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2020

SOME ASPECTS
OF THE THEORY OF DYNAMICAL SYSTEMS:
A TRIBUTE TO JEAN-CHRISTOPHE YOCCOZ

Volume I

Sylvain CROVISIER, Raphaël KRIKORIAN,
Carlos MATHEUS, Samuel SENTI, éditeurs

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**SOME ASPECTS OF THE THEORY OF DYNAMICAL SYSTEMS:
A TRIBUTE TO JEAN-CHRISTOPHE YOCCOZ**

Volume I

Sylvain Crovisier, Raphaël Krikorian, Carlos Matheus, Samuel Senti, éditeurs

Abstract. — This is the first of two volumes which celebrate the memory of Jean-Christophe Yoccoz. These volumes present research articles on various aspects of the theory of dynamical systems and related topics that were dear to him.

Résumé (Quelques aspects de la théorie des systèmes dynamiques : un hommage à Jean-Christophe Yoccoz) — Voici le premier de deux volumes qui célèbrent la mémoire de Jean-Christophe Yoccoz. Ils regroupent des articles de recherche portant sur divers aspects de la théorie des systèmes dynamiques ainsi que sur des sujets connexes qui lui étaient chers.

TABLE DES MATIÈRES

ÉTIENNE GHYS & CHRISTOPHER-LLOYD SIMON — <i>On the topology of a real analytic curve in the neighborhood of a singular point</i>	1
Statement of the main result	1
The genesis of this paper	2
1. Analytic chord diagrams: an algorithm	3
1.1. Polynomial interchanges: algorithmic description	4
1.2. Chord diagrams	5
1.3. A necessary condition	5
1.4. Proof of the fundamental lemma	8
1.5. More non-analytic diagrams	10
1.6. With a computer	12
1.7. Marked chord diagrams	13
1.8. Let us bound the number of chord diagrams	15
2. Analytic chord diagrams: interlace graphs	17
2.1. Polynomial interchanges: permutation graph	17
2.2. Collapsible graphs	19
2.3. Distance hereditary and treelike graphs	23
2.4. Some proofs	27
2.5. Appendix: completely decomposable graphs	29
References	32
ROMAIN DUJARDIN — <i>A closing lemma for polynomial automorphisms of \mathbb{C}^2</i>	35
1. Introduction and results	35
Acknowledgments	37
2. Proofs	37
2.1. Preliminaries	37
2.2. The atomic case	38
2.3. The non-atomic case	39
References	42
CARLOS GUSTAVO T. DE A. MOREIRA — <i>On the minima of Markov and Lagrange Dynamical Spectra</i>	45

1. Introduction	45
Acknowledgements	47
2. Preliminaries from dynamical systems	47
3. Proofs	50
References	57
DMITRY DOLGOPYAT & OMRI SARIG — <i>Quenched and annealed temporal</i>	
<i>limit theorems for circle rotations</i>	59
1. Introduction	60
Setup	60
Methodology	60
Known results on spatial limit theorems:	61
Known results on temporal limit theorems:	61
This paper	62
Heuristic overview of the proof	62
Functions with more than one discontinuity	62
2. Statement of results	62
3. The main steps in the proofs of Theorems 2.1, 2.2	65
Step 1: Identifying the resonant harmonics	66
Step 2: An identity for the sum of resonant terms	67
Step 3: Limit theorems for resonant harmonics	68
4. Proofs of the Key Steps.	70
4.1. Preliminaries.	70
4.2. Step 1 (Propositions 3.1 and 3.3)	72
4.3. Step 2 (Proposition 3.5)	78
4.4. Step 3 (Proposition 3.7)	80
Appendix A. Proof of Proposition 1.1	82
Appendix B. Cauchy and Poisson	83
References	84
JOEL W. FISH & HELMUT HOFER — <i>Exhaustive Gromov compactness for</i>	
<i>pseudoholomorphic curves</i>	87
1. Introduction	87
Acknowledgements	90
2. Preliminaries	90
2.1. Direct limit manifolds	91
2.2. Riemann surfaces	93
2.3. Pseudoholomorphic curves	95
2.4. Convergence of pseudoholomorphic curves	97
3. Proof of exhaustive Gromov compactness	101
Appendix A. Formula for arithmetic genus	110
References	111

