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HOLOMORPHIC FAMILIES OF IMMERSIONS AND HIGHER ANALYTIC TORSION FORMS

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Introduction

Let $i\colon W \longrightarrow V$ be an embedding of smooth complex manifolds. Let S be a complex manifold. Let $\pi_V\colon V \longrightarrow S$ be a holomorphic submersion with compact fibre X, which restricts to a holomorphic submersion $\pi_W\colon W \longrightarrow S$, with compact fibre Y. Then we have the diagram of holomorphic maps

$$(0.1)\qquad \begin{array}{ccccc} Y & \longrightarrow & W & & \\ \downarrow{\scriptstyle i} & & \downarrow{\scriptstyle i} & \overset{\pi_W}{\searrow} & \\ X & \longrightarrow & V & \underset{\pi_V}{\longrightarrow} & S \end{array}$$

Let η be a holomorphic vector bundle on W. Let (ξ, v) be a holomorphic complex of vector bundles on V, which together with a holomorphic restriction maps $r\colon \xi_{0|W} \longrightarrow \eta$, provides a resolution of the sheaf $i_*\eta$.

Let $R\pi_{V*}\xi$, $R\pi_{W*}\eta$ be the direct images of ξ, η. We make the assumption that the $R^i\pi_{W*}\eta$ are locally free. Then $R\pi_{V*}\xi$ is also locally free, and moreover we have a canonical isomorphism of $\mathbf{Z}$-graded holomorphic vector bundles on S

$$(0.2)\qquad R\pi_{V*}\xi \simeq R\pi_{W*}\eta\,.$$

Also for any $s \in S$,

$$(0.3)\qquad \begin{aligned} (R\pi_{V*}\xi)_s &\simeq H(X_s, \xi_{|X_s})\,, \\ (R\pi_{W*}\eta)_s &\simeq H(Y_s, \eta_{|Y_s}) \end{aligned}$$

(here $H(X_s, \xi_{|X_s})$ and $H(Y_s, \eta_{|Y_s})$ denote respectively the hypercohomology of $\xi_{|X_s}$, and the cohomology of $\eta_{|Y_s}$).

Let ω^V, ω^W be real (1,1) forms on V, W which are closed, and which, when restricted to the relative tangent bundles TX, TY, are the Kähler forms of Hermitian metrics g^{TX}, g^{TY} on TX, TY. Let $g^{\xi_0}, \dots, g^{\xi_m}, g^\eta$ be Hermitian metrics on $\xi_0, \dots, \xi_m, \eta$.

Let $(\Omega(Y, \eta_{|Y}), \overline{\partial}^Y)$ be the family of relative Dolbeault complexes along the fibres Y, whose cohomology is equal to $H(Y, \eta_{|Y})$.