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(le premier numéro est le plus significatif)

V. Batyrev : 14M25 ; 81T70
A. Buium : 14K02 ; 14H35
F. Catanese, P. Frediani : 14P25 ; 12E05, 14D20
L. Chiantini, C. Ciliberto : 14E25 ; 14J99
I. Dolgachev, M. Kapranov : 14J60 ; 14J26
R. Donagi : 14D20 ; 58F07
L. Ein, R. Lazarsfeld : 14J99
D. Eisenbud, J. Harris : 14N05 ; 13D40, 14H50, 14H45
H. Esnault, M. Levine : 14C25 ; 14C30
H. Esnault, V. Srinivas, E. Viehweg : 14C25 ; 14C30
D. Morrison : 14J30 ; 14J15, 22E40, 32J05, 81T40
C. Voisin : 14J30 ; 14J28, 81T40

TABLE DES MATIÈRES

INTRODUCTION

Les Journées de géométrie algébrique d'Orsay se sont déroulées du 20 au 26 juillet 1992, dans le bâtiment de Mathématiques de l'Université Paris-Sud ; elles ont réuni près de 300 participants. Leur objet était de faire le point sur l'état des connaissances en géométrie algébrique complexe, en mettant en lumière les perspectives de recherche qui semblent les plus prometteuses.

Dans ce but, les conférences du matin étaient centrées sur 4 grands thèmes de la géométrie complexe : systèmes linéaires, fibrés vectoriels, cycles algébriques, variétés de dimension 3. L'après-midi, des conférences plénières d'une heure ainsi que des conférences de 45 minutes en parallèle ont permis d'aborder des sujets plus spécialisés.

Nous espérons que ces Actes reflètent la vitalité du sujet telle qu'elle nous est apparue lors de ces journées.

Les Journées de géométrie algébrique d'Orsay ont été organisées dans le cadre du Projet européen Science "Geometry of Algebraic Varieties" (AGE) [1], et ont donc bénéficié, directement et indirectement, du soutien de l'Union Européenne (qui s'appelait encore Communauté Européenne). Nous avons d'autre part reçu un soutien financier important du Conseil Général de l'Essonne. Nous tenons à remercier chaleureusement ces deux institutions, sans l'aide desquelles ces Journées n'auraient probablement pas vu le jour. Nous remercions l'Université Paris-Sud et le C.N.R.S. qui ont également contribué au succès de cette manifestation.

Les organisateurs,

A. BEAUVILLE, O. DEBARRE, Y. LASZLO

[1] Contract SCI-0398-C(A).

RÉSUMÉS DES EXPOSÉS

V. BATYREV : *Quantum cohomology ring of toric manifolds*

We compute the quantum cohomology ring $QH^*_\varphi(\mathbf{P}_\Sigma, \mathbf{C})$ of an arbitrary d-dimensional smooth projective toric manifold $\mathbf{P}_\Sigma$ associated with a fan Σ. The multiplicative structure of $QH^*_\varphi(\mathbf{P}_\Sigma, \mathbf{C})$ naturally depends on the choice of an element φ in the ordinary cohomology group $H^*(\mathbf{P}_\Sigma, \mathbf{C})$. We check several properties of the quantum cohomology rings $QH^*_\varphi(\mathbf{P}_\Sigma, \mathbf{C})$ which are supposed to be valid for quantum cohomology rings of a wide class of Kähler manifolds.

A. BUIUM : *A finiteness theorem for isogeny correspondences*

Let Y be a curve in the moduli space of principally polarized abelian varieties of a given dimension. An isogeny correspondence on Y is by definition an (irreducible) curve $Z \subset Y \times Y$ such that for any point (y', y'') of Z the abelian varieties corresponding to y' and y'' are isogenous. There are plenty of curves Y which carry infinitely many isogeny correspondences; the union of all these Y 's is dense in the complex topology of the moduli space. However, we prove that for "most" curves Y there exist only finitely many isogeny correspondences. Here "most curves" mean "all curves belonging to a dense open subset of the space of all curves in the moduli space", where the space of curves is given a suitable topology, called the Kolchin topology, defined using algebraic differential equations.