SEBASTIÃO FIRMO & PATRICE LE CALVEZ & JAVIER RIBÓN — <i>Fixed points of nilpotent actions on \mathbb{R}^2</i>	113
1. Introduction	113
2. Proof of Theorem 3	117
2.1. Definitions	117
2.1.1. Rotation number	117
2.1.2. Blowed up annuli	118
2.1.3. Linking function	121
2.2. Preliminary results	122
2.3. Proof of the theorem	123
3. Compactly covered Nielsen classes	125
3.1. Nielsen classes	125
3.2. Existence of classes with non-vanishing Lefschetz number	126
3.3. A minimality property	128
3.4. A particular case	130
4. Proof of Theorem 1	134
5. Rotational properties of homeomorphisms of the plane	137
5.1. The function Link and Turn	137
5.2. Rotation numbers	138
6. Privileged isotopies for groups of plane homeomorphisms	141
6.1. Properties $(\mathbf{P2})'$, $(\mathbf{Q})'$, (\mathbf{R}) , (\mathbf{S})	141
6.2. Main result	142
6.3. Property $(\mathbf{P2})$ for abelian groups	147
7. Rotational properties for groups of plane diffeomorphisms	148
7.1. Proof of Theorem 2	148
7.2. Consequences of property $(\mathbf{R})_j$	148
7.3. Proof of Proposition 46 in case $\text{supp}(\mu) \subset \text{Fix}(\phi)$	149
7.4. Proof of Proposition 46 in case $\mu(\text{Fix}(\phi)) = 0$	151
References	155
CHRISTIAN BONATTI & ALEX ESKIN & AMIE WILKINSON — <i>Projective cocycles over $SL(2, \mathbb{R})$ actions: measures invariant under the upper triangular group</i>	157
1. Introduction	158
2. Applications and the irreducibility criterion	160
2.1. Linear representations of G -lattices	161
2.1.1. The suspension construction and a criterion for simplicity ...	161
2.1.2. Foliated geodesic and horocyclic flows	162
2.2. The Kontsevich-Zorich cocycle	164
3. Construction of an invariant subspace	168
3.1. The forward and backward flags	168
3.2. The forward flag and the unstable horocycle flow: defining the inert flag.	170

3.3. The inert flag is G -invariant	171
3.4. Relaxing the definition of the components of the inert flag	172
4. Proof of Theorem 1.3	173
5. Proof of Theorem 2.8	174
References	177
KRISTIAN BJERKLÖV & L. HÅKAN ELIASSON — <i>Positive fibered Lyapunov exponents for some quasi-periodically driven circle endomorphisms with critical points</i>	181
1. Introduction	181
1.1. One dimensional models	181
1.2. Skew-products	182
1.3. Our model	183
1.4. Statement of results	183
2. Building expansion for good ω	184
2.1. The functions φ_k and the sets A_k^s	184
2.2. Scales and some arithmetics	184
2.3. Good frequencies	185
2.4. Base case for the induction	185
2.5. Inductive step	186
2.6. Positive Lyapunov exponents for all good frequencies	188
3. Parameter exclusion	189
3.1. Dependence on ω	189
3.2. Parameter exclusion	191
References	193
ALBERT FATHI — <i>Recurrence on infinite cyclic coverings</i>	195
1. Introduction	195
2. The displacement function	198
3. The Pageault barrier	202
4. The ρ_+ and ρ_- functions	204
5. The fundamental proposition	206
6. The results	208
References	214
DENNIS SULLIVAN — <i>Lattice Hydrodynamics</i>	215
1. Overview	215
2. Introduction to the “momentum model”	217
3. The ideas of the construction and definitions	218
The lattice vector field V_L	218
The face velocity vectors and face normal components V_F, v_F	219
The model proposal	219
The derivative outside the nonlinear term	219

