NUMERICAL MODELS FOR FUSION

**P. Degond, V. Grandgirard, Y. Sarazin,
S. C. Jardin, C. Villani**

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Panoramas et Synthèses

Numéro 39-40

SOCIÉTÉ MATHÉMATIQUE DE FRANCE
Publié avec le concours du Centre national de la recherche scientifique

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Société mathématique de France 2013
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2000 *Mathematics Subject Classification.* — 76L05, 76N10, 76N20, 76W05, 76X05, 82C99, 82D10, 85A05.

Key words and phrases. — Algorithms, asymptotic-preserving schemes, ballooning instability, Boltzmann-Vlasov equation, computational methods, conservative schemes, cyclotron frequency, Debye length, drift-fluid regime, electron beams, electrostatics, Eulerian, fusion, galactic dynamics, gyrokinetic theory, high performance computing simulations, implicit schemes, KAM theory, Landau damping, Larmor radius, law Mach-number limit, Mach number, magnetic fields, magnetic fusion plasma, magnetohydrodynamics, Newton's methode, noise, nonlinear problems, particle-in-cell, particles, plasma, plasma confinement, plasma echo, plasma flow, plasma fluctuations, plasma fluid models, plasma instability, plasma kinetic theory, plasma Langmuir waves, plasma microinstabilities, plasma nonlinear processes, plasma nonlinear waves, plasma physics, plasma simulation, plasma temperature, plasma toroidal confinement, plasma transport processes, plasma turbulence, plasma-beam interactions, quasi-neutrality, reconnection, recurrence, semi-Lagrangian, sensitivity analysis, shock-capturing schemes, simulation, slabs, strongly anisotropic diffusion equations, tokamak, tokamak devices, turbulent transport, uses, Vlasov-Poisson, Vlasov equation.

Mots-clé et phrases. — Amortissement Landau, dynamique galactique, écho plasma, eulérien, fréquence cyclotron, fusion, limite faible nombre de Mach, longueur de Debye, magnéto-hydrodynamique, modèles fluides de plasmas, méthode de Newton, méthodes computationnelles, nombre de Mach, particle-in-cell, physique des plasmas, plasma de fusion magnétique, quasi-neutralité, régime de dérive-fluide, schémas conservatifs, schémas de capture de chocs, schémas implicites, schémas 'Asymptotic-Preserving', semi-lagrangien, simulations de calcul haute performance, théorie gyrocinétique, théorie KAM, tokamak, transport turbulent, Vlasov-Poisson.

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Abstract. — At very high temperature electrons leave the atom to which they are attached and a gas of charged particles called a plasma is obtained. Plasmas, because of their interaction with an electromagnetic field, have a global and rich behaviour, much more complex than for neutral gases. This makes their study complex and fascinating. Plasma physics and its lighthouse application, controlled thermonuclear fusion, thus offers new challenges to mathematicians. Two main models are generally used to describe the dynamics of a plasma: on the one hand the so-called kinetic models which describe the evolution of the phase space density of the different particle species, and on the other hand the fluid models which describe the evolution of macroscopic quantities, like density in physical space, mean velocity and temperature. This two types of models need to be coupled with Maxwell's equations in order to describe the non linear interactions with the electromagnetic field generated by the charged particles of the plasma. This book results from notes of the lectures that were given at CIRM in Luminy during the summer school of CEMRACS 2010. It details for topics in the mathematical and numerical study of plasmas: asymptotic preserving (AP) numerical schemes that enable to deal consistently with two space or time scales, gyrokinetic simulations of magnetic fusion plasmas, MHD simulations of magnetic fusion plasmas and the mathematical study of Landau damping.

Résumé. — **Modèles numériques pour la fusion** À très haute température les électrons quittent leur atome et on obtient un gaz de particules chargées qui s'appelle un plasma. Les plasmas en raison de leur interaction avec un champ électromagnétique ont un comportement global riche et beaucoup plus complexe que les gaz neutres, ce qui rend leur étude difficile et fascinante. La physique des plasmas et son application phare qui est la fusion thermonucléaire contrôlée offre ainsi de nouveaux défis aux mathématiciens. Les modèles principaux utilisés pour les plasmas sont d'une part les modèles cinétiques qui décrivent l'évolution de la densité dans l'espace des phases (position-vitesse) des différentes espèces de particules, et d'autres part les modèles

fluides qui décrivent l'évolution des quantités macroscopiques, densité dans l'espace physique, vitesse moyenne et température. Ces deux types de modèles doivent être couplés non linéairement aux équations de Maxwell pour décrire l'évolution du champ électromagnétique généré par les particules chargées du plasma. Ce livre fait suite aux cours qui ont été donnés au CIRM à Luminy lors de l'École d'été du CEMRACS 2010. Il présente quatre facettes de l'étude mathématique et numérique des plasmas : les schémas numériques AP qui permettent de traiter de manière consistante et stable deux échelles de temps ou d'espace, la simulation gyrocinétiques des plasmas de fusion magnétique, la simulation MHD des plasmas de fusion magnétique et l'étude mathématique de l'amortissement Landau.

