

NUMBER THEORY IN THE
NOUVELLES ANNALES DE MATHÉMATIQUES (1842–1927):
A CASE STUDY ABOUT MATHEMATICAL JOURNALS
FOR TEACHERS AND STUDENTS

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ABSTRACT. — The *Nouvelles annales de mathématiques* were a French mathematical journal, published between 1842 and 1927, intended for teachers and students. In this paper, I rely on a systematic analysis in the *NAM* of number theory content—a mathematical field that was virtually absent from French education programs during the period under consideration here. By articulating quantitative and qualitative approaches, my goal is twofold: to study what specific forms number theory takes through this specific *media* on the one hand; and to question the specific character of the functioning of a journal like the *NAM* in the case of number theory on the other. For this, I take into account the actors involved (editors, authors, readers), the various forms of texts published, and the themes studied, as well as the arithmetical practices and discourses that are brought to bear.

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RÉSUMÉ (Une étude de cas sur les journaux mathématiques pour enseignants et élèves : la théorie des nombres dans les *Nouvelles annales de mathématiques* (1842–1927))

La revue *Nouvelles annales de mathématiques* était un journal mathématique français, publié entre 1842 et 1927 à destination des enseignants et des élèves. Cet article repose sur l'analyse systématique dans les *NAM* des contenus de théorie des nombres — domaine quasiment absent des programmes d'enseignement français pendant la période considérée. En articulant approches quantitative et qualitative, mon objectif est double : étudier quelles formes spécifiques prend la théorie des nombres à travers ce *media* spécifique d'une part ; interroger la spécificité du fonctionnement d'un journal comme les *NAM* dans le cas de la théorie des nombres d'autre part. Pour cela, je m'appuie sur les acteurs impliqués (rédacteurs, auteurs, lecteurs), la diversité des formes éditoriales en jeu, les thématiques étudiées ainsi que les pratiques arithmétiques et les discours mobilisés.

1. INTRODUCTION—WHY STUDY NUMBER THEORY IN THE *NOUVELLES ANNALES DE MATHÉMATIQUES*?

The growth of periodicals containing mathematics since the 18th century has participated to the « restructuring of the mathematical communication system » [Gispert et al. 2014].¹ Since the 1840s, the rapid development of an « intermediate press » [Ortiz 1994], i.e., journals for teachers and students, contributed to this restructuring. These journals offered mathematical content that was related to education or that was considered as « elementary », and therefore accessible to the target readership. They contained multiple editorial forms, from original articles to reviews of textbooks, descriptions of the contents of the curricula of teaching institutions, and mathematical questions intended for the training of the candidates. These intermediate journals were used by various authors. For example, after the *Comptes rendus hebdomadaires de l'académie des sciences* (CRAS), the intermediate journal *Nouvelles annales de mathématiques* was the principal

¹ Since the 1990's, several works on history of mathematics have been devoted to the study of journals or have taken into account the specific form of mathematical journals in the analysis of a mathematical topic. For example, the scientific project Cirmath, funded by the *Agence nationale de recherche* between 2014 and 2019 and led by Hélène Gispert, Philippe Nabonnand and Jeanne Peiffer, studies the circulation of mathematics, especially through journals, over a long period of time (1700–1950). Two thematic volumes were recently published in this context: « Échanges et circulations mathématique. Études de cas (18^e-20^e siècles) », *Philosophia scientiæ* (19(2), 2015) and « Interplay Between Mathematical Journals on Various Scales, 1850–1950 », *Historia Mathematica* 45(4), 2018.

journal where the members of the *Société mathématique de France* (*SMF*) published mathematics [Gispert 2015].

The aim of this paper is to study a mathematical domain—number theory—through the systematic analysis of a journal of mathematics for teachers and students—the *Nouvelles annales de mathématiques* (*NAM*)—in order to identify the various forms taken by number theory in this specific context.² The *NAM* is here used as a valuable observation point in order to study the means of circulation of number theory in a *milieu* linked a priori to mathematical teaching, by articulating quantitative et qualitative approaches. Reciprocally, the results will be used to question if a teaching mathematical journal such as the *NAM* operates in a specific way through the number-theoretic lens.

The *NAM* was published between 1842 and 1927, in 84 volumes with approximately 11370 contributions by 1860 identified authors.³ It thus constitutes a relevant way to analyse mathematical content for a so-called intermediate public over a relatively long time. Its explicit target readership was students who were preparing for the admission examination of the *École polytechnique* and the *École normale supérieure* (*ENS*). From 1888, students studying for *licence* and *agrégation* were also mentioned as expected readership in the journal.⁴ The *NAM* had a unique position in the French publishing landscape for the first 25 years of its existence. However, from 1877, the growth of the number of students spurred the creation of other periodicals with the same aims, in France and abroad. With the multiple evolutions of mathematical, institutional and editorial contexts, the mathematical content, the authorship and readership of the *NAM*, and the associated dynamic of circulations changed sensibly during its publishing period. The

² All the *NAM* volumes are digitised and available at the following address: <http://www.numdam.org/journals/NAM/>

³ By identified authors, I mean authors who signed their texts even if I could not identify their profession or their birth date for example.

