

**LEONARDO FIBONACCI AND ABBACO CULTURE.
A PROPOSAL TO INVERT THE ROLES**

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In memoriam David Fowler, gentle friend and wise colleague

ABSTRACT. — Since long it has been regarded as an obvious fact in need of no argument that the mathematics of the Italian abacus school was taken over from Leonardo Fibonacci's *Liber abbaci*. What does look like an argument is that an abacus book from the outgoing 13th century (apparently the earliest extant specimen) claims to be made "according to the opinion" of Fibonacci. Close analysis of the text reveals, however, that everything basic is independent of Fibonacci, while the indubitable borrowings from the *Liber abbaci* are sophisticated matters, often copied without understanding; a text which appears to be copied from an even earlier treatise is wholly independent of Fibonacci but related to writings of abacus type from the Ibero-Provençal area. Combination of the Italian and Ibero-Provençal evidence with certain passages in the *Liber abbaci* shows that the beginnings of abacus mathematics must be traced to an environment that already existed in Fibonacci's days — an environment he knew about and of which he can be regarded an extraordinary early exponent, but no founding father.

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RÉSUMÉ (Leonardo Fibonacci et la culture de l'abbaque. Une proposition pour en inverser les rôles)

Les historiens des mathématiques étudiant l'école d'abbaque italienne prennent d'habitude pour un fait évident que les mathématiques de cette école proviennent du *Liber abbaci* de Léonard de Pise. Un des arguments avancés en faveur de ce point de vue repose sur l'affirmation, trouvée dans un livre d'abbaque datant de la fin du XIII^e siècle (probablement le premier qui nous est parvenu), qu'il a été écrit « selon l'opinion » de Léonard. Une analyse plus serrée du texte révèle cependant que toutes les parties élémentaires sont indépendantes du *Liber abbaci*, alors que les emprunts sûrs sont tous d'un niveau plus sophistiqué et ne sont souvent pas compris du compilateur. Un autre texte, sans doute une copie d'un traité encore plus ancien, est totalement indépendant du *Liber abbaci*, mais apparenté à certains égards à des traités de type abbaque provenant de l'aire ibéro-provençale. Le rapprochement de certains passages du *Liber abbaci* avec les textes italiens et ibéro-provençaux montre que les origines des mathématiques d'abbaque sont à chercher dans un milieu qui existait déjà du temps de Léonard, que celui-ci connaissait et dont il peut être considéré un représentant précoce et hors pair, mais pas un père fondateur.

THE RECEIVED VIEW

As long as the existence of the late medieval and Renaissance Italian abbaco tradition has been recognized, it has been taken for granted by almost everybody that it had to descend from Leonardo Fibonacci's writings, at most with more or less marginal additions. In particular, this has been the repeated view of those scholars who know the tradition most intimately and who have made it known to the rest of the world.

The latest phrasings of the view may be those of Elisabetta Ulivi [2002, p. 10], according to whom the *libri d'abbaco* “were written in the vernaculars of the various regions, often in Tuscan vernacular, taking as their models the two important works of Leonardo Pisano, the *Liber abaci* and the *Practica geometriae*”;¹ of Raffaella Franci [2002, p. 81], whose opinion is that the *Liber abbaci* “was the most important source for abacus teaching in Italy”, and that algebra most plausibly “entered the abacus curricula because it was the subject of a long chapter” of that work; and of Maryvonne Spiesser [2004, p. 17], who asserts that it was “*un modèle, en général inégalé, pour les arithmétiques pratiques italiennes de plusieurs générations, modèle qui, directement ou non, s'est peu à peu étendu en dehors des frontières de l'actuelle Italie. Il a aussi été un relais primordial dans la transmission*

¹ As everywhere in the following where no translator is identified, I am responsible for the translation.

des problèmes d'origine parfois très ancienne". Those of Ulivi and Franci are casual remarks, reflecting what is felt to be so obvious that no argument is needed (nor is any argument given). Spiesser gives references for Fibonacci's Arabic inspiration but no arguments for his being himself a source or a model.

Even stronger was Warren Van Egmond's statement that all abacus writings "can be regarded as [...] direct descendants of Leonardo's book" [Van Egmond 1980, p. 7]. As regards abacus algebra in particular, the same author asserted that this "tradition is logically a continuation of the work of Leonardo Pisano" [Van Egmond 1988, p. 128] though not explaining which logic should be involved.

In [Franci & Rigatelli 1985, p. 28], Raffaella Franci and Laura Toti Rigatelli had stated similarly that "the abacus schools had risen to vulgarize, among the merchants, Leonardo's mathematical works".² As regards *the algebra* contained in some of the treatises, however, Franci and Toti Rigatelli already mitigated the claim just quoted in the same article by the observation (p. 45) that

"in Florence, in the 14th century, at least two algebraic traditions coexisted. One of them was inspired by Leonardo of Pisa and was improved by Biagio the Old and Antonio de' Mazzinghi, the other, the beginning of which is unknown until now, has [Paolo] Gerardi as its first exponent."

Partial divergence from the exclusive reference to Fibonacci was also expressed by Gino Arrighi [1987, p. 10], when he suspected Paolo Gherardi's *Libro di ragioni* and another treatise which he ascribed to the same author to be either re-elaborations or translations of French writings; on the other hand he stated (p. 5) that these treatises are the only witnesses we have of important mathematical exchanges between Italy and France (i.e., the Provençal area³).

