

TEXTS & DOCUMENTS

CHARLES HERMITE'S LETTERS TO FRANCISCO GOMES TEIXEIRA

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ABSTRACT. — It is well known that Charles Hermite kept an intense correspondence with many of the world's leading mathematicians of his time. This paper focuses on Hermite's letters to Francisco Gomes Teixeira, a Portuguese mathematician, who exchanged letters with Hermite for more than twenty years.

RÉSUMÉ (TEXTES & DOCUMENTS : Les lettres de Charles Hermite à Francisco Gomes Teixeira)

Il est bien connu que Charles Hermite a maintenu une correspondance intense avec nombre des plus grands mathématiciens de son temps. Cet article est consacré aux lettres d'Hermite à Francisco Gomes Teixeira, un mathématicien portugais ; leur échange a duré pendant plus de vingt ans.

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Mots clefs. — Charles Hermite, correspondance scientifique, Francisco Gomes Teixeira.

1. INTRODUCTION

Francisco Gomes Teixeira (1851-1933) was one of the most important Portuguese mathematicians of the late nineteenth and early twentieth centuries. Having graduated from the University of Coimbra in 1875, he got his doctorate later in the same year, and, in the next year, started teaching at this university. A few years later, in 1884, he moved to the Polytechnic Academy of Porto, which would become the University of Porto in 1911. At this time, he became Rector of the newly established university.

In addition to his scientific works, mostly in analysis, published in both Portuguese and foreign journals, he innovated in the field of teaching, by writing college textbooks with ambitious content, which elevated the level of rigour of his time in Portugal.¹

One may say that Gomes Teixeira's most relevant contribution for the evolution of mathematics in Portugal was the broadening of its scope, from the national to the international level. He received two prizes from the Royal Academy of Sciences of Madrid, in 1895 and 1897, which are representative of this interest in establishing contacts at an international level. The second of these prizes was awarded for a treatise on special curves, *Tratado de las curvas especiales notables, tanto planas como alabeadas*, which was later expanded, translated to French, and republished in 1917, after receiving another prize from the French Academy of Sciences. For more on his life and work, see for instance Vilhena [1936] and Alves [2004].

Along with the activity we have mentioned, there were several aspects which are central to this internationalization of Portuguese mathematics achieved by Gomes Teixeira. One of these was the foundation, in 1877, of the scientific journal *Jornal de Ciências Matemáticas e Astronómicas*, which we will simply refer to as JSMA. For more on this journal, see Saraiva [2014] and Kharlamova [2013], a thesis that studies the importance of the journal in its time.²

The Journal benefited greatly from Gomes Teixeira's intense correspondence with some of the most renowned mathematicians of his time. The letters that Gomes Teixeira received are kept in the Archive of the University of Coimbra, which includes more than two thousand letters, indexed and catalogued by Gomes Teixeira himself. As a contribution to the anal-

¹ The textbooks were *Curso de Análise infinitesimal, cálculo diferencial*, Porto, Typ. Occidental, 1887 and *Cálculo integral*, Porto: Typ. Occidental, 1889, edited and extended in later editions.

² The volumes of this journal are now fully digitized and can be found at www.fc.up.pt/fa/index.php?p=nav&f=html.fbib-Periodico-oa.

ysis of this estate, this paper focuses on his correspondence with Charles Hermite, one of the most represented authors: there are 19 letters from Hermite (some of them photographically reproduced in Alves [2004] and Kharlamova [2013]) which we present here, and comment on for the first time.

2. THE CORRESPONDENCE

Charles Hermite (1822–1901) was known for being a very prolific correspondent. The paper Goldstein [2018] gives a general overview of Hermite's abundant correspondence (thousands of letters written). The paper also notes that since the beginning of the twentieth century, the letters Hermite sent to many mathematicians have been gradually edited—our present paper wishes to continue this trend.

Most of the correspondence received by Hermite was lost in a fire, which unfortunately makes it impossible to confront the letters in the Coimbra archive with the ones sent by Gomes Teixeira. The nineteen letters we present here go from 1872 to 1896, the number of letters for each year is as follows.

Year	72	75	79	81	85	86	88	90	91	92	96
Nb. of letters	1	2	1	2	3	2	2	2	1	2	1

We refer to the numbers of each letter in the catalogue (which can be found in Vilhena [1936]), assigned by Gomes Teixeira. However, our edition follows the chronological order, which does not correspond entirely to the order of the numeration in the catalogue.

