Astérisque

# HENRY C. PINKHAM Deformations of algebraic varieties with *G<sub>m</sub>* action

Astérisque, tome 20 (1974)

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### 1. Introduction

This thesis<sup>\*</sup> is devoted to a study of the deformations of an isolated singularity of an algebraic variety admitting a multiplicative group action. In this section we give a summary of the techniques used and the results obtained.

(1.1) First we set up some notation and review the relevant elements of deformation theory. Details can be found in
[43] and [45]. We fix once and for all an algebraically closed field k.

Let B be a local k-algebra of finite type. Let  $\mathcal{C}$  be the category of local artin k-algebras,  $\hat{\mathcal{C}}$  that of complete noetherian local k-algebras: hence if  $A \in \hat{\mathcal{C}}$  with maximal ideal m, then  $A/m^i \in \hat{\mathcal{C}}$  for all i.

<u>Definition</u> (1.2) An infinitesimal deformation of B to A  $\in \mathbb{C}$ is a cartesian diagram



where  $A \longrightarrow B'$  is flat.

\* A slightly different version of this work was submitted to Harvard University in partial fulfillment of the Ph.D. requirements in May 1974.

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<u>Definition</u> (1.3) D, the <u>local deformation functor</u> from Cto {sets} takes  $A \in C$  to isomorphism classes of deformations of B to A, isomorphism being defined in the obvious way. We extend D to  $\widehat{C}$  by taking inverse limits to get the notion of <u>formal deformation</u>.

We will usually be interested in minimal "complete" families of formal deformations, in the following sense: <u>Definition</u> (1.4) A formal deformation  $\zeta \in D(R)$ ,  $R \in \widehat{C}$ is <u>versal</u> if

(i) any formal deformation  $\zeta' \in D(S)$  may be deduced from  $\zeta$  by a base change  $R \longrightarrow S$ .

(ii) the Zariski tangent space of R is minimal for (i). R (or Spec R) is then called the <u>formal moduli space</u> of B; its tangent space (the k-vector space of first order deformations) is denoted  $T_B^1$ , or just  $T^1$  if no confusion is possible.

Lichtenbaum-Schlessinger [28] have studied  $T^1$  using the cotangent complex. Schlessinger's theorem [43] implies that when Spec B has an isolated singularity at its closed point, then B has a versal deformation.

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