Revue d'histoire des mathématiques,
7 (2001), p. 277–293.

NOTES & DÉBATS

MATHEMATICS AND MORALITY ON THE CUSP OF MODERNITY

Peter DEAR (*)

ABSTRACT. — This note suggests that a fruitful way of investigating the history of mathematics lies in consideration of its pedagogical purposes. As a general illustration of the directions that such an approach might take, the paper discusses early-modern arguments for the practical utility of mathematics and its capacity to inculcate good habits of thought, as well as the appearance of new uses for mathematical training in the later eighteenth and early nineteenth centuries that served the purpose of the modernizing state, with its characteristic emphasis on impersonal criteria of evaluation and assessment of individuals. The paper encourages an understanding of mathematical pedagogy that refuses to treat it as unproblematic, and that seeks answers in social and cultural history.

RÉSUMÉ. — MATHÉMATIQUES ET MORALITÉ À LA POINTE DE LA MODERNITÉ. — Cette note suggère qu'une façon féconde d'étudier l'histoire des mathématiques est de considérer les visées pédagogiques de ces dernières. Afin d'illustrer les grandes orientations qu'une telle approche peut définir, l'article étudie les arguments qui ont été mis en avant au début de l'époque moderne en faveur de l'utilité pratique des mathématiques et de ses capacités à inculquer de bonnes habitudes de pensée. Il examine aussi l'apparition à la fin du XVIII[e] et au début du XIX[e] siècles de nouveaux usages pour l'éducation mathématique, qui servent les intérêts de l'État en cours de modernisation, avec l'accent mis de manière caractéristique sur les critères impersonnels d'évaluation des individus. L'article vise une approche de la pédagogie mathématique, qui refuse de la traiter comme non problématique et qui cherche des réponses dans l'histoire sociale et culturelle.

The meaning of mathematics as a *pedagogical* discipline in the seventeenth and eighteenth centuries is one that seems strangely under-investigated. Perhaps this is because of an assumption that mathematics is a good thing to teach, presumably because of its associations with the rise of modern science. But there are other, more positive aspects of the teaching of mathematics in the early-modern period — a period in which the

(*) Texte reçu le 20 août 2001, révisé le 15 février 2002.
Peter DEAR, History and Science & Technology Studies, Cornell University, Ithaca, NY 14853 (USA). Courrier électronique : prd3@cornell.edu.

Keywords: mathematics, education, history, Europe.

AMS classification: 01A73, 01A40, 01A45, 01A50, 97-03.

nineteenth- and twentieth-century ideologies of modernity had not yet been formed, and in which the value of an education in mathematics had to be argued for and against in characteristically *pre*-modern terms. Early-modern mathematical pedagogy needs to be understood in the terms of its purported contributions to more dominant pedagogical aims of the period. Such aims related to the formation of good character — mathematics as a contributor to proper ways of behaving and thinking — and to broadly humanist concerns with mathematics as a source of practical utility for the good of the state. In general, therefore, it will be valuable to examine arguments that presented mathematics as a program for the development of moral virtue, whether individual or civic. This essay attempts a brief overview of some of the issues that may emerge from such an examination.

First of all, it should be understood that "mathematics" here refers to those disciplines that were regarded as constituting mathematics in this period itself. The model of the medieval quadrivium still held sway, a model comprising arithmetic, geometry, astronomy, and music. Privileging the first two, the branches of so-called "pure" mathematics, would do violence to the understanding of the category that predominated in the academic world of early-modern Europe. Mathematics was a way of doing things as much as it was a particular domain of knowledge; it proceeded by techniques of demonstration and construction, and it was concerned with magnitudes, whether abstract or embodied in matter. The domains in which mathematics was used themselves contributed to the value of a mathematical education. In the eighteenth century, in D'Alembert's *Discours préliminaire* to the *Encyclopédie*, we read the following concerning the physico-mathematical science of astronomy, the study of which

"est la plus digne de notre application par le spectacle magnifique qu'elle nous présente. Joignant l'observation au calcul, et les éclairant l'un par l'autre, cette science détermine avec une exactitude digne d'admiration les distances et les mouvemens les plus compliqués des corps célestes; elle assigne jusqu'aux forces mêmes par lesquelles ces mouvemens sont produits ou altérés. Aussi peut-on la regarder à juste titre *comme l'application la plus sublime et la plus sûre de la géométrie et de la mécanique réunies*; et ses progrès *comme le monument le plus incontestable du succès auquel l'esprit humain peut s'élever par ses efforts*" [D'Alembert 1821, p. 27].

This passage, from 1751, in a new, Newtonian universe, still sounds remarkably similar in spirit to corresponding passages from Plato and Aristotle, who also praised astronomy due to the “nobility” of its object, the heavens. D’Alembert was able simply to augment that judgement with appeals to the precision attainable by the new, physico-mathematical[1] science of Newtonian celestial mechanics; astronomy is still, nevertheless, suitably described by words such as “magnificent” and “sublime” [*ibid*].