The nonlinear term as a lattice vector field	219
The nonlinear term as a one chain	219
4. Lattice Vector calculus	219
Volume preserving	219
Divergence operator	219
Gradient of a lattice scalar field	219
Laplacian of f	220
Curl of a lattice vector field	220
5. Lattice topology, the Laplacian and the Hodge decomposition	220
6. The “potential term” and the “friction term”	221
References	222
VINCENT DELECROIX & ÉLISE GOUJARD & PETER ZOGRAF & ANTON ZORICH — <i>Contribution of one-cylinder square-tiled surfaces to Masur- Veech volumes</i>	223
Introduction	224
Siegel-Veech constants and Masur-Veech volumes	224
Equidistribution of square-tiled surfaces	224
Contribution of 1-cylinder square-tiled surfaces and large genus asymptotics of Masur-Veech volumes	225
Siegel-Veech constants and Masur-Veech volumes of strata of meromorphic quadratic differentials	226
Structure of the paper	227
Acknowledgements	227
1. Equidistribution	227
1.1. Strata of Abelian differentials	227
1.2. Strata of quadratic differentials	230
2. Contribution of 1-cylinder square-tiled surfaces to Masur-Veech volumes	232
2.1. Jenkins-Strebel differentials. Critical graphs (separatrix diagrams)	232
2.2. Contribution of 1-cylinder diagrams	234
2.3. Asymptotics in large genera	238
2.4. Application: experimental evaluation of the Masur-Veech volumes	241
2.5. Contribution of a single 1-cylinder separatrix diagram: computation	241
Choice of cyclic ordering	242
Abelian versus quadratic differentials	242
Contribution of each individual 1-cylinder separatrix diagram	243
2.6. Counting 1-cylinder diagrams for strata of Abelian differentials based on Frobenius formula and Zagier bounds	246
Frobenius formula	249
3. Alternative counting of 1-cylinder separatrix diagrams	252
3.1. Approach based on recursive relations	252

Strata of Abelian differentials	252
Strata of quadratic differentials	255
3.2. Approach based on Rauzy diagrams	259
3.3. The example of $\mathcal{Q}(1^3, -1^3)$	261
Appendix A. Impact of the choice of the integer lattice on diagram-by-diagram counting of Masur-Veech volumes	263
Appendix B. (by Philip Engel) Square-tiled surfaces with one horizontal cylinder	267
References	272

RÉSUMÉS DES ARTICLES

<i>Sur la topologie des courbes analytiques réelles au voisinage des points singuliers</i> ÉTIENNE GHYS & CHRISTOPHER-LLOYD SIMON	1
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Le but de cet article est de décrire la topologie des courbes analytiques réelles planes au voisinage d'un point singulier. Localement, une telle courbe est constituée d'un certain nombre de branches qui coupent un petit cercle centré sur la singularité en deux points. La topologie locale est décrite par un diagramme de cordes : un nombre pair de points sur un cercle, associés deux par deux. Nous montrons que la plupart des diagrammes de cordes ne proviennent pas de singularités. Quand c'est le cas nous les qualifions d'analytiques. Nous proposons d'abord une description récursive des diagrammes analytiques. Puis nous caractérisons ces diagrammes analytiques comme étant ceux qui ne contiennent pas un certain nombre de sous-diagrammes que nous explicitons.

<i>Un lemme de fermeture pour les automorphismes polynomiaux de \mathbb{C}^2</i> ROMAIN DUJARDIN	35
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Nous montrons que pour un automorphisme polynomial dissipatif de \mathbb{C}^2 , le support de toute mesure invariante est contenu dans l'adhérence de l'ensemble des points selles, à l'exception de quelques cas bien compris.

<i>Sur les minima des spectres dynamiques de Markov et Lagrange</i> CARLOS GUSTAVO T. DE A. MOREIRA	45
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Nous considérons des spectres dynamiques typiques de Lagrange et Markov associés aux fers à cheval des surfaces. Pour un grand ensemble de fonctions définies sur la surface à valeurs réelles, nous montrons que les minima des spectres dynamiques de Markov et Lagrange coïncident, sont isolés et sont l'image par la fonction d'un point périodique de la dynamique. Cela répond à une question de Jean-Christophe Yoccoz.

Théorèmes limites temporels modifiés pour les rotations du cercle

DMITRY DOLGOPYAT & OMRI SARIG 59

Soit $h(x) = \{x\} - \frac{1}{2}$. On étudie la distribution de $\sum_{k=0}^{n-1} h(x + k\alpha)$ pour x fixé et n tiré au hasard uniformément dans $\{1, \dots, N\}$, quand $N \rightarrow \infty$. Beck a montré dans [2, 3] que pour $x = 0$ et α irrationnel quadratique, ces distributions convergent, après un changement d'échelle approprié, vers une distribution gaussienne. Nous montrons que l'ensemble des α pour lesquels la distribution limite après changement d'échelle existe est de mesure de Lebesgue nulle, mais qu'on a le théorème limite modifié suivant : soit (α, n) choisi au hasard uniformément dans $\mathbb{R}/\mathbb{Z} \times \{1, \dots, N\}$, alors la distribution de $\sum_{k=0}^{n-1} h(k\alpha)$ converge après un changement d'échelle approprié quand $N \rightarrow \infty$ vers la distribution de Cauchy.

