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PROBLEMS OF METHOD IN LEVI-CIVITA'S CONTRIBUTIONS TO HYDRODYNAMICS

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ABSTRACT. — Levi-Civita made important contributions to hydrodynamics: he solved D'Alembert's paradox, introduced the "wake hypothesis", deduced the general integral of any plane motion involving a wake, and gave a rigorous proof of the existence of the irrotational wave in a canal of finite depth. In this paper, we investigate Levi-Civita's results in this area, and connect them to the methods of the new theory of integral equations. Finally, we give some information on Levi-Civita's students. In our paper, we often use letters written by and addressed to Levi-Civita.

RÉSUMÉ (Problèmes de méthode dans les contributions de Levi-Civita à l'hydrodynamique)

Levi-Civita apporta des contributions remarquables à l'hydrodynamique ; il a résolu le paradoxe de D'Alembert, introduit l'hypothèse du sillage, déduit l'intégrale générale d'un mouvement plan avec sillage et démontré de manière rigoureuse l'existence de l'onde irrotationnelle dans un canal de profondeur finie. Dans notre article, nous présentons les résultats de Levi-Civita dans cette discipline et en montrons le lien avec les méthodes de la nouvelle théorie des équations intégrales. Enfin, nous donnons quelques informations sur les étudiants de Levi-Civita. Dans notre article, nous employons souvent des lettres écrites par et adressées à Levi-Civita.

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INTRODUCTION

In their well-known contribution to *Handbuch der Physik*, "The classical field theories", Truesdell and Toupin [1960] give an overview of the history of hydrodynamics, without quoting either Levi-Civita's paper on the "wake hypothesis" [Levi-Civita 1901] (see section 1) or his subsequent article in which this hypothesis leads to the general integral of any plane motion involving a wake, by means of an adequate conformal transformation [Levi-Civita 1907a]. However, they do often quote some of Levi-Civita's students – mainly Umberto Cisotti[1] – who extended and deepened his results.

Other sources do reference Levi-Civita's method [Levi-Civita 1907a]; for example, the classical works of Villat [1920; 1930] and Lamb [1932] on hydrodynamics, Gurevich's treatise on the theory of jets [Gurevich 1966], and Weinstein's paper on Levi-Civita's contribution to the theories of jets and wakes [Weinstein 1975]. More recent expositions of the history of hydrodynamics do not quote Levi-Civita's 1901 paper. There, he solved D'Alembert's paradox – namely, if a solid body moves in a perfect fluid (originally motionless), then the resisting force acting on the body is always zero – using his wake hypothesis, and deduced the law for the resistance on a body due to the fluid. Levi-Civita assumes that a solid body moving in a fluid separates the fluid into two regions – one in front of the body and one behind it (the wake) – and that the separation surface is a discontinuity surface (see section 1 for details).

The wake hypothesis was well-known to Levi-Civita's contemporaries, for instance Cisotti [1912a] and Villat [1918], who considered it very important for new and fruitful research. Today, it is Cisotti who tends to be referenced relative to D'Alembert's paradox, since it was Cisotti who clarified and developed the ideas in Levi-Civita's 1901 paper

[1] Umberto Cisotti (1882-1946) was one of Levi-Civita's students at the University of Padua, where he graduated in 1903. From 1907 to 1913 he was Levi-Civita's assistant. He recalled his studies at the University of Padua in a letter to Levi-Civita on March 28, 1935: "The 29th of March 1903, in the morning, I was in your old study in via Altinate, in Padua, and you carefully looked over my dissertation. In the meantime, the door suddenly opened and your father, so distinctive and nice, with a cigar in his hand entered the room [...]" [Nastasi & Tazzioli 2003, p. 69-70]. Cisotti became full professor of rational mechanics at the Polytechnic of Milan. His main research fields concern plane hydrodynamics, which he (and his students) studied by means of complex variables.

[Levi-Civita 1901]. Anderson [1997, p. 252], however, provides a single (incorrect) reference to a nonexistent paper of Levi-Civita said to have been published in 1901 in the *Comptes rendus* de l'Académie des sciences. Anderson also quotes Levi-Civita in a free (and not very faithful) translation of lines from [Levi-Civita 1907a, p. 522].

Levi-Civita's method [Levi-Civita 1907a] is a fundamental contribution to hydrodynamics and provided the starting point for much research – research directly suggested by Levi-Civita to his students and research, as in the case of H. Villat and M. Brillouin, inspired by his papers. It is interesting to quote here what Olivier Darrigol recently wrote en passant on Levi-Civita on the basis of indirect reports (by Hadamard and Brillouin): "Jacques Hadamard [1903, pp. 355-361] gave a proof that surfaces of discontinuity cannot be formed in a perfect fluid as long as cavitation is excluded. This proof, however, does not exclude the growth of a pre-existing, tiny surface of discontinuity. Marcel Brillouin [1911] made this point, described the growth process, and extended the conformal methods of Kirchhoff, Rayleigh, and Levi-Civita to curved obstacles devoid of angular points." [Darrigol 2002, p. 46, footnote 48]. As shown in sections 1 and 2 below, the two fundamental works by Levi-Civita [1901; 1907a] are based on the existence of this surface of discontinuity.

