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INVENTIVE INTERPRETATIONS

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Suppose we are investigating some substantial corpus of historical texts, without any access to notes, drafts, letters, conversations, etc. regarding the original composition of this material. Also suppose that, in this corpus, we find no reference whatsoever to some feature that we regard as intimately connected with the subject matter, something indeed that we find hard to put aside, or that we may not even realise is only in our mind. How might we react to this ?

The paradigm example I am thinking of here is Greek mathematics, in particular the deductive geometry of Euclid, Archimedes, and Apollonius and, to a lesser extent, parts of some later mathematicians and commentators. I am specifically excluding the later Heronian corpus and Ptolemy and the astronomical tradition, which have different goals from the deductive concerns of my authors. And the missing topic in this synthetic geometry is any involvement with any kind of numbers beyond the *arithmoi*, $1, 2, 3, \ldots$, conceived very concretely.[1]

Contrast that with the dominant approach to geometry today. For us a line has a length, a number ; a plane region has an area, a number, of which the most basic example is the area of a rectangle, the product of the lengths of its two sides. Curves often have equations, some arithmetical way of describing them ; for example $y^2 = ax$ is almost irresistible to us as shorthand for Apollonius' lengthy description of a parabola in his *Conics* — and this step illustrates another almost irresistible tendency, the move from arithmetic to algebra. A good illustration of this is given by T.L. Heath, introducing his translation of the *Method* and explaining his procedure:

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[1] Two mild exceptions to this are described briefly below and discussed at length in my book [Fowler 1987/1999].

"*In accordance with the plan adopted in* The Works of Archimedes*, I have marked by inverted commas the passages which, on account of their importance, historically or otherwise, I have translated literally from the Greek; the rest of the tract is reproduced in modern notation and phraseology*" [Heath 1912, p. 11].

The only passage thus marked with inverted commas is Archimedes' opening letter to Eratosthenes, together with the opening citations of the enunciations of propositions from other of his works. The rest (which, on a strong interpretation of Heath's words, are historically unimportant, or at least less important) are the propositions themselves, which he gives in modern notation.[2] It must be emphasised, however, that Heath is quite explicit about this, even explaining it in the full title of his book, and the same can be said of his earlier book on Apollonius' *Conics*, which used this approach to an even greater extent. There he chose groups of propositions, not necessarily consecutive, reformulated and rearranged them into a form which he thought most intelligible to his modern reader, and then, again with this modern reader in mind, composed their proofs himself and presented them algebraically. Heath does a great service in giving us Apollonius' material in a generally comprehensible way — Apollonius' own expositions are almost impossibly long and verbose for most readers today — but he does not give us any idea of how Apollonius presented, or perhaps even conceived, his own material.

A further example, isolated, excusable, but significant, can be found in Heath's translation of the *Elements* (which, again, has an accurate full title). Here he follows the Greek text very closely, but there is a slip in Book XIII, Proposition 11, where [Heath 1926, vol. 3, p. 464, 5 lines up] he translates "square root", a number, for what he elsewhere calls a "square line" ; he explains this usage in [Heath 1926, vol. 3, p. 13].[3]

What, then, is the principal interpretation today of this feature, the complete absence of anything like the rational or real numbers from this corpus ? To describe it briefly and brutally, it is that the early

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[2] Although translations into several languages have been published, no purely English-reading student has yet seen a translation of Archimedes' works. This may be a result of Heath's widely circulated, convenient, and readable account, but at last a real translation is now in progress, by Reviel Netz.

[3] I know of no other such examples in his translation, but they can be very difficult for a modern reader to spot for precisely the reasons I am explaining.

Greek mathematicians did start with an arithmetised geometry, but then reformulated it in this non-arithmetical way. Moreover, though this is not often said, they did this so thoroughly that no trace of this earlier arithmetic survives in this corpus of deductive mathematics. In other words, we have here an invention of its supposed presence, followed by an invention of its complete disappearance!

Of course there are reasons for these proposals. There is the existence of the earlier sophisticated Babylonian mathematics, which is thoroughly and visibly arithmetised, though more subtly and less obviously geometrical. Here the next invention of modern scholarship is of a Babylonian influence on the origins of Greek deductive mathematics, and hence its arithmetical basis at this earliest stage, and again I say "invention" because the evidence is indirect, mainly by comparing some Babylonian procedures with those of some Euclidean propositions. But, just to point to one missing feature, there is no evidence known so far of any sexagesimal numbers, the formulation of the Babylonian mathematics, to be found in Greek scientific thought before the time of Hypsicles in the 2nd century BC. What arithmetical procedures we do find in the whole spectrum of Greek life, from mathematics through general calculation along to land measurement and taxation, are done using the "Egyptian" procedure of sums of parts, of "unit fractions". The very few instances where we find what have been interpreted as common fractions are examined in [Fowler 1987, Chapter 7] where they are shown not to be so. The only places where sexagesimal fractions are to be found, consistently and thoroughly, is in astronomy after the time and works of Ptolemy, in the second century A.D., although Ptolemy's immensely successful *Almagest* certainly absorbed and rendered obsolete the works of earlier astronomers, for example Hypsicles.[4] The "Egyptian" parts even make a fleeting kind of appearance in the *Elements*, at the end of Book VII in Propositions 37 & 38, which refer to "the number which … will have a part called by the same name", like the third (as in a queue) and the third (as a part of the unit). In fact Greek writers comment on significant contacts with the Egyptians, but say almost nothing about the Babylonians.[5]

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[4] This is not unlike our own culture today, where we find sexagesimals only in the measurement of angles and times, which have their origin in astronomy.

[5] The exception is the celebrated passage in Herodotus, Histories ii, p. 109, where he

Now the disappearance. Here, again briefly and brutally, the explanation is that the discovery that $\sqrt{2}$ is an irrational number put in question this arithmetised approach to geometry, so the subject had to be reformulated non-arithmetically. There is evidence for this story, but all of it is late (4th century AD onwards) and found only in generally unreliable sources (mainly Iamblicus). A detailed analysis of this topic would be much too long to give here, so I refer the reader to my treatment of it in [Fowler 1999, Chapter 10.1, Appendix], which is presented there as a new introduction to the book itself. The conclusion is that such a statement cannot be justified by what we know.

So, in summary, we have very little if any evidence for the influence of Babylonian arithmetical procedures on early Greek mathematics, and only very unreliable evidence concerning its subsequent disappearance. So why do modern historians need to make this kind of interpretation? I tried to give one reason at the beginning: it is so much a part of our own culture that it is difficult for us to conceive of a geometry arising naturally without it, and we project our own approach back onto this ancient material. And I would like to finish by emphasising once again that, although this interpretation may eventually be found to be justified, at present it is invention, pure and simple.

I have only considered this one instance of Greek mathematics here, but there are many others. For another kind of example, observe how my introductory paragraph made no mention of mathematics, and the whole of historical interpretation of the past is candidate for discussion. For example, what might we say of some interpretations of parts of the Bible, as represented by the doctrines of different religions?

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talks of how "*the twelve divisions of the day came to Greece not from Egypt but from Babylonia*". This also is discussed in [Fowler 1987, Chapter 8.1], where it is suggested that what Herodotus is saying here is that the Greeks learned the measurement of land from the Egyptians and of time from the Babylonians. We can also add to this that this passage is not referring to the kind of deductive mathematics we are considering here.

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