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These notes are essentially an expanded version of a few talks given at the Ecole Normale Supérieure (Verdier Seminar) in the spring of 1980 by the undersigned as well as one talk by M.S Narasimhan. Some unpublished material figures in the notes :

- i) the moduli of vector bundles on singular (not necessarily irreducible) curves. Here one finds a complete generalisation of D'Souza's thesis [5] as well as a partial generalization of Oda-Seshadri [30] (it would be interesting to carry out a more detailed generalization of Oda-Seshadri). It would also be interesting to make a detailed study of the relation between this moduli space and the one on the normalization of the curve. Some work has been done by Gieseker in this connection (lectures of TIFR in 1979-80) ; in fact this work of Gieseker served as the catalyst for this investigation.
- ii) the non-existence of certain Poincaré families. This proof is due to M. Nori.
- iii) the moduli space of vector bundles with level structures. This is related to the work of Drinfeld. We follow the definition of D. Mumford in an unpublished manuscript. D. Mumford defines this concept only for vector bundles (i.e only for torsion free sheaves). It would be interesting to investigate the connection between these varieties and those on singular curves.

These notes have been written by J.M Drezet who has done a tremendous job. He has carried out many improvements and most often he had only to rely on sketches of proofs.

It is great pleasure to thank J.L Verdier and the audience for the interest shown by them.

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INTRODUCTION

One is interested here in vector bundles on a projective curve over k , an algebraically closed field, and more generally in coherent sheaves (possibly with some extra structures). One tries to classify such objects. For example one wants to build an algebraic variety whose geometric points are isomorphism classes of vector bundles. This variety should satisfy some "universal property", that is, represent some "natural" functor from the category of noetherian k -schemes to the category of sets.

This construction is possible only for a kind of vector bundles (or sheaves ...), which are said to be *stable*. To get complete varieties, one needs to consider also *semi-stable* bundles.

First are given the definition and elementary properties of (semi-)stable vector bundles (or sheaves...). Then existence theorems of moduli varieties are obtained. Next we study these moduli varieties (are they reduced, have they singular points ...).

The demonstrations of the existence theorems are very similar, so the calculations are given only in the cases of level structures and coherent sheaves on a reduced curve, which are not treated in the literature. The "theoretical" part is detailed only in the cases of vector bundles, and vector bundles with parabolic structures.

In the first part we are interested in (semi-)stable vector bundles on a smooth projective curve over k . We give some results of theories (Grothendieck schemes, Mumford's geometric invariant theory) which are used in the other parts.

The second part deals with the deformation of the moduli varieties obtained in the first part, and follows a talk of M.S Narasimhan.

In the third part we study (semi-)stable vector bundles with parabolic structures. Langton's work (adapted to bundles with parabolic structures) is exposed only here to shorten the first part. The results of this part follow a paper of Mehta and Seshadri.

The fourth part treats of level structures. They appear in the study of (semi-)stable sheaves on a reduced projective curve.

In the fifth part we build a "natural" desingularisation of the moduli varieties of semi-stable vector bundles.

The sixth part gives a partial demonstration of a result of S. Ramanan about the inexistence of Poincaré bundles on a non empty open set of some moduli varieties. The way to prove this is different from this of Ramanan, and is due to M. Nori.

The seventh part deals with coherent (semi-)stable sheaves on a reduced projective curve over k . It is a generalization of the first part.

In the eighth part we study some properties of the moduli of coherent semi-stable sheaves built in the seventh part, when the singular points of the curve are ordinary double points.

The results of the fourth, fifth, seventh and eighth part are due to C.S Seshadri.