Astérisque

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Astérisque, tome 192 (1990), p. 7-10
<http://www.numdam.org/item?id=AST_1990__192__7_0>

Non-trivial projections of the trivial knot

By Mitsuyuki Ochiai

T. Homma, the author and M. Takahashi proved in [HOT] that there is a good algorithm for recognizing the standard 3-sphere S^3 in 3-manifolds with Heegaard splittings of genus two, and later T. Homma and the author proved in [HO] that any nontrivial 3-bridge knot diagrams of the trivial knot T always have waves but generally speaking there are many knot diagrams of T without waves (see also [Mo]). In this paper, we define the concept of n-waves and 0-waves mean waves. Then it is shown that there exist knot diagrams of T with no n-waves, where n is an arbitrary non-negative integer. Furthermore we consider a method to distinguish by the computer whether knot diagrams with a certain range of crossings give the trivial knot. Of course, the method does permit us to distinguish whether 3-bridge knot diagrams to be trivial and so at this present a plenty of knot diagrams including the one given by Figure 2 (but not Figure 3 and Figure 4) are distinguished to be trivial by the computer.

The author would like to thank S. Suzuki for helpful discussions and S. Yamada, T. Yamada and Y. Ozaki for programming aids.

1. n-waves of knot diagrams.

Let K be a knot in R^3 and $P(K)$ be a regular projection ρ of K on a 2-plane R^2 in R^3 (see [CF]). Then an arc τ in R^2 is called an n-wave if following conditions hold;

(1) $\partial\tau \cap P(K) = \partial\tau$ and it is disjoint from double points of $P(K)$.

(2) $\mathrm{int}(\tau)$ intersects $P(K)$ transeversely at n interior points which are disjoint from double points of $P(K)$.

(3) one of $\rho(K_1)$ and $\rho(K_2)$, say $\rho(K_1)$ is either an overpath or an underpath, where K_1 and K_2 are two connected components of $K - \rho^{-1}(\partial\tau)$. And if $\rho(K_1)$ is overpath (resp. underpath), then τ may be thought to consist of only an overpath (resp. underpath) with respect of $P(K)$.

(4) n is less than the number of double points of $P(K)$ in $\rho(K_1)$.

Let τ be a n-wave of $P(K)$. Then it is easily seen that $\tau \cup \rho(K_2)$ is a regular projection of K which has crossing points less than them of which $P(K)$ has. As result, the existence of n-waves does give us a method to simplify knot diagrams (regular projections). And so, if knot diagrams of the trivial knot always have n-waves, then we get an algorithm for recognizing whether knots are trivial or not. But it is impossible as follows;

Theorem 1. *There exist knot diagrams of the trivial knot with no n-waves, where n is any non-negative integer.*

To prove the theorem, we construct such examples of three different types. At first, we give a knot diagram of T without 1-waves but with a 2-wave. As such an example, we give the Figure 1. We can make such examples by the computer to do knot diagrams and to compute their Jones polynomials [J] [FYHLO]. Next we change the knot diagram illustrated in Figure 1 by deformations and get the first example of a knot diagram of T without n-waves illustrated in Figure 2. Next we can get the second such example by k-cabling of the knot diagram given in Figure 1. Figure 3 gives such an example with 2-cabling. Finally we give the third such example as illustrated in Figure 4. It will be noticed that the last example is constructed by S. Suzuki. To verify that the three knot diagrams of T given by Figure 2, Figure 3 and Figure 4 have no n-waves, it is enough to verify that all overpaths and underpaths have no n-waves. It is easily checked by case by case. As what follows, Theorem 1 has been established.

Remarks. We will now describe our computer software mentioned above. Our present system , "KNOT THEORY BY COMPUTER", has following facilities;

(1) to make regular projections (P-DATUM, see Figure 5) of knots and links using very simple datum about one among general planar graphs, 4-regular planar graphs (see [DW]) and braid presentations,

(2) to draw smoothly and rapidly knots and links by using P-DATUM to compute inverse matrices of incident matrices associated with crossing points,

(3) to simplify knot and link diagrams with n-waves to those without n-waves by cancelling n-waves and deforming link diagrams to make use of Reidemeister's move of type III [CF],

(4) to compute Jones, two variable Jones, Conway and Alexamder polynomials of knots and links.

The system is programmed by C language, had been implemented on MS-DOS machines and SUN-3 and later has been implemented on Macintosh to be integrated with MiniEdit programmed by S. Chernicoff [C]. Our programming system is available either as a complete description or else on disk.

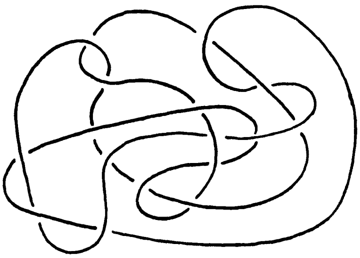


Figure 1.

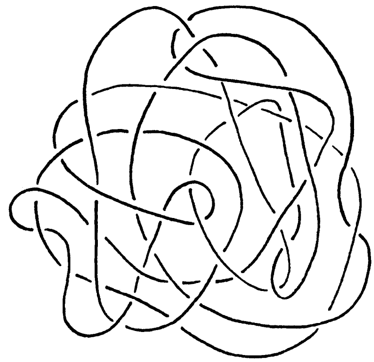


Figure 2.

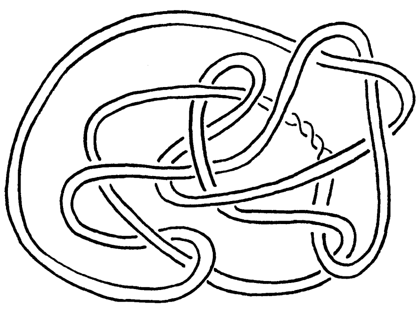


Figure 3.

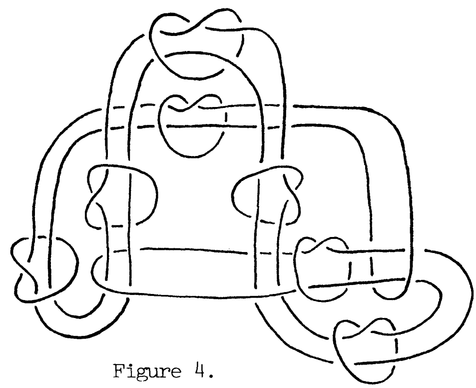
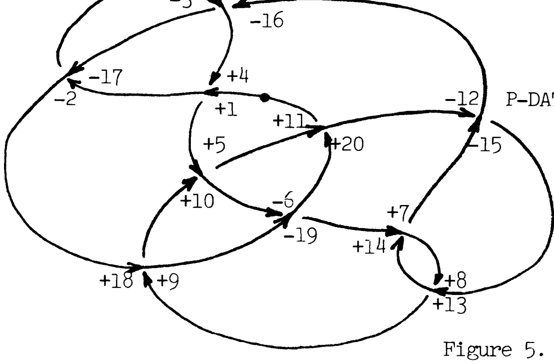


Figure 4.



P-DATA : $n \quad c$
$n_1 \; n_2 \; \ldots \; n_c$
$a_1^1 \; a_2^2 \; \ldots \; a_{n/2}^c$

20 1
20
4 -16 10 14 20 8 -12 -2 9 -6

(n : 2 x crossings,
c : numbers of connected components,
n_i: 2 x crossings of i-th component)

Figure 5.

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