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ONE-DIMENSIONAL GENERAL FOREST FIRE PROCESSES

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Xavier BRESSAUD
Nicolas FOURNIER

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Maison de la SMF
B.P. 67
13274 Marseille Cedex 9
France
smf@smf.univ-mrs.fr

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Secrétariat : Nathalie Christiaën

Mémoires de la SMF
Société Mathématique de France
Institut Henri Poincaré, 11, rue Pierre et Marie Curie
75231 Paris Cedex 05, France
Tél : (33) 01 44 27 67 99 • Fax : (33) 01 40 46 90 96
revues@smf.ens.fr • http://smf.emath.fr/

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Xavier Bressaud

Xavier Bressaud: Université P. Sabatier, Institut de Maths de Toulouse, F-31062 Toulouse Cedex, France.

E-mail : `bressaud@math.univ-toulouse.fr`

Nicolas Fournier

Nicolas Fournier: Laboratoire d'Analyse et de Mathématiques Appliquées, CNRS UMR 8050, Université Paris-Est, 61 avenue du Général de Gaulle, 94010 Créteil Cedex, France.

E-mail : `nicolas.fournier@univ-paris12.fr`

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ONE-DIMENSIONAL GENERAL FOREST FIRE PROCESSES

Xavier Bressaud, Nicolas Fournier

***Abstract.* —** We consider the one-dimensional generalized forest fire process: at each site of $\mathbb{Z}$, seeds and matches fall according to i.i.d. stationary renewal processes. When a seed falls on an empty site, a tree grows immediately. When a match falls on an occupied site, a fire starts and destroys immediately the corresponding connected component of occupied sites. Under some quite reasonable assumptions on the renewal processes, we show that when matches become less and less frequent, the process converges, with a correct normalization, to a limit forest fire model. According to the nature of the renewal processes governing seeds, there are four possible limit forest fire models. The four limit processes can be perfectly simulated. This study generalizes consequently previous results of **[15]** where seeds and matches were assumed to fall according to Poisson processes.

***Résumé* (Processus de feux de forêt généraux en dimension 1)**

Nous étudions le processus des feux de forêt généralisé en dimension 1 : sur chaque site de $\mathbb{Z}$, des graines et des allumettes tombent suivant des processus de renouvellement stationnaires i.i.d. Quand une graine tombe sur un site vide, un arbre pousse immédiatement. Quand une allumette tombe sur un site occupé, un feu démarre et brûle immédiatement la composante connexe occupée autour de ce site. Nous montrons — sous des hypothèses raisonnables sur les processus de renouvellement — que lorsque la fréquence des allumettes tend vers zéro, le processus converge, correctement renormalisé, vers un processus limite. Suivant la nature des processus de renouvellement gouvernant l'apparition des graines, quatre processus limites sont possibles. Les quatre modèles limites peuvent être simulés parfaitement. Cette étude généralise des résultats de **[15]**, où nous supposions que graines et allumettes tombaient suivant des processus de Poisson.

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CHAPTER 1

INTRODUCTION

1.1. Introduction

Consider a graph $G = (S, A)$, S being the set of vertices and A the set of edges. Introduce the space of configurations $E = \{0, 1\}^S$. For $\eta \in E$, we say that $\eta(i) = 0$ if the site $i \in S$ is vacant and $\eta(i) = 1$ if i is occupied by a tree. Two sites are neighbors if there is an edge between them. We call forests the connected components of occupied sites. For $i \in S$ and $\eta \in E$, we denote by $C(\eta, i)$ the forest around i in the configuration η (with $C(\eta, i) = \varnothing$ if $\eta(i) = 0$). We consider the following (vague) rules:

- ▷ vacant sites become occupied (a seed falls and a tree immediately grows) at rate 1;
- ▷ occupied sites take fire (a match falls) at rate $\lambda > 0$;
- ▷ fires propagate to neighbors (inside the forest) at rate $\pi > 0$.

Such a model was introduced by Henley [**37**] and Drossel and Schwabl [**27**] as a toy model for forest fire propagation and as an example of a simple model intended to clarify the concept of *self-organized criticality.*

The order of magnitude of the rate of growth is much smaller than the propagation rate, $\pi \gg 1$. We will focus here on the limit case where the propagation is instantaneous: when a tree takes fire, the whole forest (to which it belongs) is destroyed immediately. The model is thus:

- ▷ vacant sites become occupied (a seed falls and a tree immediately grows) at rate 1;
- ▷ matches fall on occupied sites at rate λ and then burn instantaneously the corresponding forest.

The features of the model depend on the geometry of the graph; we only consider in this paper the case $S = \mathbb{Z}$ (with its natural set of edges). They also depend on the laws of the processes governing seeds and matches; the standard case is when these are Poisson processes so that the forest fire process is Markov. We deal here with the most general (stationary) case; Poisson processes are replaced by stationary renewal processes.

Our main preoccupation is the behavior of this model in the asymptotic of rare seeds, namely when $\lambda \to 0$. We present four possible limit processes (depending on the tail properties of the law of the stationary processes governing seeds) arising when we suitably rescale space and accelerate time while letting $\lambda \to 0$. This is a considerable generalization of the results obtained in [**15**].

This introduction consists of six subsections.

(i) In subsection 1.1.1, we briefly recall the concept of *self-organized criticality* and recall a certain number of models supposed to enjoy self-organized critical properties.

(ii) We present in subsection 1.1.2 a quick history of the forest-fire process, its other possible interpretations and its links with other models.

(iii) subsection 1.1.3 explains the importance of the geometry of the underlying graph G and the links of the forest-fire model with *percolation*.

(iv) In subsection 1.1.4, we recall what has been done for the (Markov) forest-fire process on $\mathbb{Z}$ from a rigorous mathematical point of view.

(v) subsection 1.1.5 is devoted to a brief exposition of the main ideas of the present paper.

(vi) Finally, we give the plan of the paper in subsection 1.1.6.

1.1.1. Self-organized criticality. — One of the successes of statistical mechanics is to explain how local interactions generate macroscopic effects through simple models on lattices. Among the most striking phenomena are those observed around so-called *critical values* of the parameters of such models, such as scale-free patterns, power laws, conformal invariance, critical exponents or universality.

1.1.1.1. Paradigm. — The study of self-organized critical systems has become rather popular in physics since the end of the 80's. These are simple models supposed to clarify temporal and spatial randomness observed in a variety of natural phenomena showing *long range correlations*, like sand piles, avalanches, earthquakes, stock market crashes, forest fires, shapes of mountains, clouds, etc. It is remarkable that such phenomena reminiscent of critical behavior arise so frequently in nature where nobody is here to finely tune the parameters to critical values.

An idea proposed in 1987 by Bak-Tang-Wiesenfeld [**5**] to tackle this contradiction is, roughly, that of systems *growing* toward a *critical state* and relaxing through *catastrophic* events: avalanches, crashes, fires, etc. If the catastrophic events become more and more probable when approaching the critical state, the system spontaneously reaches an equilibrium *close* to the critical state. This idea was developed in [**5**] through the study of the *archetypical* sand pile model.