⁴ The *Licence* is one of the degrees awarded by the *Université* in France and the *agrégation* is the competitive examination by which French teachers were recruited. It is difficult to estimate the number of potential student readers of a journal like the *NAM* over the entire period considered, but some quantitative data are nevertheless available in the secondary literature. The number of students in preparatory classes increased throughout the 19th century [Nabonnand & Rollet 2013, 1] and Bruno Belhoste estimated that there were at least 700 students in *mathématiques spéciales* in public preparatory education in 1843 [Belhoste 2001]. In addition, between 1875 and 1913, the number of students was just over 500 per year at the *École polytechnique*, between 500 and 800 at the *École centrale*, between 40 and 70 at the *ENS* and between 120 and 5800 (following major university reforms: see below) in the science faculties [Lundgreen 1980, 328–329].

choice of the *NAM* is also based on the existence of a collective research work engaged in a global study of the *NAM* and led by H el ene Gispert, Philippe Nabonnand and Laurent Rollet, which permits us to put the results obtained for a specific domain in perspective.⁵

Number theory represents an interesting case study for several reasons. First, number theory had a marginal position in the mathematical literature during the period considered here: as an example, between 1870 and 1914, the average proportion of pages on number theory in the German reviewing journal *Jahrbuch  uber die Fortschritte der Mathematik* was between 3.5 and 4% [Goldstein 1999, p. 196]. In a certain way, this marginality allows us to develop a systematic study of every occurrence of number theory because the corresponding corpus is of a reasonable size. Secondly, and related to the previous point, number theory had an even more marginal position in French curricula during the period. As the content of the *NAM* is known not to be limited to the curricula for mathematics in French schools, it is interesting to see how a field like number theory was treated in this type of medium. Thirdly, if number theory was mostly absent from the curricula, the borderline between number theory and algebra, which was useful for students, was often blurred. Furthermore, number-theoretical statements could be elementary and thus met the requirements of the *NAM* editors, namely to publish elementary content, « at the scope, at the level of the students ». ⁶ This specificity of number theory was also commented upon by one of the editors of the *NAM* in 1900, Raoul Bricard. In his review of a treatise on number theory, he recalled that « number theory frequently has statements of striking simplicity, intelligible almost without mathematical initiation, even when their demonstration has required the efforts of the most subtle invention » and that « the newcomer, bringing together the conditions of knowing how to read, having a mathematical mind and a taste for the pleasures of the abstract, can approach number theory and take part, after a few days, in the highest speculations of the human mind » [Bricard 1900, p. 477].⁷

⁵ One result of this group research project is a database <http://nouvelles-Annales-Poincare.univ-lorraine.fr/> containing the number of entries published by every author and for each year, and a collective work on the *NAM* is currently being written. I will specify the methods and content of this database below.

⁶ «   la port ee, et   la couleur des  l eves ». This quote is issued from a letter from Terquem to Eug ene Catalan dated August 31, 1849, and reproduced in [Verdier 2009, p. 252]. Unless otherwise indicated, all the translations are mine.

⁷ « [...] les propositions de th eorie des nombres ont fr equemment des  nonc es d'une simplicit e frappante, intelligibles presque sans initiation math ematique, m eme quand leur d emonstration a exig e les efforts de l'invention la plus subtile [...] Le pre-

In this paper, I focus on the multiple mathematical circulations, among authors, journals, articles, methods, and results, that emerge from the analysis of the number-theoretical content of the *NAM*, between authors, journals, articles, methods, results. I mainly rely on a corpus obtained from a systematic analysis, page by page, of the *NAM* during the period of its existence and on the recent historiography on number theory, mathematical teaching and mathematical periodicals. I will occasionally sketch out comparisons between two types of mathematical journals: French intermediate journals created from the end of the 1870s and German journals whose target audience was quite similar.⁸

2. WHAT WAS NUMBER THEORY? SOME PICTURES BETWEEN 1800 AND 1930

Before considering number theory in the *NAM*, it is important to question what the term “number theory” referred to over the period considered, particularly in France. This puts into context the results obtained for the *NAM*. Furthermore, the definitions of number theory were multiple according to the period and the mathematical communities involved. The consideration of these different definitions was the basis for the construction of the database used here.

Around 1800, two books dedicated to number theory were published, proposing two very different definitions of number theory. Adrien-Marie Legendre’s *Essai sur la théorie des nombres* [Legendre 1798] was a synthesis of results conjectured and/or proved by Pierre de Fermat, Leonhard Euler or Joseph-Louis Lagrange and is focused on indeterminate analysis, identified by Legendre with number theory. In Carl Friedrich Gauss’ *Disquisitiones arithmeticae* [Gauss 1801], number theory was defined as the study of the domain of integers. The book was organised according to two main arithmetical objects: congruences and forms. In the first quarter of the nineteenth century, mainly the algebraic content of Gauss’ work—the algebraic solution of binomial equations $x^n = 1$ —was considered; thus its influence touched mostly algebraic texts. Between 1825 and the 1860’s, a research domain, entitled *Arithmetic Algebraic Analysis* in [Goldstein & Schappacher 2007a], was developed by an international network of schol-

mier venu, réunissant les conditions de savoir lire, d’avoir l’esprit mathématique et de goûter les jouissances abstraites, peut aborder la théorie des nombres et prendre part, après quelques jours, aux spéculations les plus élevées de l’esprit humain. »

⁸ The various journals consulted and their abbreviations are listed in the appendix: see page 64.