Compacité de Gromov pour courbes pseudoholomorphes au sens exhaustif
 JOEL W. FISH & HELMUT HOFER 87

On s'intéresse à la convergence de courbes pseudo-holomorphes vers une variété au sens de Gromov et dans un sens local. On étend cette notion de convergence au cas où la variété cible n'est pas supposée compacte mais recouvrable par une suite de voisinages compacts. On suppose que dans chaque voisinage compact, les courbes considérées ont une aire et un genre borné (ce qui n'est pas le cas au sens global) et on prouve l'existence d'une sous-suite qui converge au sens « exhaustif » de Gromov.

Points fixes des actions nilpotentes de \mathbb{R}^2
 SEBASTIÃO FIRMO & PATRICE LE CALVEZ & JAVIER RIBÓN 113

Nous montrons plusieurs résultats d'existence de points fixes pour des groupes nilpotents de difféomorphismes de classe C^1 du plan \mathbb{R}^2 . Les cas principaux sont ceux de groupes de difféomorphismes de la sphère fixant le point à l'infini, de groupes de difféomorphismes fixant un compact donné du plan, et finalement de groupes de difféomorphismes préservant une mesure de probabilité.

Cocycles projectifs au-dessus d'actions de $SL(2, \mathbb{R})$: mesures invariantes au-dessus du groupe triangulaire supérieur
 CHRISTIAN BONATTI & ALEX ESKIN & AMIE WILKINSON 157

Nous considérons l'action de $SL(2, \mathbb{R})$ sur un fibré vectoriel H , préservant une mesure de probabilité ergodique ν sur la base X . Soit $\hat{\nu}$ un relevé quelconque de ν qui est une mesure de probabilité sur le fibré projectivisé $\mathbb{P}(H)$, invariante sous l'action du sous-groupe triangulaire supérieur. Sous une hypothèse d'irréductibilité de l'action, nous prouvons que toute mesure $\hat{\nu}$ comme ci-dessus est supportée par le projectivisé $\mathbb{P}(E_1)$ de l'espace de Lyapunov associé à l'exposant de Lyapunov le plus grand pour l'action du semi-groupe diagonal positif. Nous en déduisons deux applications : Premièrement, les exposants du cocycle de Kontsevich-Zorich dépendent continûment des mesures affines, ce qui répond à une question de [57]. Deuxièmement, soit $\mathbb{P}(V)$ un fibré projectif irréductible, plat, au dessus d'une surface fermée hyperbolique Σ , et soit \mathcal{F} le feuilletage à

feuilles hyperboliques, tangent à la connection plate ; alors le flot horocyclique sur $T^1(\mathcal{F})$ est uniquement ergodique sous l'hypothèse que le plus grand exposant de Lyapunov du flot géodésique est simple. Ceci généralise le résultat principal de [13] en dimension arbitraire.

Exposants de Lyapunov fibrés pour certains produits-croisés d'endomorphismes du cercle avec points critiques et force quasi-périodique

KRISTIAN BJERKLÖV & L. HÅKAN ELIASSON 181

Dans cet article, nous donnons des exemples de produits croisés $T : \mathbb{T}^2 \rightarrow \mathbb{T}^2$ de forme $T : (x, y) = (x + \omega, x + f(y))$, où $f : \mathbb{T} \rightarrow \mathbb{T}$ est un endomorphisme C^1 explicite de degré deux avec un point critique unique et où ω appartient à un ensemble de mesures positives, dont l'exposant de Lyapunov fibré est positif pour presque tout $(x, y) \in \mathbb{T}^2$. Le point critique est de type $f'(\pm e^{-s}) \approx e^{-\frac{\beta s}{(\log s)^2}}$ pour tout s grand, où $\beta > 0$ est une petite constante numérique.

