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MATHEMATICS AND HYDRAULICS BETWEEN TURIN AND FERRARA IN THE 18TH CENTURY: THE WORKS BY F. D. MICHELOTTI AND T. BONATI

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Abstract. — During the 18th century mathematical studies devoted to hydraulic and hydrodynamics applications become relevant. The importance of studies both from a theoretical and a practical point of view is well documented by a remarkable increase of papers about the motion of waters. Many mathematicians were involved in this research field. They were asked to describe the motion of waters by means of mathematical formulas. Unfortunately, the motion of water can't be described by Euler's equations. That's why in this period many practical experiments were conducted in order to find a better description of the motion of waters in rivers and streams. After a brief overview of the condition of Italian studies about hydraulics and hydrodynamics in the 17th century, we will focus on case studies of Turin and Ferrara. In 1763 the King Carlo Emanuele III of Savoy financially supported the construction of a laboratory for hydraulic experiments in Turin and appointed Francesco Domenico Michelotti (1710–1787), professor of hydraulics at the university of Turin, with these experiments. In the same period another mathematician from Ferrara, Teodoro Bonati (1726–1820), was involved in similar studies and experiments on behalf of the Papal States. The scientific relationship

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Mots clefs. — Mathématiques appliquées au mouvement de l'eau, enseignement de l'hydraulique dans les universités et collèges italiens, consultant en hydraulique, Francesco Domenico Michelotti, Teodoro Bonati, expériences sur l'eau.

between Michelotti and Bonati will be presented in this article, starting from the examination of the correspondence between the two mathematicians in the period 1768–1772, soon after the publication of the first volume of the *Sperimenti idraulici* by Michelotti.

Résumé (Mathématiques et hydraulique entre Turin et Ferrare au XVIII^{e} siècle : les travaux de F. D. Michelotti et de T. Bonati)

Le XVIII^{e} siècle a été marqué par un intérêt croissant pour les études mathématiques concernant les applications hydrauliques et hydrodynamiques. L'importance de ces études, tant d'un point de vue théorique que pratique, est documentée par un grand nombre d'œuvres décrivant le mouvement des eaux par des formules mathématiques. Les équations formulées par Euler ne pouvant rendre compte de la complexité du mouvement des eaux dans les rivières et les canaux, la conduite d expériences pratiques s'est avérée nécessaire. Après une rapide vue d'ensemble des études italiennes concernant l'hydraulique et l'hydrodynamique au cours du XVII^{e} siècle, notre attention se focalisera sur les études menées à Turin et Ferrare. En 1763 le Roi Charles Emmanuel III de Savoie finance la construction d'un laboratoire pour des expériences hydrauliques à Turin tout en confiant sa direction à Francesco Domenico Michelotti (1710–1787), professeur d'hydraulique à l'université de Turin. Au même moment, un autre mathématicien de Ferrare, Teodoro Bonati (1726–1820) mène des études et des expériences au nom de l'État pontifical. Nous examinerons la relation scientifique entre Michelotti et Bonati aux moyen de la correspondance qu'entretiennent les deux hommes entre 1768 et 1772, à la suite de la publication du premier volume de *Sperimenti idraulici* de Michelotti.

1. INTRODUCTION

At the beginning of the 18th century the science of waters represented a new section of mathematical sciences, whose principles and topics came from physics and whose method of research was based on observations and experiments, according to the Galilean tradition from which it derived. In such a discipline two different parts—theoretical and practical—had to "coexist". Two different professional figures corresponded to these two different traditions: the technical consultant, usually a mathematician, who was asked to find solutions to problems of practical hydraulics, and the so-called "perito", an expert, who had to put it into practice. The role of the hydraulic consultant was often played by the most relevant members of the Galilean school. [1]

As far as studies on fluvial hydraulics, Italy has led the field since the 16th century, thanks particularly to the works by Galileo and his school. Since

[1] Many works published in the last twenty years deal with the science of waters in Italy: Fiocca [1998]; Fiocca et al. [2003]. On the theoretical contribution of Italian scientists to the science of waters see Maffioli [1994].

