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GEOMETRICAL PATTERNS IN THE PRE-CLASSICAL GREEK AREA. PROSPECTING THE BORDERLAND BETWEEN DECORATION, ART, AND STRUCTURAL INQUIRY

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ABSTRACT. — Many general histories of mathematics mention prehistoric "geometric" decorations along with counting and tally-sticks as the earliest beginnings of mathematics, insinuating thus (without making it too explicit) that a direct line of development links such decorations to mathematical geometry. The article confronts this persuasion with a particular historical case: the changing character of geometrical decorations in the later Greek area from the Middle Neolithic through the first millennium BCE.

The development during the "Old European" period (sixth through third millennium BCE, calibrated radiocarbon dates) goes from unsystematic and undiversified beginnings toward great phantasy and variation, and occasional suggestions of combined symmetries, but until the end largely restricted to the visually prominent, and not submitted to formal constraints; the type may be termed "geometrical impressionism".

Since the late sixth millennium, spirals and meanders had been important. In the Cycladic and Minoan orbit these elements develop into seaweed and other soft, living forms. The patterns are vitalized and symmetries dissolve. One might speak of a change from decoration into *art* which, at the same time, is a step *away* from mathematical geometry.

Mycenaean Greece borrows much of the ceramic style of the Minoans; other types of decoration, in contrast, display strong interest precisely in the formal properties of patterns – enough, perhaps, to allow us to speak about an authentically mathematical interest in geometry. In the longer run, this has a certain impact on the style of vase decoration, which becomes more rigid and starts containing non-figurative elements,

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without becoming really formal. At the breakdown of the Mycenaean state system around 1200 BCE, the "mathematical" formalization disappears, and leaves no trace in the decorations of the subsequent Geometric period. These are, instead, further developments of the non-figurative elements and the repetitive style of late Mycenaean vase decorations. Instead of carrying over mathematical exploration from the early Mycenaean to the Classical age, they represent a gradual sliding-back into the visual geometry of earlier ages.

The development of geometrical decoration in the Greek space from the Neolithic through the Iron Age is thus clearly structured when looked at with regard to geometric conceptualizations and ideals. But it is not linear, and no necessity leads from geometrical decoration toward geometrical exploration of formal structures (whether intuitive or provided with proofs). Classical Greek geometry, in particular, appears to have its roots much less directly (if at all) in early geometrical ornamentation than intimated by the general histories.

RÉSUMÉ. — MOTIFS GÉOMÉTRIQUES DANS L'AIRE DE LA GRÈCE PRÉ-CLASSIQUE. EXPLORATION DES FRONTIÈRES ENTRE DÉCORATION, ART ET RECHERCHE DE STRUCTURES. — Nombre d'histoires générales des mathématiques évoquent aux tout débuts des mathématiques les décorations « géométriques » de la préhistoire, en même temps que l'opération de compter et les baguettes à encoches, suggérant ainsi (sans que ce soit dit explicitement) qu'une ligne de développement directe lie ces décorations à la géométrie en tant que branche des mathématiques. L'article confronte cette conviction à un cas historique particulier: le caractère changeant des décorations géométriques dans ce qui sera l'aire grecque, du néolithique moyen au premier millénaire av. J.-C.

Pendant la période « européenne ancienne » (du sixième au troisième millénaire av. J.-C., dates obtenues à l'aide du carbone 14 et calibrées), le développement va de débuts non systématiques et non diversifiés vers un déploiement d'imagination et de variation, suggérant parfois des symétries combinées, mais ressortissant toujours au visuel sans être soumises à des contraintes formelles; ce type de décoration pourrait être appelé « impressionisme géométrique ».

Depuis la fin du sixième millénaire, les spirales et méandres y occupent une place importante. Dans l'orbite cycladique et minoenne, ces éléments se sont transformés en algues et autres formes souples. De la vie est insufflée dans ces dessins et les symétries se dissolvent. On pourrait parler d'une rupture, la décoration devenant art tout en s'éloignant simultanément de la géométrie.

La céramique de la Grèce mycénienne emprunte beaucoup au style minoen; d'autres types de décoration, en revanche, exhibent un fort penchant pour les propriétés formelles des dessins – suffisamment peut-être pour nous permettre de parler d'un intérêt authentiquement mathématique dans la géométrie. Sur la longue durée, ceci aura un certain impact sur le style des poteries décorées, qui devient plus rigide et commence à inclure des éléments non figuratifs, sans qu'ils soient purement formels. Lors de l'effondrement du système étatique mycénien, vers 1200 av. J.-C., cette formalisation « mathématique » disparaît et ne laisse pas la moindre trace dans les décorations de la période suivante, dite géométrique. Celles-ci résultent, en revanche, d'autres développements, ceux d'éléments non figuratifs et répétitifs présents sur les vases décorées de la période mycénienne tardive. Loin de transférer l'exploration mathématique présente au début de l'époque mycénienne à l'âge classique, elles représentent plutôt un retour progressif vers la géométrie visuelle des périodes antérieures.

Examiné à la lumière des conceptualisations et idéaux géométriques, le développement de la décoration géométrique dans l'aire culturelle grecque, du néolithique à l'âge de fer, apparaît ainsi clairement structuré. Mais il n'est pas linéaire, il ne mène pas nécessairement d'une décoration à caractère géométrique à l'exploration systématique de structures formelles (qu'elles soient intuitives ou accompagnées de preuves). En particulier, la géométrie grecque classique semble plonger ses racines moins directement que ne le suggèrent les histoires générales, dans les anciennes ornementations géométriques (si toutefois il y en a).