The other hydrodynamic subjects studied by Levi-Civita concern the theory of waves, where he restated intuitions and previous problems in terms of rigorous mathematical formalism. [Levi-Civita 1925] is his main work on the subject and concerns irrotational waves with finite amplitude. In particular, it deals with periodic waves that propagate without changing their shape. Levi-Civita deduced rigorous solutions instead of the (second-order) approximations obtained by Stokes and Lord Rayleigh. Their method did not lead naturally to further approximations and a fortiori did not prove the convergence of the approximation algorithm.

Lord Rayleigh, after a sequence of not quite satisfactory attempts, doubted the real existence of the phenomenon, that is, the rigorous solution of hydrodynamic equations corresponding to periodic and permanent waves (of Airy). However, towards the end of his life, he changed his mind because of new intuitions that made the existence of this wave type plausible from a physical point of view (as noted by Levi-Civita [1925, pp. 201-202]). Rayleigh was impressed by the results obtained by Korteweg and de Vries, and especially by the celebrated Korteweg-de Vries equation[2]. In their paper, Korteweg and de Vries wrote: "[...] we find that, even when friction is neglected, long waves in a rectangular canal must necessarily change their form as they advance, becoming steeper in front and less steep behind. Yet since the investigations of De Boussinesq, Lord Rayleigh and St. Venant on the solitary wave, there has been some cause to doubt the truth of this assertion" [Korteweg & de Vries 1895, p. 422]. In one of his later works, Rayleigh [1917] studied the solitary wave again and made new calculations to the sixth order of approximation.

Levi-Civita developed and solved the problem with the utmost rigour, by means of a new approximating expression that he named the "stokian" (in honour of Stokes). In so doing, he then, in Lamb's words, closed "an historic controversy" [Lamb 1932, p. 420] by representing both the exact outline of such waves and the mathematical equation linking height, length, transport, and velocity of propagation of waves by means of simple formulae. His students broached and solved many other problems on waves.

We plan to investigate Levi-Civita's contribution to this area of fluid mechanics in detail in a subsequent work. In the present paper, we mainly consider Levi-Civita's contribution to hydrodynamics in the first period of his scientific career, when he taught at the University of Padua (from 1892 to 1918). In the first part of our paper (sections 1, 2, 3), we consider the wake hypothesis, Levi-Civita's method, and some further developments. In the second part (section 4), we concentrate on the role of Dini's formula and – more generally – of the theory of integral equations in solving hydrodynamic problems concerning wake. Finally, we add some information on Levi-Civita's school in the concluding remarks (section 5).

Dini's formula – which connects the values of a function f on the circumference of a circle with the values of its normal derivative on the same circumference, if f is assumed to be harmonic in the circle – is an analytical relation which seems far from any hydrodynamic application. In reality, some questions of hydrodynamics (for example, certain

[2] See [Blij 1978], [Darrigol 2003] on the Korteweg-de Vries equation and its history.

two-dimensional problems involving a wake) can be reduced to a particular conformal mapping by means of the theory of complex analysis and, in particular, using Dini's formula. A deeper analysis of Dini's formula has allowed us to draw attention to a new approach to the study of the partial differential equations of mathematical physics. This new study developed from the first decade of the 20-th century onward, and used the theory of integral equations. In fact, some students and correspondents of Levi-Civita, also influenced by his ideas, considered the theory of integral equations as the best approach to the study of the equations of mathematical physics. In this paper, we also aim to show that there were close connections between hydrodynamic problems and the emergence of the new methods of integral equations.

1. D'ALEMBERT'S PARADOX

In 1901, Levi-Civita published "Sulla resistenza dei mezzi fluidi", a paper that left an important mark on the history of hydrodynamics. It is part of a letter by Levi-Civita to Francesco Siacci (1893-1907), who communicated it to the *Accademia dei Lincei*. By using his hypothesis concerning the wake, Levi-Civita was able to overcome all the theoretical difficulties connected with the so-called D'Alembert paradox: namely, if one assumes that in a perfect fluid a body produces a continuous motion, then – as a consequence of Bernoulli's theorem - the resistance on the body due to the fluid will be zero for any shape of the body. Many scientists pointed out that the underlying assumptions were illegitimate and probably responsible for D'Alembert's paradox: fluids were supposed to be ideal and without friction. But not everyone shared this idea.

Helmholtz [1868] assumed that the region between the wake and the region outside of it was a discontinuity surface formed at any sharp angle of the walls along which the fluid moved[3]. Kirchhoff [1869] and Rayleigh [1876] developed the dead-water theory of resistance according to which the body in motion drags behind it an infinite liquid column that moves with it. Therefore, there are two different regions in the fluid – the wake and the region outside of it – which are divided by a (vortical) surface of

[3] On the prehistory of discontinuity surfaces, see [Darrigol 2002].