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RADICAL, IDEAL AND EQUAL POWERS: NEW DEFINITIONS IN MODERN GEOMETRY 1814–1826

Jemma Lorenat

Abstract. — Alongside new practices, early nineteenth-century geometers in France and Germany developed a variety of new definitions with which to designate their objects of study. Accompanying arguments for choosing or creating new names or rejecting alternative designations reveal careful attention to epistemic values, potential user experiences, and evolving academic careers. To observe the effects of these arguments within the practice of geometry, we will focus on the nomenclature introduced in research articles by Louis Gaultier, Jean-Victor Poncelet, and Jakob Steiner to designate what were respectively called radical axes, ideal common chords, and lines of equal powers. During the 1820s each of these terms found currency in overlapping contexts, signifying both the geometrical value of the terminology and the proliferation of texts. We will show that naming was perceived as an important aspect in successfully introducing what these geometers declared to be modern geometry.

Résumé (Radicaux, idéaux et puissances égales : nouvelles définitions en géométrie moderne)

Parallèlement à de nouvelles pratiques, les géomètres du début du dix-neuvième siècle en France et en Allemagne développèrent une variété de nouvelles définitions pour désigner leurs objets d'étude. Les arguments donnés pour choisir ou créer de nouveaux noms ou pour rejeter des désignations alternatives révèlent l'attention portée aux valeurs épistémiques, aux expériences d'un utilisateur possible et aux évolutions des carrières académiques.

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Pour observer les effets de ces arguments dans la pratique de la géométrie, nous nous concentrerons sur la nomenclature introduite dans des articles de recherche par Louis Gaultier, Jean-Victor Poncelet et Jakob Steiner pour se référer à ce qu'ils appelaient respectivement axes radicaux, cordes communes idéales et lignes d'égales puissances. Pendant les années 1820, chacun de ces termes fut utilisé dans des contextes en chevauchement, témoins à la fois la valeur de la terminologie pour la géométrie et de la prolifération des textes. Nous montrerons que nommer était perçu comme un aspect important dans l'introduction réussie de ce que ces géomètres voyaient comme la géométrie moderne.

1. INTRODUCTION

During the early nineteenth century, the language of geometry evolved and expanded. In 1827, an anonymous reviewer in the *Bulletin des sciences mathématiques, astronomiques, physiques et chimiques* (also referred to as the *Bulletin de Férussac* after the editor) praised the recent changes in pure geometry. Additions to vocabulary, he wrote, had clarified, condensed, and simplified the science. The reviewer recognized this simplification as analogous to the symbolic abbreviations used by analysts.

> Analysts perceiving that certain quite complicated functions are reproduced frequently in their calculations, have called them exponentials, logarithms, sines, tangents, factorial derivatives, etc.; they have created abbreviated signs to designate them, and their formulas have acquired greater clarity and conciseness. And thus for certain points, certain lines and certain circles whose consideration is frequently represented in geometric speculations, it is natural to do the same with respect to them, and to call them, following their properties, similitude centers, radical centers, polars, similitude axes, radical axes, circles of common power, etc. This attention must inevitably introduce analogous simplifications in the statement of theorems and in the solution of problems which belong to the science of magnitude.[1] [*Bulletin* 1827, p. 279]

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1 "Les analystes s'étant aperçus que certaines fonctions assez compliquées se reproduisaient fréquemment dans leurs calculs, les ont appelées exponentiels, logarithmes, sinus, tangentes, dérivées factorielles, etc. ; ils ont créé des signes abréviatifs pour les désigner, et leurs formules en ont acquis beaucoup de clarté et de concision. Puis donc qu'il est certains points, certaines droites et certains cercles dont la considération se représente fréquemment dans les spéculations de la géométrie, il est naturel d'en user de même à leur égard, et de les appeler, suivant leurs propriétés, centres de similitude, centres radicaux, polaires, axes de similitude, axes radicaux, cercles de commune puissance, etc. Cette attention doit introduire inévitablement des simplifications analogues dans l'énoncé des théorèmes et dans la solution des problèmes qui appartiennent à la science de l'étendue."

By focusing on their interrelated properties, points, lines, and circles received new names. Somewhat paradoxically, an increase in new terms seemed to simplify a geometry that increasingly focused on these previously unnamed relationships.

Geometers presented their emerging vocabulary as improving the practice of geometry. The success of new terms depended on the strength of these presentations, but also on the status of the publications in which they appeared and the existent contemporary vocabulary and research questions. Like solving problems or proving theorems, definitions served as a potential medium with which a geometer might illustrate the advantages of his approach and make a name for himself. However, unlike other geometrical contributions, new definitions could render old definitions obsolete. While a geometer might introduce his new solution as equivalent to past solutions, definitions needed to be better.

This paper considers a series of definitions proposed between 1814 and 1826: Louis Gaultier's radical axes, Jean-Victor Poncelet's ideal common chords, and Jakob Steiner's lines of equal powers. The definitions and their respective receptions will be traced chronologically as each geometer responded to his predecessors. As we will see, these definitions relied on numerical properties, figure-based constructions, and a careful elaboration of key terms. A brief outline here will illuminate the main features.

Gaultier defined radical axes in 1814.[2] The radical axis was the locus of points o such that for secants g_1k_1o and g_2k_2o drawn respectively to two given circles $\sqrt{g_1o \cdot k_1o} = \sqrt{g_2o \cdot k_2o}$.

Several years later, Poncelet defined ideal common chords. Any common chord MN to two conic sections possessed two defining properties. First, the direction of MN determined a unique diameter to each conic section, and the respective conjugate diameters AB and $A'B'$ intersected at the center O of the chord.[3] Secondly, for uniquely defined constant values p and p' associated with the respective conic sections, $p \cdot OA \cdot OB = p' \cdot OA' \cdot OB'$. An ideal common chord was a common chord with imaginary points of intersection with the given conic.

Shortly after Poncelet's definition appeared, but with limited knowledge of contemporary French geometry, Steiner defined lines of equal powers as the line PG with respect to given circles centered at M and m with radii R and r. The line of equal powers was the locus of points of

[2] Gaultier's name often appeared as Gaultier de Tours in early nineteenth century citations, including this article. As there is no risk of ambiguity here, he will be referred to simply as Gaultier.

[3] Conjugate diameters will be discussed at greater length in Section 4.2