PRELIMINARY REMARKS

How did mathematics begin? And why did the ancient Greeks develop their particular and unprecedented approach to geometry? Such questions are probably too unspecific to allow any meaningful (not to speak of a simple) answer; even if meaningful answers could be formulated, moreover, sources are hardly available that would allow us to ascertain their validity.

In the likeness of the grand problems of philosophy (Mind-Body, Free Will, and so forth), however, such unanswerable questions may still engage us in reflections that illuminate the framework within which they belong, thereby serving to develop conceptual tools that allow us to derive less unanswerable kindred questions. The pages that follow are meant to do this.

They do so by analyzing a collection of photographs which I made in the National Archaeological Museum and the Oberländer Museum in Athens in 1983, 1992 and 1996, representing geometrical decorations on various artefacts, mostly ceramics; those of them which are essential for the argument are reproduced below.[1] All the artefacts in question were found within, and thus connected to cultures flourishing within, the confines of present-day Greece (Crete excepted). The earliest were produced in the sixth millennium BCE (calibrated radiocarbon date); the youngest belong to the classical age.

General histories of mathematics often identify geometrical patterns along with counting and tally-sticks as the earliest beginnings of the field.[2]

[1] All items are already published and on public display. The photos used here are all mine.

[2] In a sample of eleven works which I looked at, six began in that way: [Smith 1923], [Struik 1948], [Hofmann 1953], [Vogel 1958], [Boyer 1968] and [Wußing 1979]. [Cantor 1907], [Ball 1908] and [Dahan-Dalmedico & Peiffer 1982] take their beginnings with the scribes of the Bronze Age civilization. So does [Kline 1972] on the whole, even though he does discuss pre-scribal mathematics on half a page, and mentions "geometric

Mathematicians (and in this respect historians of mathematics belong to the same tribe) tend to assume that what *we* describe in terms of abstract pattern and shape was also somehow meant by its producers to deal with pattern and shape *per se*, or was at least automatically conducive to interest in these; this is never stated explicitly, but it is an implied tacit presupposition. At least for members of our mathematical tribe it seems a reasonable presupposition.

When first running into the objects rendered in my photographs, I was indeed struck by the easily distinguishable trends in the changing relation of these patterns to geometrical inquiry and thought (what I mean by this beyond "interest in pattern and shape *per se*" will be made more explicit in the following); I also noticed, however, that development over time could as easily lead away from mathematical geometry as closer to it. Mathematics is no necessary, not even an obvious consequence of the interest in visual regularity (which, on its part, appears to be rather universal). Not every culture aims at the same type of regularity, and the interest in precisely mathematical regularity is a choice, one possible choice among several.

On the other hand, the universal human interest in regularity – that "sense of order" of which Gombrich [1984] speaks – *may* certainly lead to systematic probing of formal properties of symmetry, similarity, etc. Whether such inquiry is connected to some kind of proof or argument or not (which mostly we cannot know), there is no reason to deny it the label of "mathematics" (or, if we prefer this distinction and that use of the term, "ethnomathematics", as an element of mathematical thought integrated in an oral or pre-state culture). In order to distinguish these cases from such uses of patterns and shapes whose intention and perspective we are unlikely to grasp through a characterization as "mathematics", we need to develop concepts that reach further than the conventional wisdom (or, with Bacon, "idols") of our tribe.

My purpose is thus primarily a clarification of concepts which may permit us to look deeper into the relation between decorative patterns and mathematics; it is neither the history of artistic styles nor the links

decoration of pottery, [and] patterns woven into cloth" in these eight words. Chapter 1 on "Numeral Systems" of [Eves 1969] contains half a page of speculations on "primitive counting".

between cultures. For this reason I do little to point out the evident connections between, for example, the decorations found on Greek soil and the styles of the Vinča and other related Balkan cultures.

The gauge is deliberately anachronistic, and I make no attempt to interpret the artefacts which I discuss in their own practical or cultural context (although I do refer occasionally to their belonging within a specific framework – deliberate anachronism should never be blind to *being* anachronism). My purpose is, indeed, not to understand this context but to obtain a better understanding of the implications of that other blatant anachronism which consists in reading early decorations in the future-perfect of mathematics – an anachronism which can only (and should only, if at all) be defended as a way to understand better the nature *of mathematics* and the conditions for its emergence.

Though this was not on my mind when undertaking the investigation, my approach can be described as a *hermeneutics of non-verbal expression* – "hermeneutics" being so far taken in Gadamer's sense that the expression of "the other" is *a priori* assumed, if not to be "true" (obviously, expressions that do not consist of statements possess no truth value) then at least to be "true to an intention". Whereas the habitual ascription of a "mathematical intention" to every pattern and symmetry can be compared with that reading of a foreign text which locates it straightaway within the "horizon" of the reader, my intention here may be likened to Gadamer's *Horizontverschmelzung*, "amalgamation of horizons". In agreement with Gadamer's notion of the hermeneutic circle I presuppose that such an amalgamation is possible, that our present horizon can be widened so as to encompass that of the past "dialogue partners" (yet without sharing Gadamer's teleological conviction that this amalgamated horizon can also be said to be the *true* implied horizon of the partners; the wider horizon remains ours, and remains anachronistic).[3] As we shall see, this requires that our wider horizon transcends that of the mathematical tribe.

As affirmed emphatically by Gadamer, hermeneutics is no method, no prescription of the steps that should be taken in the interpretation of a

[3] See [Gadamer 1972, pp. 289f and *passim*]. The stance that the amalgamated horizon is the true implied horizon of the partners corresponds to that kind of historiography of mathematics according to which contemporary mathematicians, those who have insight into the tradition as it has unfolded, are the only ones that are able to understand the ancient mathematicians and thus those who should write the history of mathematics.