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Schémas 'Asymptotic-Preserving' pour les modèles fluides de plasmas

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Ces notes résument une série de travaux concernant l'approximation numérique de modèles fluides de plasmas. Nous construisons des schémas appelés 'Asymptotic-Preserving' qui sont valides pour une large plage de valeurs (depuis des valeurs très petites jusqu'a des valeurs d'ordre un) des paramètres adimensionnels qui apparaissent dans les modèles fluides de plasmas. Plus spécifiquement, nous nous intéressons à deux paramètres : la longueur de Debye adimensionnelle qui quantifie à quelle distance le plasma se trouve du régime quasineutre, et la période cyclotron adimensionnée, qui est inversement proportionnelle à l'intensité du champ magnétique. Nous nous focaliserons essentiellement sur les idées, de manière à donner les moyens au lecteur d'appliquer ces concepts à d'autres situations.

Simulations gyrocinétiques des plasmas de fusion magnétique

VIRGINIE GRANDGIRARD & YANICK SARAZIN 91

Ce cours présente l'approche gyrocinétique et les différents schémas numériques utilisés dans les simulations pour calculer le transport turbulent dans les plasmas de fusion par confinement magnétique. Les caractéristiques essentielles de la configuration magnétique des tokamaks et des plasmas de fusion sont rappelées. Les élements fondamentaux de la théorie gyrocinétique sont introduits, dont en particulier les équations des trajectoires et l'électro-neutralité. Du point de vue numérique, les trois classes de méthodes numériques existantes sont présentées en détail, à savoir Particle-In-Cell, eulerienne et semi-lagrangienne. Leurs propriétés sont discutées, et leurs forces et faiblesses respectives sont passées en revue.

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STEPHEN C. JARDIN 177

La fusion nucléaire promet une énergie propre et sécuritaire capable de soutenir la demande mondiale en énergie dans les années futures sans la production de déchets radioactifs de longue durée ou de matières fissiles. La configuration la plus avancée pour le confinement magnétique d'un plasma à haute température est le tokamak, lequel est caractérisé par un plasma toroïdal conduisant un courant électrique et soumis à de forts champs magnétiques externes. Les températures, densités et courants diffusent à travers les lignes de champ magnétique à un taux qui détermine les propriétés de confinement du tokamak. Des instabilités globales peuvent cependant se développer à l'intérieur du tokamak lorsque le courant et/ou la pression dépassent un certain seuil. Cette série de cours à pour but de décrire les formulations analytiques, ainsi que leurs méthodes numériques correspondantes, pour le mouvement lent de diffusion associé au transport ainsi que le mouvement ondulatoire rapide associé aux instabilités. La première formulation utilise une échelle de temps longs afin d'éliminer le mouvement ondulatoire rapide, ainsi qu'une transformation de coordonnées alignée avec le champ magnétique et évoluant temporellement afin d'isoler le transport lent perpendiculaire aux lignes de champ du transport rapide le long des lignes de champ. La seconde formulation utilise une combinaison d'éléments finis d'ordre élevé, une représentation judicieuse des champs vectoriels de vélocité et de champ magnétique, ainsi qu'un algorithme implicite avancé ayant les propriétés désirées pour l'intégration temporelle.

Amortissement Landau
CÉDRIC VILLANI 237

Ce cours a été enseigné à l'été 2010 au Centre International des Rencontres Mathématiques (CIRM), dans le cadre d'un programme sur l'étude mathématique des plasmas, en liaison avec le projet ITER ; c'est une introduction au phénomène d'amortissement Landau en régime linéarisé et non-linéaire perturbatif, basé sur le travail récent de Mouhot & Villani.

ABSTRACTS

Asymptotic-Preserving Schemes for Fluid Models of Plasmas

P. Degond 1

These notes summarize a series of works related to the numerical approximation of plasma fluid problems. We construct so-called 'Asymptotic-Preserving' schemes which are valid for a large range of values (from very small to order unity) of the dimensionless parameters that appear in plasma fluid models. Specifically, we are interested in two parameters, the scaled Debye length which quantifies how close to quasi-neutrality the plasma is, and the scaled cyclotron period, which is inversely proportional to the magnetic field strength. We will largely focus on the ideas, in order to enable the reader to apply these concepts to other situations.