Récurrence sur les revêtements infinis cycliques

ALBERT FATHI 195

Nous étudions les propriétés de récurrence par chaînes du relevé d'un homéomorphisme à un revêtement infini cyclique. Cette étude est connectée au théorème de Poincaré-Birkhoff, voir les travaux de John Franks [2, 3].

Hydrodynamique sur les réseaux

DENNIS SULLIVAN 215

Nous utilisons la combinatoire de deux réseaux cubiques à faces centrées s'interpénétrant pour construire un calcul vectoriel discret qui permet la construction de deux modèles de l'hydrodynamique incompressible de l'espace tridimensionnel tripériodique, l'une fondée sur la conservation de l'élan et l'autre sur le principe du transport de la vortacité. Sans passer à la limite différentielle, on arrive néanmoins dans le langage du calcul vectoriel exactement aux formulations du modèle continu, d'abord celle de Jean Leray, où la dérivation agit à l'extérieur du terme non linéaire, et ensuite celle plus habituelle où elle agit à l'intérieur. Des études numériques montrent que ces deux modèles diffèrent au niveau discret.

Contribution des surfaces à petits carreaux à un cylindre aux volumes de Masur-Veech

VINCENT DELECROIX & ÉLISE GOUJARD & PETER ZOGRAP & ANTON ZORICH 223

Nous établissons une formule pour la contribution des surface à petits carreaux formées d'un seul cylindre horizontal au volume de Masur-Veech des strates de différentielles abéliennes. Nous en déduisons le comportement asymptotique lorsque le genre des surfaces grandit. À la lumière des résultats récents de Aggarwal et Chen-Möller-Zagier sur l'asymptotique des volumes de Masur-Veech, nous en déduisons que la contribution relative est de l'ordre de $1/d$ où d est la dimension de la strate. De manière similaire, nous donnons une formule pour la contribution

des surfaces à petits carreaux formées d'un seul cylindre horizontal au volume de Masur-Veech des strates de différentielles quadratiques. En combinant cette formule avec nos résultats récents sur l'équidistribution des surface à un cylindre horizontal, nous proposons une méthode empirique pour le calcul des volumes de Masur-Veech des strates de différentielles quadratiques. Cette dernière s'avère être efficace en petites dimensions.

ABSTRACTS

On the topology of a real analytic curve in the neighborhood of a singular point
 ÉTIENNE GHYS & CHRISTOPHER-LLOYD SIMON 1

The purpose of this paper is to describe the topology of real analytic planar curves in the neighborhood of a singular point. Locally, such a curve consists of a number of branches that intersect a small circle centered on the singularity at two points. The local topology is described by a chord diagram: an even number of points on a circle, associated two by two. We show that most chord diagrams do not come from singularities. When this is the case, we call them analytical diagrams. First, we propose a recursive description of analytical diagrams. Then we characterize these analytical diagrams as those that do not contain a number of subdiagrams that we describe explicitly.

A closing lemma for polynomial automorphisms of \mathbb{C}^2
 ROMAIN DUJARDIN 35

We prove that for a dissipative polynomial diffeomorphism of \mathbb{C}^2 , the support of any invariant measure is, apart from a few well-understood cases, contained in the closure of the set of saddle periodic points.

On the minima of Markov and Lagrange Dynamical Spectra
 CARLOS GUSTAVO T. DE A. MOREIRA 45

We consider typical Lagrange and Markov dynamical spectra associated to horseshoes on surfaces. We show that for a large set of real functions on the surface, the minima of the corresponding Lagrange and Markov dynamical spectra coincide, are isolated, and are given by the image of a periodic point of the dynamics by the real function. This solves a question by Jean-Christophe Yoccoz.

Quenched and annealed temporal limit theorems for circle rotations
 DMITRY DOLGOPYAT & OMRI SARIG 59

Let $h(x) = \{x\} - \frac{1}{2}$. We study the distribution of $\sum_{k=0}^{n-1} h(x + k\alpha)$ when x is fixed, and n is sampled randomly uniformly in $\{1, \dots, N\}$, as $N \rightarrow \infty$. Beck proved in [2, 3] that if $x = 0$ and α is a quadratic irrational, then these distributions converge, after proper scaling, to the Gaussian distribution. We show

that the set of α where a distributional scaling limit exists has Lebesgue measure zero, but that the following *annealed* limit theorem holds: Let (α, n) be chosen randomly uniformly in $\mathbb{R}/\mathbf{Z} \times \{1, \dots, N\}$, then the distribution of $\sum_{k=0}^{n-1} h(k\alpha)$ converges after proper scaling as $N \rightarrow \infty$ to the Cauchy distribution.

