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Induced representations and classifications for $GSp(2, F)$ and $Sp(2, F)$

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Résumé. Soit F un corps p-adique de charactéristique différente de 2. On caractérise la réducibilité des représentations de $GSp(2, F)$ et $Sp(2, F)$ qui sont induites paraboliquement par des représentations irréductibles. On donne aussi une classification (modulo les représentations cuspidales) de différentes classes de représentations irréductibles de ces groupes. Un cas spécial est la classification des représentations irréductibles unitaires.

Abstract. Let F be a p-adic field whose characteristic is different from 2. The reducibilities of the representations of $GSp(2, F)$ and $Sp(2, F)$ which are parabolically induced by the irreducible representations are described. We obtain also classifications (modulo cuspidal representations) of various classes of irreducible representations of these groups. In particular, the classification of the irreducible unitary representations is obtained.

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Introduction

Let F be a p-adic field. We shall assume that the characteristic of F is different from two. Denote by R the direct sum of the Grothendieck groups of the categories of all smooth representations of finite length of the groups $GL(n,F)$'s. The functor of the parabolic induction defines a multiplication $\times$ on R. In this way R becomes a ring ([Z1]). Obviously, one can define an additive mapping

$$m : R \otimes R \to R$$

which satisfies $m(r_1 \otimes r_2) = r_1 \times r_2$. A comultiplication

$$m^* : R \to R \otimes R.$$

is defined in [Z1]. The definition of the comultiplication involves the Jacquet modules for the maximal parabolic subgroups. In this way R becomes a Hopf algebra ([Z1]). This structure can be very helpful in the representation theory of the groups $GL(n,F)$. Some examples of the use of this structure can be found in [Z2] and [T2]. The crucial property of this structure is that the mapping $m^* : R \to R \otimes R$ is multiplicative. In the other words, we have a simple formula for the composition

$$m^* \circ m.$$

Let $R(S)$ (resp. $R(G)$) be the sum of the Grothendieck groups of the categories of the smooth representations of finite length of the groups $Sp(n,F)$'s (resp. $GSp(n,F)$'s). Using the functor of the parabolic induction one can define a structure of R-modules on $R(S)$ and $R(G)$ (see the first section). These multiplications are denoted by $\rtimes$. They induce biadditive mappings

$$\mu : R \otimes R(S) \to R(S)$$

and

$$\mu : R \otimes R(G) \to R(G).$$

Using the Jacquet modules for the maximal parabolic subgroups, one can define a comodule structures

$$\mu^* : R(S) \to R \otimes R(S)$$

and

$$\mu^* : R(G) \to R \otimes R(G)$$

(see the first section). The first question may be what is the formula for

$$\mu^* \circ \mu.$$

Formulas for these compositions were obtained in [T6]. A usefulness of such formulas could be seen from the paper [T5] where some results about the square integrable representations and the irreducibility of the parabolically induced representations were announced. An essentially new situations was treated there. We have obtained that results using the formulas for $\mu^* \circ \mu$. Examples of the use of such formulas, and outlines of proofs of some of the results announced in [T5] can be found in [T7]. A complete proofs will appear in the forthcoming papers.

In this paper we apply this type of approach to the representation theory of the groups $GSp(2, F)$ and $Sp(2, F)$. We study first the questions of the reducibility of the representations parabolically induced by the irreducible representations. Then we get the classification of various classes of irreducible representations, in particular, the classification of the irreducible unitary representations. Such questions were settled for the unramified representations by F. Rodier in [R2]. Because of that, our attention in this paper is directed more to the remaining irreducible representations and this paper completes F. Rodier's investigation. For the representations supported in the two intermediate parabolic subgroups, such questions were solved by F. Shahidi and J.-L. Waldspurger. We do not use in this paper arguments specific for the spherical representations. Also, we give very often alternative proofs to the Rodier's proofs. The main part of the paper is the analysis of the parabolically induced representations. The case corresponding to the regular characters is relatively easy. It was settled in a