Gyrokinetic simulations of magnetic fusion plasmas

Virginie Grandgirard & Yanick Sarazin 91

This lecture presents the gyrokinetic framework and details the various numerical schemes used in nonlinear simulations to compute turbulent transport in magnetic fusion plasmas. The basic features of tokamak magnetic configuration and of fusion plasmas are recalled. Fundamental elements of the gyrokinetic theory are carefully introduced, including the derivation of velocity drifts and of the quasi-neutrality. From the numerical point of view, the main focus is put on the three existing classes of numerical methods, namely Particle-In-Cell, Eulerian and semi-Lagrangian. Their properties are discussed, and their strengths and weaknesses are exhaustively reviewed.

MHD Simulations for Fusion Applications
STEPHEN C. JARDIN 177

Nuclear fusion holds forth the promise of being a clean and safe solution to meet the world's energy demand in the foreseeable future without producing long-lived radioactive waste or weapons-grade material. The most mature configuration for magnetically confining a fusion plasma is the tokamak; a current carrying toroidal plasma characterized by strong externally produced magnetic fields. The temperatures, densities, and current will diffuse across the magnetic field lines at some rate, determining the confinement properties of the tokamak. The tokamak can also develop global instabilities if the current and/or pressure exceed certain instability thresholds. This set of lectures is aimed at describing analytical formulations and associated numerical methods for quantitatively describing both the slow (diffusive) motion associated with transport and the faster (wavelike) motion associated with instabilities. The former uses slow time scale ordering to remove the wavelike motion, and a time-dependent field-aligned coordinate transformation to isolate the cross-field transport from the faster transport along the magnetic field lines. The latter uses a combination of high-order finite elements, a particular representation of the magnetic and velocity vector fields, and an implicit time advance algorithm with desirable properties.

Landau damping
CÉDRIC VILLANI 237

This course was taught in the summer of 2010 in the Centre International des Rencontres Mathématiques as part of a program on mathematical plasma physics related to the ITER project; it constitutes an introduction to the Landau damping phenomenon in the linearized and perturbative nonlinear regimes, following the recent work by Mouhot & Villani.

PREFACE

This book contains four lectures given in the Summer School of CEMRACS 2010 which took place at CIRM in Luminy in July 2010. It is devoted to Numerical Models for Fusion. These lectures were meant to present the state of the art in several numerical and mathematical topics related to controlled thermonuclear fusion. Before introducing the contents of the different contributions to this book, let me give a little bit of background to the physics of controlled thermonuclear fusion and the roadmap towards fusion based power plants.

Due to the evolution of energy needs and the vanishing of fossil fuels, the development of new sources of energy is essential. Next to wind and solar energy, which depend on non controllable parameters, a reliable source of energy that can be turned on and off depending on the demand and that is environmentally neutral is needed. The well known formula $E = mc^2$ tells us that it is possible to produce energy from mass. This can be done in two types of nuclear reactions. The fission reaction which produces two light nuclei from a heavier nucleus and the fusion reaction which produces a heavier nucleus from two smaller ones. The fission reaction is used industrially in nuclear power plants, but has important safety and waste disposal issues. Controlled fusion is in principle very interesting but is still an area of active research.

The most accessible fusion reaction uses Deuterium and Tritium nuclei, which are isotopes of Hydrogen, in order to produce a Helium nucleus and a very energetic neutron, which will be captured to generate heat that will be the source of electricity. The temperatures required for thermonuclear fusion are over a hundred million degrees. At these temperatures, the electrons leave their atom and a globally neutral gas of charged ions and electrons called plasma is obtained. In order to produce energy, the amplification factor Q, which is the ratio of produced power to supplied power, needs to be larger than one. An energy balance yields the Lawson criterion which links the amplification factor Q to the product nTt_E, where n is the plasma density, T its temperature and t_E the energy confinement time. Fusion is the source for the energy of the stars in which the confinement at a sufficient density is ensured due to their mass. Laboratory research on confined fusion on earth is based on two different approaches: inertial confinement which consists in realising the conditions of the Lawson criterion by confining a target of Deuterium of Tritium at a very high density for a relatively short time by shooting at it with laser beams; magnetic confinement fusion consists in confining the plasma using a very large magnetic field at a lower density than inertial fusion but for a longer time. The plasma is confined in a toroidal chamber called Tokamak. This last approach is adopted in the international experimental project ITER. Experimental installations for inertial fusion are also being built (NIF in Livermore, USA and LMJ near Bordeaux, France). More information on these programs can be found at `http://www.ilp.u-bordeaux1.fr`.