the twenties of the 17th century, Benedetto Castelli (1578–1643) began to study problems related to practical hydraulics. As a consultant for the Papal States he was involved in the debate regarding the regulation of the rivers between the provinces of Bologna and Ferrara. The result of these studies was the publication of the work, *Della misura delle acque correnti* (Rome, 1628). The treatise by Castelli deals with specific issues related to running waters, such as determining the mathematical relationship between the section of a river and its speed, flooding rivers and streams and methods for reducing or preventing floods. The text consists of two parts, the second provides an exact formulation of the law of continuity of fluid motion:

> For ideal fluids, in a continuous current in continuous motion the flow rate is constant in each section. [2]

In the Papal States the main centres for hydraulic studies were Bologna and Ferrara and the main representatives had local origins. Since the end of the 17th century the hydraulic tradition in Bologna was carried on first by Gian Domenico Cassini [3] (1625–1712) and Domenico Guglielmini (1655–1710), then by Eustachio (1674–1739) and Gabriele (1681–1761) Manfredi, Vittorio Francesco Stancari (1678–1709) and later by Giuseppe Venturoli (1768–1846). The origin of the hydraulic tradition in Ferrara may be traced back to Giambattista Aleotti (1546–1636) and Niccolò Cabeo (1586–1650). [4] During the 18th century the main representatives were Romualdo Bertaglia (1688–1763) and Teodoro Bonati (1726–1820). [5]

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[2] Thanks to the Galilean school, Tuscany attracted many scientists from other parts of Italy, such as Castelli, Giovanni Alfonso Borelli (1608–1679), Famiano Michelini (1604–1665), Guido Grandi (1671–1742), Leonardo Ximenes (1716–1786), in addition to local scientists like Vincenzo Viviani (1622–1703), Tommaso Perelli (1704–1783), Vittorio Fossombroni (1754–1844), Vincenzo Brunacci (1768–1818). See Barsanti [1988]; Barsanti & Rombai [1987]; Maccagni [1987].

[3] See Pallotti [1983]. Between the fifties and the sixties of the 17th century Gian Domenico Cassini was the leading actor of the Bolognese delegation. He took part in the conferences in Rome promoted by the Holy Congregation of Waters and wrote many hydraulic papers about the river Reno, who were collected in Collectives [1682]. On the figure of Gian Domenico Cassini see Giuntini [2001–2; 2006–2]. The role played by Eustachio Manfredi as hydraulic consultant for the city of Bologna clearly emerged by his correspondence with Guido Grandi in the period 1701–1738. See Basta [1992]; Forlani [1992].

[4] Giambattista Aleotti worked in Ferrara as architect and hydraulic engineer during all his life. See Fiocca [1998]; Fiocca et al. [2003]. On the Jesuit Niccolò Cabeo see Fiocca [2002].

[5] See Borgato et al. [1992]. Romualdo Bertaglia was a surveyor and hydraulic technician. Since the first decades of the 18th century he took part in many visits to the

As regards the motion of waters, a general theory that enabled a description of even easily observable phenomena did not exist. The lack of valid instruments to measure the velocity of waters contributed to support an empirical and local method. The technicians had to apply some empirical formulas, by adapting them to suit concrete situations. The greatest difficulty, using both equations and empirical formulas, was represented by the whirling motion of waters that flow in rivers and canals. Euler's equations for fluid dynamics, expounded in three essays published in the *Mémoires* of the Berlin Academy in 1757, could not be useful to describe the motion of waters because they referred to ideal conditions and did not consider whirling motions and frictional forces of the water in the river bed. Euler himself was aware of the limits of his work. In his essay about fluvial hydraulics, entitled *Recherches sur le mouvement des rivières*, he said:

> C'est peu de chose ce que les Auteurs ont écrit jusqu'ici sur le mouvement des rivières, et tout ce qu'ils en ont dit n'est fondé que sur des hypothèses arbitraires, et souvent même tout à fait fausses. [...] pour chercher le mouvement de l'eau dans une rivière, il faut abandonner les hypothèses auxquelles on a attaché jusqu'ici toutes les recherches hydrauliques, pour remonter aux premiers principes de Mécanique, par lesquels tous les mouvemens des corps tant solides que fluides sont déterminés. [6]