F. CATANESE, P. FREDIANI : *Configurations of real and complex polynomials*

The present paper is devoted to the combinatorial descriptions of the connected components of certain open sets of the space of real or complex polynomials of a fixed degree. One instance is the open set of generic real polynomials (i.e. with distinct critical values). Describing the connected components of the open set of real lemniscate generic polynomials (i.e. with critical values with distinct non-zero absolute values), we give in particular a geometric proof of the equality between the number of connected components of the space $\mathcal{L}_n$ of complex lemniscate generic polynomials of degree $n+1$ and the number of connected components of the space of real monic polynomials of degree $n+1$ with n distinct real critical values, the lemniscate configurations occurring from real polynomials.

L. CHIANTINI, C. CILIBERTO : *A few remarks on the lifting problem*

We start with a projective variety X in $\mathbf{P}^r$ and a family W of projective subvarieties of $\mathbf{P}^r$, parametrized by the space B, such that for any $t \in B$ the corresponding fibre W_t of W is contained in some h-plane L_t and $W_t \supseteq X \cap L_t$; we assume that the L_t 's for variable t fill an open dense subset of the corresponding Grassmannian. We give conditions on the degrees of X and W_t which imply that the varieties W_t glue together to give a variety W (containing X) such that $W_t = W \cap L_t$ for all t. The proofs are based on the classical differential theory of "foci" introduced by C. Segre. Our results generalize the theorems of Laudal and Gruson-Peskine, which deal with the case X is a curve in $\mathbf{P}^3$.

I. DOLGACHEV, M. KAPRANOV : *Schur quadrics, cubic surfaces and rank 2 vector bundles over the projective plane*

Let $\Sigma \subset \mathbf{P}^3$ be a smooth cubic surface. It is known that S contains 27 lines. Out of these lines one can form 36 Schläfli double-sixes, i.e., collections $\{l_1, ..., l_6\}, \{l'_1, ..., l'_6\}$ of 12 lines such that each l_i meets only l'_j, $j \neq i$ and does not meet $l_j, j \neq i$. In 1881 F. Schur proved that any double-six gives rise to a certain unique quadric Q, the *Schur quadric*, characterized as follows : for any i the lines l_i and l'_i are orthogonal with respect to Q.

The aim of the paper is to relate Schur's construction to the theory of vector bundles on $\mathbf{P}^2$. In fact, we show that the whole theory of Hulek of rank 2 vector bundles on $\mathbf{P}^2$ with odd c_1 can be given a "geometric" interpretation involving some natural generalizations of cubic surfaces, double-sixes and Schur quadrics.

R. DONAGI : *Decomposition of spectral covers*

A G-principal Higgs bundle over a variety X (with values in an arbitrary line bundle on X) determines a family of spectral covers $\widetilde{X}_\rho$ of X, one for each irreducible representation ρ of G. We show that each of the $\mathrm{Pic}(\widetilde{X}_\rho)$ is isogenous to the sum, with multiplicities, of a finite collection of abelian varieties, obtained as isotypic pieces for the action of the Weyl group W on $\mathrm{Pic}(\widetilde{X})$, where $\widetilde{X}$ is the cameral, or W-Galois, cover of X, independent of ρ. The piece $\mathrm{Prym}(\widetilde{X})$, corresponding to the reflection representation of W, is distinguished : it occurs in $\mathrm{Pic}(\widetilde{X}_\rho)$ for each ρ (this characterizes Prym for classical G but not for exceptional groups such as G_2, E_6), and is essentially the moduli space of Higgs bundles with spectral data $\widetilde{X}$. Various Prym identities are recovered as the case $X = \mathbf{P}^1$, G simply laced, studied previously by Kanev.

L. EIN, R. LAZARSFELD : *Seshadri constants on surfaces*

Let L be an ample line bundle on a smooth projective variety X of dimension n. Demailly has introduced the *Seshadri constant* $\epsilon(L, x)$ of L at x, which roughly speaking measures how positive L is at x. For example, if L is very ample, then $\epsilon(L, x) \geq 1$ for all $x \in X$. We study these invariants in the first non-trivial case, when X is a smooth surface. We prove (somewhat surprisingly) that in this case $\epsilon(L, x) \geq 1$ for all except perhaps countably many $x \in X$, and moreover if $L \cdot L > 1$ then the exceptional set is finite. On the other hand, simple examples due to Miranda show that $\epsilon(L, x)$ can take on arbitrary small positive values at isolated points. The paper also contains some related examples and open problems.

