the distribution function at each grid point by following the particle trajectories backward in time. To compute the origin of the characteristics, high-order

interpolation methods are needed. E. Sonnendrücker et al. proposed the cubic spline reconstruction which gives very good results [**126**], at the expense of the loss of the local character of the reconstruction. Nakamura and Yabe also presented the cubic interpolated propagation (CIP) method based on the approximation of the gradients of the distribution function in order to use a Hermite interpolation [**107**]. This method is very stable but very expensive in terms of computational memory since it requires the storage not only of the distribution function f, but also of its gradients $\nabla_x f$ and $\nabla_v f$. Such a constraint is also present for the Morinishi's scheme [**104**]. As will be detailed in these notes, all the various schemes have both advantages and drawbacks. The diversity of numerical approaches existing now in gyrokinetic codes clearly constitute a strength with regard to the complexity of the physical problem. As a result, benchmarks become crucial, as exemplified by the growing number of papers devoted to this difficult task in the community of gyrokinetic code development.

The reminder of the paper is organized as follows. After a general introduction in Section 1, we briefly describe the basic features of magnetic fusion research and of fusion plasmas in Section 2. In Section 3, we describe a theoretical hierarchy of plasma physics, with a focus on plasma kinetic theory in Section 4. The gyrokinetic approach specific to magnetic fusion plasmas is described in Section 5. Section 6 is devoted to the description of the numerical methods used in gyrokinetic codes, divided in three categories PIC, Eulerian and Semi-Lagrangian approach. In Section 7, some differences among the various methods regarding the treatment of some of the main physical plasma properties are highlighted. Finally, a summary is given in Section 8.

2. Brief introduction to plasma fusion

The increase in energy needs and the fact that fossil fuels are running out make indispensable the development of new sources of energies. To produce energy out of matter, it is necessary to carry out a transformation in which, between the initial and final state, a small proportion of the body mass involved disappears. This mass defect may then be found in the form of energy through the well-known formula $E = mc^2$, where E is the produced energy, m the mass that has disappeared and c the speed of light. Chemical reactions involve atoms or molecules and their electrons. As such, the corresponding energies lie in the tens of electron-Volt range (the ionisation energy of hydrogen is 13.6eV). As far as nuclear reactions are concerned, there exist two main types, both in the range of Mega electron-Volt[(1)]. The fission reaction consists in splitting the nucleus of a sufficiently heavy atom (such as the uranium or plutonium

(1) The ratio between the energy retrieved from chemical and nuclear reactions directly relates to the characteristic interaction lengths of the underlying forces: the Coulomb interaction links electrons to the nucleus on distances r^{Coulomb} of the order of a few tens of Angström ($1 = 10^{-10}m$), while the nuclear force (also called residual strong force) binds neutrons and protons in nuclei at distances r^{nuclear} of the order of one Fermi ($1Fermi = 10^{-15}m$). Since the potential energy associated to each of these central forces (there also exists a weak noncentral component of the nuclear force) decays like $1/r$, with r the distance between the two interacting particles, it comes: $E_{\text{pot}}^{\text{Coulomb}}/E_{\text{pot}}^{\text{nuclear}} \sim$