Let us remark that there are nine other letters in Gomes Teixeira's archives, referring to the homage organized for Hermite on the occasion of his 70th birthday, in 1892—a subscription was made to present Hermite with a gold medal struck in his honor. Gomes Teixeira was invited by Gösta Mittag-Leffler to be a member of the honor committee for this event. The letters are from Mittag-Leffler, Gaston Darboux, Miguel Merino, Juan Jacobo Duran Loriga, Zoel Garcí de Galdeano, Lauro Clariana Ricart and other undisclosed senders, most of them asking for participation in the homage.

2.1. July 17, 1872

This is letter 141, and is the earliest letter from Hermite that can be found in this archive. Gomes Teixeira was still a university student at this point, finishing his 3rd academic year and starting the 4th. This letter, including a clear reference to the journal he would establish only five years later, reveals that, even as a 21 year-old student, Gomes Teixeira already planned to launch a new Portuguese mathematical journal and did not hesitate to write to one of the most famous mathematicians of his time asking for collaboration.

The paper contained in the letter and which we do not reproduce, is Hermite [1878], on the Frenet-Serret formulas for curves in 3-dimensional space. It was the first paper by a non-Portuguese author published in the JSMA. It can also be found in [Hermite 1917, pp. 508-511].

Monsieur,

Vous m'avez demandé, en commençant la publication de votre Journal des sciences mathématiques et astronomiques, de vous donner ma collaboration ; je viens remplir l'engagement que j'ai pris envers vous en vous adressant la note ci-jointe, concernant un point élémentaire de calcul différentiel.

Veillez agréer Monsieur, l'expression de mes sentiments les plus distingués,

Ch. Hermite

Paris 17 Juillet 1872

2.2. June 10, 1875

This is letter 140 in the archive. In this letter, Hermite thanks Gomes Teixeira for sending him his inaugural dissertation (this is the name used for the Ph.D. thesis, see [Alves 2004, pp. 34ff.]), entitled *Integração das equações às derivadas parciais de 2a. ordem* (Integration of 2nd order partial differential equations). He takes the opportunity to send some of his own publications, without indication of their content.

Monsieur,

Je viens de parcourir la thèse inaugurale que vous m'avez fait l'honneur de m'adresser et quoique le sujet important et difficile que vous avez traité ne rentre point dans le cercle habituel de mes études, j'ai acquis l'assurance que vous avez fait un travail très sérieux et approfondi.

Veillez accepter, Monsieur, les opuscules qui accompagnent cette lettre comme un témoignage de ma sympathie et recevoir l'expression de ma considération distinguée.

Ch. Hermite

Paris, 10 juin 1875

2.3. December 3, 1875

This is letter 765 in the archive, and contains an exercise given by Hermite to his students on the subject of continued fractions, one of Teixeira's interests. The exercise has some interest outside the context of student competitions: it provides a new proof of a result by Johann Lambert, quoted in the letter. Lambert is credited with the first proof that π is irrational (a proof which also uses continued fractions) in Lambert [1768]. This paper also studies powers of e , so this is probably the paper Hermite refers to in the letter.³

We note that the exercise in this letter, with its solution, does not appear among Hermite's contributions to the *Nouvelles annales de mathématiques*, nor is it published in Gomes Teixeira's JSMA.

Monsieur,

Je m'empresse de me conformer à vos intentions en vous accusant réception du mémoire que vous m'avez adressé. Le sujet que vous avez traité est d'une grande importance et touche à des questions dont je me suis moi-même occupé. C'est vous dire que je lirai votre travail avec beaucoup d'intérêt aussitôt que mes devoirs d'enseignement m'en donneront le loisir. J'aurai terminé au mois de Mars mon cours à l'Ecole polytechnique, et si d'ici là je n'ai pas pu trouver le temps d'étudier votre travail, vous pouvez compter qu'à cette époque du mois de Mars je m'en occuperai immédiatement.

Je saisis Monsieur cette occasion pour vous donner communication d'une petite question qui a été le sujet de la dernière composition donnée à mes élèves, et qui se rapporte précisément à la théorie des fractions continues. J'ai proposé de démontrer qu'en posant

$$F(x) = 1 + \frac{x}{1} + \frac{x^2}{1 \cdot 2} + \cdots + \frac{x^n}{1 \cdot 2 \cdots n},$$

de sorte que

$$e^x = F(x) + \frac{x^{n+1}}{1 \cdot 2 \cdots (n+1)} + \text{etc.},$$

la dérivée d'ordre n de l'expression :

$$\frac{e^x - F(x)}{x^{n+1}}$$

a la forme suivante :

$$\frac{e^x \varphi(x) - \psi(x)}{x^{2n+1}},$$

³ As is well-known, Hermite himself proved in 1873 that e is transcendental, Hermite [1873a], see also his allusions to Lambert in two letters he sent the same year to Paul Gordan and Carl Borchardt, Hermite [1873b;c].