D. EISENBUD, M. GREEN, J. HARRIS : *Some conjectures extending Castelnuovo theory*

We propose a series of conjectures concerning the Hilbert functions of points (or more generally zero-dimensional subschemes) in projective space. We begin by extending the results of Castelnuovo and others on points in uniform position, and then consider the corresponding problem without the hypothesis of uniform position. A special case is a

conjectured extension of the classical Cayley-Bacharach theorem. We prove this conjecture in projective space $\mathbf{P}^r$ for all $r \leq 7$. Finally we make a conjecture extending Macaulay's theorem on the Hilbert function of graded rings, and discuss its relation to the previous conjectures.

H. ESNAULT, M. LEVINE : *Surjectivity of cycle maps*

Let X be a smooth proper complex variety. We consider the cycle map from the Chow ring to the ring of the Deligne cohomology. If this cycle map is injective (modulo torsion), then it has to be surjective as well, and the groups $\mathrm{H}^p(\mathrm{X}, \mathcal{K}_{p+1})$ are generated by constant functions on codimension p cycles (modulo torsion). This generalizes Jannsen's results concerning the cycle map with values in the Betti cohomology.

H. ESNAULT, V. SRINIVAS, E. VIEHWEG : *Decomposability of Chow groups implies decomposability of cohomology.*

Let X be a smooth proper complex n-dimensional variety. We consider the cup product map from the product of the Chow groups (modulo torsion)

$$\mathrm{CH}^{n_1}(\mathrm{V}) \otimes \cdots \otimes \mathrm{CH}^{n_r}(\mathrm{V}) \to \mathrm{CH}^n(\mathrm{X}) ,$$

where $\sum_{i=1}^{i=r} n_i = n$ and V is a non empty Zarisky open set in X. If it is surjective (modulo torsion), then the corresponding map from the "edge" Hodge groups

$$\mathrm{H}^{n_1}(\mathrm{X}, \mathcal{O}_\mathrm{X}) \otimes \cdots \otimes \mathrm{H}^{n_r}(\mathrm{X}, \mathcal{O}_\mathrm{X}) \to \mathrm{H}^n(\mathrm{X}, \mathcal{O}_\mathrm{X})$$

is surjective. We give variants and discuss some problems.

D. MORRISON : *Compactifications of moduli spaces inspired by mirror symmetry*

We study moduli spaces of nonlinear sigma-models on Calabi-Yau manifolds, using the one-loop semiclassical approximation. The data being parameterized includes a choice of complex structure on the manifold, as well as some "extra structure" described by means of classes in H^2. We formulate a simple and compelling conjecture about the action of the automorphism group on the Kähler cone, which would enable the construction of a partial compactification of the moduli space using Looijenga's "semi-toric" method. We then explore the implications which this construction has concerning the properties of the moduli space of complex structures on a "mirror partner" of the original Calabi-Yau manifold.

C. VOISIN : *Miroirs et involutions sur les surfaces K3*

On construit une série d'exemples de "symétrie miroir" en considérant des variétés de Calabi-Yau du type $(\mathrm{E} \times \mathrm{S})/(j, i)$, où S est une surface K3 munie d'une involution i agissant par (-1) sur $\mathrm{H}^{2,0}(\mathrm{S})$, et E une courbe elliptique munie d'une involution j telle que $\mathrm{E}/j \cong \mathbf{P}^1$. On utilise les travaux de Nikulin pour construire l'involution miroir sur $\mathrm{H}^2(\mathrm{S}, \mathbf{Z})$, et le théorème de Torelli pour construire l'application miroir holomorphe

$$\big((\mathrm{E} \times \mathrm{S})/(j, i), \alpha\big) \longmapsto \big((\mathrm{E}' \times \mathrm{S}')/(j', i'), \alpha'\big) .$$