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POSITIVE THINKING CONCEPTIONS OF NEGATIVE QUANTITIES IN THE NETHERLANDS AND THE RECEPTION OF LACROIX'S ALGEBRA TEXTBOOK

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Abstract. — The beginning of the 19th century witnessed the emergence of several new approaches to negative numbers. New notions of rigour made the 18th century conceptions of negative quantities unacceptable. This paper discusses theories of negative numbers emerging in the Netherlands in the early 19th century. Dutch mathematicians then opted for a different approach than that of their contemporaries, in Germany or France. The Dutch translation (1821) of Lacroix's *Élémens d'algèbre* illustrates the 'Dutch' notion of rigour.

Résumé. — Penser positivement. Conception des nombres négatifs aux Pays-bas et réception du traité d'algèbre de Lacroix. — Au début du XIX^{e} siècle, des attitudes nouvelles par rapport aux nombres négatifs émergent. La notion de rigueur en mathématique se renouvelle, rendant inacceptables les approches qui s'étaient développées au XVIII^{e} siècle. Cet article présente les théories des nombres négatifs qui avaient cours aux Pays-bas au début du XIX^{e} siècle. Les mathématiciens néerlandais optaient pour une conception différente de celles de leurs contemporains en Allemagne et en France. La traduction néerlandaise des *Elémens d'algèbre* de Lacroix illustrera cette approche « néerlandaise » de la notion de rigueur.

Mathematical theories developed around 1800 were not readily accepted, and mathematics in the various European countries met with rather different epistemological approaches. Different institutional contexts, where mathematical knowledge was pursued in France and Germany for instance, gave rise to different notions of rigour [Schubring 1996]. Sylvestre F. Lacroix's textbooks provide examples to test this assertion, as they were very popular in France and kept in line with the ideas of

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rigour which prevailed there at the time. Moreover, they were translated into several European languages and during the process of translation the texts were adapted to fit other standards of rigour. This paper discusses the Dutch translation of Lacroix's algebra textbook and reveals which ideas on negative numbers then prevailed in the Netherlands. A map of the European background and 18th century Dutch works on negative numbers come first in order.

1. EUROPEAN BACKGROUND

During the 18th century many continental mathematicians began viewing algebra as a universal language. While the British chose a more careful approach to algebra and adopted the geometrically inspired Newtonian theory of fluxions, Leibnizian calculus, on the other hand, is often the exponent of the continental faith in results obtained from algebraic manipulations. The proof of the rule $\mathrm{d}y/\mathrm{d}x = (\mathrm{d}y/\mathrm{d}z)(\mathrm{d}z/\mathrm{d}x)$, for instance, relies heavily on the notation which is chosen (*cf.* [Grabiner 1990] and [Ferraro 1998]). Negative quantities in those days were regarded as "less than nothing", metaphorically linked to debt, as opposed to possession. By the end of the 18th century mathematicians generally agreed that this view of negative quantities was no longer befitting.[1]

The British university curriculum stimulated a stark focus on foundations, by regarding mathematics as the paradigm of human reasoning. Mathematicians worried that algebra somehow fell short of geometry as a logic [Pycior 1981]. Rethinking algebra (or treating it anew from the field and concentrating on geometry) was the obvious thing to do in Britain [Pycior 1997, pp. 309–310]. Francis Maseres, in his 1758 dissertation [Maseres 1758], denounced the way negative quantities were defined in algebra. By the end of the 18th century Maseres and William Frend seriously attacked the use of negative quantities and proposed to cast algebra as a general arithmetic in the strictest sense [Frend 1796; Maseres

[1] According to Schubring the problem with accepting negative numbers had to do with mathematicians moving from the concept of quantity, as something feasible, of a physical nature, e.g. an object or its weight, or trajectory, to a more theoretical concept of number [Schubring 1986, pp. 7–10]. 'Feeling' the need to accept this abstraction or reject it and cling to some sense-related interpretation might have been the main stake in the different 19th-century attempts to rigorize mathematics in general and algebra in particular.

1800]. There, only operations which are meaningful in ordinary arithmetic are admitted and a negative solution to an equation is regarded as a solution to another problem. Many British scholars did not abandon algebra in such a radical way. Frend and Maseres were not a part of the establishment at the time (partly due to their religion, *cf.* [Pycior 1987]), but their work did help to set the stage for a rethinking of algebra as a whole [Pycior 1997, pp. 307–316]. The work by George Peacock [Peacock 1830, 1833] and its reception by Augustus de Morgan, however severely critized, even ridiculed at first [Pycior 1982a,b], introduced to the British a more abstract approach to algebra in which the manipulation of symbols obeyed a set of arbitrary laws – that, of course, 'happened' to be the laws for elementary arithmetic.

Mathematics played a marginal role at the French universities. By the 1750s, however, military education played a serious role and attracted able students. In military education mathematics was a subject taken very seriously, but in general teachers did not want to frighten students away by giving them difficult problems, and such were foudations of negative numbers [Schubring 1996, pp. 365–366]. Apart from the military schools there existed in France a broad intellectual élite whose reception of negative numbers was more ambiguous. On the one hand, the *Encyclopédie* contained an article by d'Alembert who thought that negative quantities – as quantities less than zero – were absurd (since $1 : -1 = -1 : 1$, which meant that the larger was to the smaller in the same proportion as the smaller to the larger). On the other hand, the *Encyclopédie* sported an article that simply defined negative quantities as less than zero.

