JOSEPH LIPMAN

Dualizing sheaves, differentials and residues on algebraic varieties

Astérisque, tome 117 (1984)

<http://www.numdam.org/item?id=AST_1984__117__1_0>

117 ASTÉRISQUE

1984

DUALIZING SHEAVES, DIFFERENTIALS AND RESIDUES ON ALGEBRAIC VARIETIES

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(*) Supported by NSF grant MCS-8200624 at Purdue University.

SOCIÉTÉ MATHÉMATIQUE DE FRANCE
Publié avec le concours du CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE

AMS Subjects Classification : 14 B 15, 14 F 99

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Leitfaden

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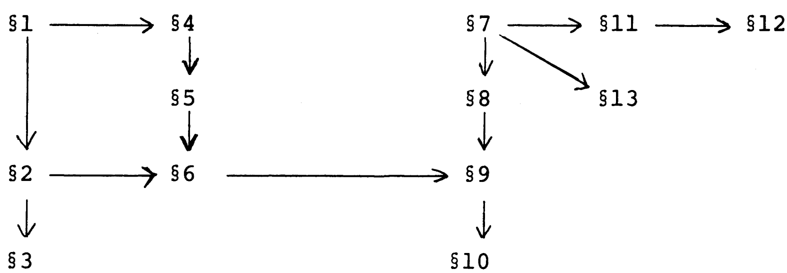

INTRODUCTION

This is a semi-expository account about the role of differential forms and residues in the duality theory of algebraic varieties over a perfect field. The main results are summarized in the Residue Theorem (0.6) stated near the end of §0, and generalized in §10.

In some sense there is little here which cannot be dug out from other sources: the basic ideas involved were announced by Grothendieck in [G2]; the foundations of duality theory were then worked out from a very general point of view (derived categories dualizing complexes, etc.) by Hartshorne [RD], Deligne [RD, Appendix] and Verdier [V]; and the "fundamental class", a canonical map from differential forms to dualizing complexes, was studied by El Zein and Angéniol [E]. But, fundamental, powerful, and beautiful as the resulting theory is, the formalism in which it is ensconced and some lack of detail in the literature(1) have prevented it from becoming as well-known and understood as it should be. For me at any rate, reaching even the level of understanding represented by these notes has been a long and arduous process. And the reactions of audiences to lectures which I have given over the past twelve years on this subject have suggested that an exposition in the spirit of [S, pp. 76-81] (case of curves, after Rosenlicht) and [K2] (case of projective Cohen-Macaulay varieties)- i.e. accessible in principle to someone familiar with, say, Chapter III of [H]- may not be superfluous.

*   *   *

Various forms of the main results to be presented have appeared in the literature. In this Introduction, and in §0, we gather some variants together and point out their interconnections. In the process indications about the contents of the paper will emerge.

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(1) Of course in the writing of any exposition (this one included) the choices about which details to include and which to leave to the reader are a matter of taste, judgement, mood,...