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THE *GAṆITAPAÑCAVIṂŚĪ* ATTRIBUTED TO ŚRĪDHARA

Takao Hayashi

Abstract. — This paper provides a detailed study of the *Gaṇitapañcaviṃśī* attributed to the famous eighth-century mathematician Śrīdhara with a view to restore it to its original form. It consists of a revision of the text edited by David Pingree, an English translation of the whole text, and a mathematical commentary.

Résumé (Le *Gaṇitapañcaviṃśī* attribué à Śrīdhara). — Ce document présente une étude détaillée de la *Gaṇitapañcaviṃśī* attribuée au célèbre mathématicien Śrīdhara du huitième siècle en vue de lui redonner sa forme originale. Il se compose d'une révision du texte édité par David Pingree, d'une traduction en anglais de l'ensemble du texte, et d'un commentaire mathématique.

INTRODUCTION

David Pingree discovered a manuscript of a small arithmetical text named *Gaṇitapañcaviṃśī* (hereafter GP) in the library of the Wellcome Institute for History of Medicene, London. The manuscript consists of three folios, but the second folio is missing. The first and the penultimate verses mention the name of the author as Śrīdhara; the last verse here

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occurs also at the end of most of the extant manuscripts of the *Triśatikā* (Tr) written by the eighth-century mathematician Śrīdhara. Pingree has published an edition based on this single manuscript under the title 'The *Gaṇitapañcaviṃśī* of Śrīdhara' [Pingree 1979].

In an earlier paper [Hayashi 1995], I expressed doubts about Śrīdhara's authorship of the GP. My main arguments were: (1) the GP contains more verses (estimated to be about eighty-seven) than its title ('Mathematics in Twenty-Five ⟨Ślokas⟩') and the introductory stanza (*ślokānāṃ pañcaviṃśatyā* 'by twenty-five ślokas') claim; (2) it contains no stanzas of the previously known two works in the same field by Śrīdhara, namely the Tr and the *Pāṭīgaṇita* (PG), which in turn have a number of verses in common (these two points had already been briefly mentioned by Pingree 1979, 888); (3) it contains half a verse of Śrīpati's *Gaṇitatilaka* (GT, 11th century) and eight and a half verses of Bhāskara II's *Līlāvatī* (L, AD 1150) and some of these fit metrically better in the GT and in the L than in the GP; (4) it contains half a verse prescribing a formula for the area of a circle-segment, which Gaṇeśa in his commentary (AD 1545) on the L attributes to his father Keśava and which causes inconsistency in the GP with regard to π; and (5) it uses, in addition to the ordinary numerals, the word-numerals which Śrīdhara avoids in the Tr and PG. My conclusion in that paper was that 'the present GP is not the original work itself (GP_0) of Śrīdhara, although we cannot of course deny the possibility that the GP_0 existed' [Hayashi 1995, 247].

Table 1 shows the number of verses in the Tr, PG, and GP as well as a conjectural number of verses to be added to, or removed from, each chapter or section of the GP in order to get GP_0. This has been determined as follows.

First, I regarded the 'twenty-five ślokas' as referring to the number of the verses for the *sūtras* (rules) only, because it is too small to include the *saṃjñāḥ* (terminology) and the *udāharaṇāni* (examples) or even one of them. Note that the anonymous *Pañcaviṃśatikā* ('⟨Mathematical Text⟩ Consisting of Twenty-Five ⟨Verses⟩', PV) is so called without terminology and examples.

Second, I estimated the number of verses in the chapters and sections which must have been contained in the lost folio as follows: half a verse each for the barter and the selling of living beings in the lost portion of the chapter on 'Three-quantity operation etc.'; one verse each for the interest, the purity of gold, and the investment in the lost chapter on Mixture; one verse each for the sum, the first term, the common difference, and the number of terms of an arithmetical progression in the

lost chapter on Mathematical Series; and one verse each for equi- and inequi-perpendicular quadrilaterals in the lost portion of the chapter on Plane Figures. Thus, the total number of the verses to be restored for GP_0 is twenty, the average being ten verses per page. On the other hand, the extant three pages contain fifty-one verses, the average being seventeen verses per page. The gap can be explained by interpolations in those lost chapters and sections.

Third, I omitted from the extant portion of the GP those verses which have obviously been borrowed from other works. They are designated by 'F + serial number' in Table 2. They are half a verse (F1) of the chapter on Classes (from GT), four (F5-8) of the chapter on Plane Figures (from Keśava and L), and one (F9-10) of the chapter on Shadow (from L). When F7-8 and FE6 are omitted, the GP_0 would be without the rules and examples related to the Pythagorean Theorem which is one of the most basic propositions of geometry. But this would not be so strange as it appears to be at first sight, because the same is also the case with the PV. I also omitted three more verses (F2-4) in the chapter on Classes because the problems they treat are too sophisticated to be included in a small text like GP.

The total number of the verses for *sūtras* of the GP_0 thus obtained is exactly twenty-five without the introductory and concluding verses (S1, 26). The GP_0 still contains poetic meters other than *śloka* in its narrow sense, that is, the Anuṣṭubh meter, (Āryā in 23cd and 26 and Upajāti in 12-3ab), but this difficulty is resolved by taking the word in its broader sense, that is, 'a verse in general.'

The possibility, however, still remains that even the GP_0 is not the original work of Śrīdhara but a counterfeit. The following are the several points unfavorable to Śrīdhara's authorship of GP_0. For the details see the relevant places in the Commentary.

(1) The terminology of the weights and measures of the GP_0 shows affinities with the L, rather than with the Tr and PG, in the use of the second weight system (S3ab), of the monetary unit *dramma* (S5), and of the area unit *nivartana* (S7bcd). (2) The sum and difference in the GP_0 (1ab) is the ordinary sum and difference of positive integers by means of the decimal place-value notation while those in the Tr and PG are the sum of a finite series of the natural numbers and the difference between two of them. (3) In the inverse problem class (10cd–11ab), the GP_0, like the L, includes the case where the unknown number is increased or decreased by its own part, but the PG does not do so (the Tr does not deal with this class at all). (4) The Tr and PG do not contain the optional-quantity operation or the so called *regula falsi*, but the GP_0 (11cd), like the L, does. Moreover,