² More recently, Franci [forthcoming] has downplayed the importance of the *Liber abbaci* significantly while maintaining that of Fibonacci, suggesting that the inspiration was derived from a lost *liber minoris guise*, "book in a smaller manner", which Fibonacci says to have written [Boncompagni 1857, p. 154]. I shall return to my reasons for finding this implausible in note 11, cf. also note 25.

³ Politically, Montpellier was only definitively integrated in the French Kingdom in 1349 (which did not in itself make it culturally French), after having been bought from the Aragon-Majorcan king; Avignon and the surrounding Comtat Venaissin were only absorbed by France in 1791. Thirteenth-century practical arithmetic from France proper, as known

Franci [2002, p. 82] sharpened her dissent from the prevailing view somewhat – still only with reference to fourteenth-century algebra. Now she accepted that its “authors may have had access to Arabic sources different from those used by Leonardo”. Still partial but none the less more general divergence from the conventional wisdom was expressed by Enrico Giusti [2002, p. 115], according to whom some of the abacus writings

“were genuine and proper vernacular versions of [Fibonacci’s] works, made easier by elimination of the most abstract and theoretical parts; in other cases the author limits himself to dig in the mine of examples and problems from the *Liber abaci*, in order to find material he could insert in his own treatise.”

Before the autonomous existence of the abacus tradition was recognized, it was even more obvious to those few who did work on abacus material that it could only belong within a current leading from Fibonacci to Luca Pacioli, Tartaglia and Cardano. One clear enunciation is due to Louis Karpinski [1929, p. 177], who ends his description of Jacopo da Firenze’s *Tractatus algorismi* from 1307 with the observations that the

“treatise by Jacob of Florence, like the similar arithmetic of Calandri, marks little advance on the arithmetic and algebra of Leonard of Pisa. The work indicates the type of problems which continued current in Italy during the thirteenth to the fifteenth and even sixteenth centuries, stimulating abler students than this Jacob to researches which bore fruit in the sixteenth century in the achievements of Scipione del Ferro, Ferrari, Tartaglia, Cardan and Bombelli.”

One reason for the persistence of this belief (which, as I shall argue, is largely illusory) is probably the *principle of the great book*, to which scholars are prone to fall victims: the belief that everything in a book, if not an innovation, must be derived from a *famous book* that is *known to us* – known at least by name and fame if no longer extant.

In a way, this principle can be seen as a sound application of Occam’s razor: explanatory entities in the shape of marvellous secret traditions that have left no traces should not be multiplied without necessity. But if

from the last part of the *Pratike de geometrie* [Victor 1979] was very different in character from what we know from Jacopo da Firenze’s *Tractatus algorismi* and Paolo Gherardi’s *Libro di ragioni* (both written in Montpellier, in 1307 and 1328, respectively), and also from a *Trattato di tutta l’arte dell’abaco* (Rome, Biblioteca dell’Accademia Nazionale dei Lincei, Cors. 1875, with parallel manuscripts) written in Avignon in the 1330s (see [Cassinet 2001]; the ascription of the latter treatise to Paolo dell’Abbaco, *e.g.* in [Van Egmond 1977], is apparently based solely on a probably ill-founded guess by a fifteenth-century owner of one of the manuscripts).

applied without attention to the copious evidence that is offered by less famous sources, without regard for the details of the material and without recognition of the fact that this extant material may contain more holes than cheese, then it can at best be compared to Kepler's explanation of planetary movements by means of magnetism, the only force acting at a distance he knew.

However, the creed of modern scholars is only half of the explanation. Early sources also seem to suggest a key role for Fibonacci. In the *Ars magna*, Cardano [1663, p. 222] tells that algebra took its beginning with al-Khwārizmī and was copiously developed by Fibonacci; much later, as he further relates, three new derivative chapters were added by an unknown author, being put together with the others by Luca Pacioli.

We may go even further back. The rather few abacus writers of the mature tradition who refer to intellectual ancestors tend to mention Fibonacci together with more recent *maestri d'abbaco*. Moreover, already (what is likely to be) the oldest extant abacus treatise presents itself as a *Livro de l'abbecho* “secondo la oppenione de maistro Leonardo de la chasa degli figluogle Bonaçie da Pisa” [Arrighi 1989, p. 9], an “*Abacus book* according to the opinion of master Leonardo Fibonacci”. This seems to leave little doubt that Fibonacci was indeed a founding father of abacus mathematics, if not *the* father.⁴

THE UMBRIAN EVIDENCE

This earliest extant *libro d'abbaco* (Florence, Riccardiana, MS 2404, fols 1^r–136^v) appears from internal evidence to have been written in c. 1288–1290 in Umbria.⁵ Whoever starts reading attentively beyond the introductory lines that were just quoted will discover that it contains material that

⁴ In the interest of moral balance I shall cite my own [Høyrup 2000, p. 56] as an example of a scholar taken in by this title and the identification of some indubitable borrowings.

⁵ The actual date of the original may be slightly later, *cf.* note 37, and the vellum manuscript we possess is so beautiful that it is likely to be a *de luxe* copy and not the original; it may thus be even later. Improved understanding of the coin list contained in the “Columbia Algorithm” (Columbia University, MS X 511 A13, [Vogel 1977]) due to Lucia Travaini [2003, p. 88–92] shows that at least this list (which is not annexed to the text but integrated) was made in the years between 1278 and 1282. The manuscript has habitually been ascribed to