The motion of waters did not need the knowledge of refined analytical tools (partial differential equations, calculus of variation and so on), that were developed during the 18th century. In 1777 Joseph Louis Lagrange, writing to Anton Maria Lorgna, said that the principles of this discipline were still vague and that there still did not exist a geometric theory on this subject:

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waters on behalf of the city of Ferrara. See [Lugaresi 2014, pp. 213–216]. Also the Duchies of Modena and Mantua had a local tradition in hydraulic studies. Geminiano Montanari (1633–1687), Domenico Corradi d'Austria (1677–1756) and Giambattista Venturi (1746–1822) came from Modena. Geminiano Montanari was professor of mathematics at the university of Bologna from 1664 to 1679, then he moved to Padua where he kept the chair of astronomy and meteors. See Cattelani & Barbieri [1992]. See also [Maffioli 1994, pp. 129–163]. Domenico Corradi d'Austria was hydraulic engineer and mathematician of the Duke of Modena. He was involved in many problems related to the waters. See Pantanelli [1911]; [Pepe 1981, pp. 84–85]. On the figure of the Modenese Giambattista Venturi see Spaggiari [1984].

6 [Euler 1767, pp. 101–102]. In the middle of the 18th century fluid mechanics was organized as autonomous discipline thanks to the works by Daniel Bernoulli, *Hydrodynamica* (1738), Johann I Bernoulli, *Hydraulica* (1742), D'Alembert, *Traité de l'équilibre et du mouvement des fluides* (1744) and *Essai d'une nouvelle théorie de la résistance des fluides* (1752). The major contribution in this field came from three memoirs by Euler that contained the general equations of hydrodynamics for incompressible fluids. See Euler [1757a;b;c].

> With the exception of infrequently used general principles, I have found too much vague reasoning and experience, that cannot be used as a foundation for a strict and geometric theory. This science may be compared to practical medicine, which, despite its importance and the many things that have been discovered in anatomy, chemistry and natural history, has not progressed since the times of Hippocrates, in fact, it may even have regressed. [7]

Lagrange's words clearly shed light on the prevalence of practical point of view in the study of hydraulic matters, as will be explained in the next sections of this work.

2. MATHEMATICAL STUDIES APPLIED TO THE MOTION OF WATERS IN ITALY (17TH-18TH CENTURY)

In Italy subjects related to the regulation of waters had long-standing roots. Just think of the Roman systems of water supply, the building of irrigation and navigable canals or the reclamation of wide marshlands since the Middle Ages.

From a geographical point of view, the main locations of hydraulic interventions were in northern and central Italy. From a political point of view, the main states involved were the Grand Duchy of Tuscany, the Papal States, with particular focus on the provinces of Bologna, Ferrara and Ravenna, the Republic of Venice and to a lesser extent the Austrian Empire, whose territory extended to Lombardy. From a hydraulic point of view, the main interventions dealt with the regulation of some rivers and streams—the prime example was represented by the river Reno, whose path was very irregular in the Po river plane between the provinces of Bologna, Ferrara and Ravenna—, the reclamation of wide marshlands (such as the Pontine marshes in Lazio or some Tuscany wetlands like the

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7 "Fatta eccezione per qualche principio generale la cui applicazione ha raramente luogo, non vi ho trovato che ragionamenti ed esperienze troppo vaghi per poter servire da fondamento ad una teoria rigorosa e geometrica. Fin'ora è di questa scienza come della medicina pratica che, nonostante la sua estrema importanza e nonostante le belle scoperte che sono state fatte in Anatomia, in Chimica, in Storia Naturale ecc. non è più progredita dal tempo di Ippocrate, se anche non è regredita". J. L. Lagrange, *Œuvres*, Paris, Gauthier-Villars, t. XIV, p. 260. A fundamental contribution to the study of the history of fluids mechanics was given by Truesdell in Truesdell [1954]. Some years later, Roger Hahn provided a wide summary of the development of hydrodynamics in the 18th century in Hahn [1965]. General studies on hydrodynamics have been resumed by different authors over the past few years: Blay [2007]; Calero [2008]; Darrigol [2005]; Darrigol & Frisch [2008]. Hydrodynamics in France has been recently studied by A. Guilbaud. See Guilbaud [2007; 2008; 2012; 2013].