The point of view expressed by d'Alembert is interesting in that it influenced French mathematicians then and later. According to d'Alembert a proportion such as the above only makes sense in absolute values; the signs do not affect the quantities themselves, but only the 'state' they are in, which is not conceivable in an abstract sense, but only in relation to a specific problem. For example, someone owing another person -3 écus was intelligible, but "-3 pris abstraitement ne présente à l'esprit aucune idée".[2] The establishment in 1794 of the *École Polytechnique* and the French project of making mathematical knowledge more elementary and appealing to the beginner encouraged rethinking mathematics from a

[2] Literally from the *Encyclopédie*, v. 11, p. 73. Quoted from [Schubring 1986, p. 9].

different perspective. This is how a new concept of negative numbers was fostered [Schubring 1982, 1988].

In his posthumous treatise *La langue des calculs* (1798) E.B. de Condillac claimed that algebra was the result of a number of abstractions severed from simple counting and believed that d'Alembert was wrong in denying the existence of negative quantities [Condillac 1981]. For Condillac, algebra served as the fundament of mathematics [Schubring 1986, pp. 10–12]. D'Alembert nevertheless found a follower in Lazare Carnot, who reinforced the rejection of negatives and his ideas about negative quantities were to become very influential. He denied to algebra the fundamental role it played in Condillac's view, restricting it to a mere translation of geometrical propositions [Carnot 1801]. Subtraction as a general algebraic operation was thereby unacceptable: $a-b$ is meaningful only in case $a > b$. Being convinced that geometry predominated algebra he replaced the notion of negative quantity by the direct and inverse lines which were the basic notions of the geometrical framework which he proposed, instead of the algebraic theory of negative quantities, in his *Géométrie de position* [Carnot 1803]. Reckoning with negative numbers is an operation which Carnot considered not rigorous and thus he made algebra subordinate to geometry by basing (among other things) the theory of negative quantities on geometrical constructions [Dhombres 1997, pp. 514–519].

Carnot's ideas found their way into the textbooks of S.F. Lacroix. His textbook on elementary algebra (itself a new edition of a much older textbook by Clairaut), was one of the most popular elementary textbooks published in late 18th century France. After having read Carnot's *Géométrie de position* Lacroix revised his textbook – which he did many times – to fit this new point of view [Schubring 1996, pp. 368–369]. In this revised edition he avoided the use of negative numbers. When a problem (an equation) resulted in a negative solution, Lacroix simply re-formulated the initial problem (this topic will be further elaborated on in section 4). This made solving equations a highly complicated and very annoying business.

Unlike France, the Germanic countries had no significant system of military schools. Almost all scientific activity took place at the universities, where mathematics was represented by particular professorships within a philosophical faculty. During the second half of the 18th century these professors began to view mathematics as an interesting subject on its own accord, and started performing research. The foundations of mathematics became a very popular subject [Schubring 1991]. Euler had regarded negative (abstract) numbers as existing, though he did not pay much attention to them. He simply viewed multiplication with a negative quantity as changing the quantities' 'state' and, when solving equations, he simply mentioned – or did not mention – negative solutions, depending on the domain of his problem [Euler 1773, vol. 1, p. 21; vol. 2, pp. 72–82].[3]

Negative numbers became mathematised in a different fashion by German professors than by their French colleagues. Based on the philosophical notion of opposition, quantities were conceived as provided not only with a quantitative, but also with a qualitative (positive or negative) attribute. Thus, for every quantity a there was also a quantity $\overline{a}$ of the same nature, but of an opposed quality, such that $a + \overline{a} = 0$. Matthias Metternich, in 1811, published a German translation of the algebra textbook by Lacroix [Lacroix 1811] – mandatory literature since at the time his hometown Mainz was within French territory. He unabashedly adapted Lacroix's text to the German point of view, correcting Lacroix with footnotes, omitting or adding several paragraphs on the theory of negative quantities [Schubring 1996, pp. 367–369]. During the early 19th century, the research paradigm extended to the gymnasia, where able and properly trained mathematicians taught. Gymnasium teachers contributed their part to the development of foundations of negative numbers [Förstemann 1817] as well as to the advancement of many other fields: they worked in a context that valued formal mental training and thus strove for clarity in the foundations. This led to a fairly wide acceptance of mathematical rigorisation in the German states: engineers like A.L. Crelle and J.A. Eytelwein felt the need for a rigorous theory in mathematics as much as did the university professors, while in France two contrasting views (the technicians *versus* the theoretically minded mathematicians) competed until the late 19th century [Schubring 1996, pp. 371–373].

During the first half of the 19th century algebra evolved from a technical subject concerned with solving equations to a much wider

[3] The Dutch translation of the algebra textbook is (at least at this point) in accordance with the German version of the *Vollständige Anleitung zur Algebra* from 1771 to which I don't have easy access to verify the page numbers.