Exhaustive Gromov compactness for pseudoholomorphic curves

JOEL W. FISH & HELMUT HOFER 87

Here we extend the notion of target-local Gromov convergence of pseudoholomorphic curves to the case in which the target manifold is not compact, but rather is exhausted by compact neighborhoods. Under the assumption that the curves in question have uniformly bounded area and genus on each of the compact regions (but not necessarily global bounds), we prove that a subsequence converges in an exhaustive Gromov sense.

Fixed points of nilpotent actions on \mathbb{R}^2

SEBASTIÃO FIRMO & PATRICE LE CALVEZ & JAVIER RIBÓN 113

We show several results providing global fixed points for nilpotent groups of orientation-preserving C^1 diffeomorphisms of the plane \mathbb{R}^2 . The main cases are namely groups of diffeomorphisms of the sphere such that ∞ is a global fixed point, groups of diffeomorphisms preserving a non-empty compact set and finally groups of diffeomorphisms preserving a probability measure.

Projective cocycles over $SL(2, \mathbb{R})$ actions: measures invariant under the upper triangular group

CHRISTIAN BONATTI & ALEX ESKIN & AMIE WILKINSON 157

We consider the action of $SL(2, \mathbb{R})$ on a vector bundle \mathbf{H} preserving an ergodic probability measure ν on the base X . Under an irreducibility assumption on this action, we prove that if $\hat{\nu}$ is any lift of ν to a probability measure on the projectivized bundle $\mathbb{P}(\mathbf{H})$ that is invariant under the upper triangular subgroup, then $\hat{\nu}$ is supported in the projectivization $\mathbb{P}(\mathbf{E}_1)$ of the top Lyapunov subspace of the positive diagonal semigroup. We derive two applications. First, the Lyapunov exponents for the Kontsevich-Zorich cocycle depend continuously on affine measures, answering a question in [57]. Second, if $\mathbb{P}(\mathbb{V})$ is an irreducible, flat projective bundle over a compact hyperbolic surface Σ , with hyperbolic foliation \mathcal{F} tangent to the flat connection, then the foliated horocycle flow on $T^1\mathcal{F}$ is uniquely ergodic if the top Lyapunov exponent of the foliated geodesic flow is simple. This generalizes results in [13] to arbitrary dimension.

Positive fibered Lyapunov exponents for some quasi-periodically driven circle endomorphisms with critical points

KRISTIAN BJERKLÖV & L. HÅKAN ELIASSON 181

In this paper we give examples of skew-product maps $T : \mathbb{T}^2 \rightarrow \mathbb{T}^2$ of the form $T(x, y) = (x + \omega, x + f(y))$, where $f : \mathbb{T} \rightarrow \mathbb{T}$ is an explicit C^1 -endomorphism of degree two with a unique critical point and ω belongs to a set of positive measure, for which the fibered Lyapunov exponent is positive for a.e. $(x, y) \in \mathbb{T}^2$. The critical point is of type $f'(\pm e^{-s}) \approx e^{-\beta s / (\ln s)^2}$ for all large s , where $\beta > 0$ is a small numerical constant.

Recurrence on infinite cyclic coverings

ALBERT FATHI 195

We study the chain-recurrence properties of the lift of a homeomorphism to an infinite cyclic cover. This is related to the Poincaré-Birkhoff theorem, see the work of John Franks [2, 3].

Lattice Hydrodynamics

DENNIS SULLIVAN 215

We construct a lattice model of 3D incompressible hydrodynamics on triply periodic three space.

It is not possible to have a discrete finite dimensional model of exterior d , wedge product of forms and hodge star [equivalently of vector calculus] satisfying all of the identities used to manipulate the NSE = Navier Stokes Equation.

This means the *same* discretization method applied to different but equivalent versions of NSE should be considered to be *different* when the identities used to prove the equivalence do not all hold for the discrete replacements.

Thus we do not derive the lattice model by directly discretizing the NSE, but rather use the derivation of the NSE based on momentum transfer and creation in small regions of fixed size as in a finite volume method (see reference).