The ITER project is a partnership between the European Union, Japan, China, South Korea, Russia, USA and India. The international agreement has been signed on November 21, 2006 in Paris. Its objective is to demonstrate the scientific and technical feasibility of the production of electricity with fusion energy benefiting from large fuel resources and little impact on the environment.

The construction of the ITER Tokamak has just started in Cadarache near Aix-en-Provence in the south-east of France. The exploitation phase should start around 2020 and last for about 20 years. The main objectives for ITER are on the one hand to attain an amplification factor of 10 at least and thus to really allow for energy production, and on the other hand to develop and test technologies which will be needed for a fusion power plant. In case of success, the next step will be to build a fusion plant, called DEMO, actually producing electricity before the construction of commercial fusion power plants. More information can be found on the site `http://www.iter.org`.

At the temperature required by fusion, plasmas are essentially fully ionized gases consisting of several species of ions and electrons, which are all charged particles, interacting with each other mostly through the electromagnetic field they generate. So a good model to describe the self consistent evolution of a plasma would be to use the electromagnetic field generated by all the particles to get the force field needed in Newton's law to model the dynamics of the particles. However, due to the huge number of particles in a plasma this is numerically unbearable, so that reduced models are necessary. Two levels of reduced models are generally used in practice and are the subject of these lecture notes. The first one yields the so-called kinetic models, which are obtained by a statistical approach from the particle to particle interaction. In this kind of model, each species of particles is represented by a distribution function, which gives the amount of particles at each position of phase space at a given time. The kinetic equations describing the evolution of the distribution function can be of Vlasov type if no collisions are involved or of Boltzmann type if particle collisions are involved. These equations also involve a self-consistent coupling with Maxwell's equations for the evolution of the electromagnetic fields generated by the charged particles. These models are the subject of the studies presented in the lecture notes of C. Villani, V. Grandgirard a,d Y. Sarazin. The second level, called fluid models, can be obtained by taking the velocity moments of the Vlasov (or Boltzmann) equations, thus yielding equations for the conservation of mass, momentum, energy and sometimes higher order moments. Closure relations are needed to express the higher-order correlations present in the governing equations after having taken a finite number of such moments.

In magnetized plasmas, a very common fluid model is the so-called Magneto-Hydro-Dynamics (MHD) model that describes the evolution of a single charged fluid. The numerical approximation of these models is the subject of the lectures by P. Degond and S. Jardin.

The lecture by Pierre Degond addresses an important new class of numerical schemes which is essential in physics problems, like fusion physics, involving many different space and time scales, namely asymptotic preserving schemes. Such numerical schemes have the property to remain consistent with the limit model when a small parameter in the starting model for which the scheme is build tends to zeros. In the notes he first explains the ideas of these schemes and then applies them to different limits in plasmas, in particular the quasi-neutral limit of Euler-Poisson and the drift fluid limit of Euler-Lorentz.

The contribution of Virginie Grandgirard and Yanick Sarazin is devoted to turbulence simulations in tokamaks. Such simulations are based on an approximation of the Vlasov equation in a large magnetic field which is very realistic in a Tokamak plasma. This model called the gyrokinetic model averages the particle trajectories around the magnetic field lines and thus enables to use much larger time steps in the simulation which imply a huge gain in computational time. They introduce the Vlasov-Maxwell system and its mathematical properties and explain the derivation of the gyrokinetic model. Then an important section is devoted to explaining different numerical methods for their approximation.

After that Steve Jardin presents the other main workhorse for Tokamak simulations, namely the MHD model and its numerical approximation. First he introduces the mathematical model and several of its variants typically used for magnetic fusion simulations. Then he introduces typical coordinate transformations used to better take into account the anisotropic phenomena in Tokamaks. Long time approximations which are essential for the Tokamak operation are derived. The last part is devoted to the numerical approximation of these models using the Finite Element method.

Finally, the lecture notes of Cédric Villani are devoted to a fundamental process in plasma physics, Landau damping. This phenomenon where a small perturbation to an equilibrium plasma is completely damped without any dissipative mechanism was discovered for a linear approximation by Landau in the 1940s. A full mathematical proof in the nonlinear case was only provided very recently by Mouhot and Villani. For this proof along with other fundamental contributions Cédric Villani was awarded the Fields medal in 2010 shortly after giving this lecture. This lecture gives a very interesting and readable introduction to their theory. It also serves as an excellent introduction in the theory of collisionless kinetic equations.

Eric Sonnendrücker