I. MORAL WORTH AND INTELLECTUAL VALUE

Around the beginning of the seventeenth century, the prominent Jesuit mathematician and pedagogue Christopher Clavius had repeated an even more conventional praise of astronomy in his widely-used textbook on the subject, his commentary on Sacrobosco’s *De sphaera*. Astronomy, he says [Clavius, *Opera* 3, p. 3], is the noblest of all the mathematical disciplines, because it fulfills Aristotle’s criteria of excellence better than any other: not only does it use demonstrations from geometry of the greatest certainty, but it also deals with the most noble subject-matter, namely the heavens. Nobility, a moral evaluation, played a major role in Clavius’s promotion of the mathematical sciences as a whole. Clavius wrote the following as part of his enormously influential attempts to raise the status of mathematical teaching in the Jesuit colleges:

“Since therefore the mathematical disciplines in fact require, delight in, and honor truth — so that they not only admit nothing that is false, but indeed also nothing that arises only with probability, and finally, they admit nothing that they do not confirm and strengthen by the most certain demonstrations — there can be no doubt that they must be conceded the first place among all the other sciences.”[2]

Clavius was the prime mover in encouraging the teaching of mathematics as part of the curriculum in the European-wide network of Jesuit

[1] On “physico-mathematical”, see [Dear 1995, chap. 6].

[2] Clavius, “In disciplinas mathematicas prolegomena”, in [Clavius, *Opera* 1, p. 5]: “Cum igitur disciplinae Mathematicae veritatem adeo expetant, adament, excolantque, ut non solum nihil, quod sit falsum, verum etiam nihil, quod tantum probabile existat, nihil denique admittant, quod certissimis demonstrationibus non confirment, corroborentque, dubium esse non potest, quin eis primus locus inter alias scientias omnes sit concedendus.” My translation.

colleges, and the moral status of mathematical knowledge and its use clearly played an important role in the techniques by which Jesuit mathematicians continued to promote their subject in the colleges during the course of the seventeenth century.

The most important elaboration on Clavius's apologia for mathematics was written by a former student of his, Giuseppe Biancani, in a text of 1615, *De natura mathematicarum.* While largely a work of epistemology, the text makes powerful use of moral evaluations. Biancani describes earlier claims (including, especially, those of certain Jesuit philosophers) that attempted to downgrade the status of mathematical knowledge as "calumnies", and, like Clavius, he protests indignantly against them. Plato is a useful resource here; Biancani quotes Ficino on Plato's position concerning the educational value of mathematical training. Plato's Academy, of course, was said to have used the motto "Let no one ignorant of mathematics enter here", and Biancani writes the following:

"Therefore Socrates rightly said in the *Republic* that while the mind's eye is blinded, indeed, is gouged by other pursuits, the mathematical disciplines restore it and elevate it to the contemplation of Him Who Is, and from the imitations to the true things, for the beauty and order of mathematical reasonings, and the firmness and stability of contemplation join us and perfectly attach us to the intellects, which always remain the same, shine together with divine beauty, observing their mutual order."[3]

Another point that Biancani borrows from Clavius concerns the criticism that mathematics is inferior to other disciplines, and is not a true part of philosophy, because it "abstracts from the good" — that is, it fails to concern itself with "the good." Biancani cites Aristotle's *Metaphysics* in response, where Aristotle writes that "those who claim that mathematics says nothing about good or the beautiful speak falsely, for it does say, and it does show a great deal about them; for even if it does not mention them by name, by showing the works and reasons [of the good and the beautiful], does it not say anything about them? For the species of beauty are order, symmetry and shapeliness, which are shown especially

[3] Translation adapted from [Mancosu 1996, p. 198]. This text is discussed more fully in [Dear 1995, chap. 2]. See also, on the general issue of the contemporary controversy over the scientific status of mathematical knowledge, [Jardine 1988, pp. 685–711], with many further references.

by mathematical sciences."[4]

There is much more along similar lines, including a description of algebra as equalling "no human ingenuity, but what you would rather call heavenly revelation" [*Ibid.*, p. 205]. And, naturally, Biancani notes that mathematics is relevant to things mentioned in the Scriptures [*Ibid.*, p. 207]. In general, Biancani, like Clavius, is concerned to stress the certainty of mathematical demonstrations in relation to the Aristotelian ideal, and in fact to characterize them as "perfect" demonstrations — a technical term, to be sure, that described the fact that they fitted all of Aristotle's criteria for demonstration; but also one that carried a valuable rhetorical function, in associating mathematics with perfection itself.

Biancani's text quickly became in the seventeenth century a standard source for discussions of the nature of mathematical knowledge, not just among Jesuit mathematicians but among mathematicians in general, including Protestant mathematicians (who were hardly able to ignore the widespread and influential Jesuit writings on the subject). So this Jesuit doctrine on the value of mathematical studies as part of a thorough liberal education was a widely-known attempt at selling mathematics for educational purposes, and resembles in many ways the by-then standard arguments for the moral value of a regular *humanist* education.[5] In that respect, it is of course no coincidence that Biancani made a point of citing classical authorities like Plato, whose pronouncements were largely irrelevant to the technical philosophical views of Aristotle on mathematics and demonstration.

Nonetheless, those Aristotelian views were of fundamental importance. In using mathematics (primarily geometry) to shape his account of deductive axiomatic systems in the *Posterior Analytics*, Aristotle had attempted to lay out the formal structure of any ideal science whatsoever, regardless of its subject matter. In the later sixteenth and seventeenth centuries, it had become a widely held belief that this Aristotelian deductive structure was in fact a representation of the best way of teaching a subject (this is also the view of most present-day scholars of Aristotle's philosophy). At the same time, the reverse of this kind of deductive inference, often referred to as "analysis," was held to be the best way

[4] Translation Mancosu (adapted), see [Mancosu 1996, p. 202].

[5] See on this subject [Grafton and Jardine 1986].