Besides the perspective on discretization mentioned above the new point and the main point is to express the finite scale calculation in terms of optimally chosen operations of combinatorial topology that are discrete analogs of the continuum ones.

Such exist in this special combinatorics consisting of eight translated cubical decompositions of edge size twice the spacing of the basic lattice. The discrete analogs of d , wedge and star will satisfy all of the familiar identities except that the Leibniz rule for d acting on a product is deformed.

The discrete operators are the *coboundary* of algebraic topology, the *Poincare dual cell* operator and the *lattice site* wedge product.

The calculus limit need not be taken and the model closure point happens when one replaces the average of a product [the velocity at a face times its

normal component] by the product of these averages. At a *physical scale* where the velocity is not varying the model would be *physically correct*.

The derived ODE for the lattice velocity vector field resembles the Leray form of NSE with the derivative outside the nonlinear term, but it cannot be manipulated into the other familiar form with the derivative inside the nonlinear term because Leibniz has changed.

Contribution of one-cylinder square-tiled surfaces to Masur-Veech volumes

VINCENT DELECROIX & ÉLISE GOUJARD & PETER ZOGRAF & ANTON

ZORICH 223

We compute explicitly the *absolute* contribution of square-tiled surfaces having a single horizontal cylinder to the Masur-Veech volume of any ambient stratum of Abelian differentials. The resulting count is particularly simple and efficient in the large genus asymptotics. Using the recent results of Aggarwal and of Chen-Möller-Zagier on the long-standing conjecture about the large genus asymptotics of Masur-Veech volumes, we derive that the *relative* contribution is asymptotically of the order $1/d$, where d is the dimension of the stratum.

Similarly, we evaluate the contribution of one-cylinder square-tiled surfaces to Masur-Veech volumes of low-dimensional strata in the moduli space of quadratic differentials. We combine this count with our recent result on equidistribution of one-cylinder square-tiled surfaces translated to the language of interval exchange transformations to compute empirically approximate values of the Masur-Veech volumes of strata of quadratic differentials of all small dimensions.



JEAN-CHRISTOPHE YOCCOZ (1957–2016)

Jean-Christophe Yoccoz was without a doubt one of the great mathematicians of his generation. His work in the field of dynamical systems is of considerable importance and depth and it has shed new light on many problems of this theory. Together with his thesis advisor, Michel Herman, he helped carry the French school of dynamical systems to its highest level yet.

His results covered an impressive number of areas: his construction with P. Arnoux of exotic pseudo-Anosov dynamics (“Arnoux-Yoccoz pseudo-Anosov systems”), his relentless quest for optimal linearization conditions for circle diffeomorphisms and holomorphic germs, his development of a revolutionary combinatorial concept (the “Yoccoz puzzles”) for the fine study of the topology of the Mandelbrot set related to Douady and Hubbard’s MLC conjecture, his detailed investigation with J. Palis and C. G. Moreira of the dynamics of systems obtained by homoclinic and heteroclinic bifurcations, a series of works with S. Marmi and P. Moussa describing the surprising similarities and differences between the linearization problems of circle diffeomorphisms and generalized interval exchange maps... His ease of calculation and the power of his combinatorial analysis were enthralling. His major achievements earned him the Fields Medal in 1994, elected him to the Académie des Sciences, and subsequently led to a professorship at the Collège de France. But honors and distinctions never changed his simple, humble, generous and optimistic view on life.

Throughout his career, he developed close ties with numerous colleagues around the world. Following his military service in Brazil, he would very regularly visit the Instituto de Matemática Pura e Aplicada (IMPA) in Rio de Janeiro. He also continuously influenced the dynamical systems community through his courses in Orsay and at the Collège de France, through the seminar he hosted with H. Eliasson, through the many conferences he organized, and finally through the advice he always gave so generously.

He was looking forward to having his 60th birthday celebrated by a conference, and to have the proceedings on topics that were dear to him appear as a special volume of *Astérisque*. His premature death forced a modification of these plans: his memory was honored in May 2017 during a colloquium organized at the Collège de France. A special issue of the *Gazette des Mathématiciens* published in May 2018 pays tribute to this Mathematician through various texts of reminiscences as well as texts presenting his most important scientific works.

The present two volumes of proceedings extend the scope of the conference and gather research articles which we hope would have pleased Jean-Christophe.

S. Crovisier
R. Krikorian
C. Matheus
S. Senti