Revue d'histoire des mathématiques
17 (2011), p. 175–210

ÉVARISTE GALOIS AND THE SOCIAL TIME OF MATHEMATICS

Caroline Ehrhardt

Abstract. — The thrust of this article is to offer a new approach to the study of Galois's *Mémoire sur les conditions de résolubilité des équations par radicaux*. Drawing on methodology developed by social and cultural historians, it contextualizes Galois's work by situating it in the parisian mathematical milieu of the 1820s and 1830s. By reconstructing the social process whereby a young man became an established mathematician at the time, this article shows that Galois's trajectory was far from unusual, and most importantly, that he was not treated differently from other aspiring mathematicians.

Second this article seeks to operate a shift from the writing of biographies of mathematicians to biographies of mathematical texts. Indeed, the meaning of a mathematical text is the product of a long social and scientific process, one that, in the case of Galois's text, took over one hundred years. During this long period, Galois's text was read, interpreted and recast by a large number of actors who did not agree as to its meaning and mostly construed it through local lenses. Only at the beginning of the 20th century, when Galois theory entered the realm of teaching in European countries, did it acquire a more unified meaning. By then, Galois, the aspiring mathematicians who had failed to convince the members of the *Académie des sciences*, was becoming a legend.

Résumé (Évariste Galois et le temps social des mathématiques)

Cet article propose une nouvelle approche de l'étude du *Mémoire sur les conditions de résolubilité des equations par radicaux* de Galois. En se fondant sur la méthodologie de l'histoire sociale et culturelle, il contextualise le travail de Galois en le situant dans le milieu mathématique parisien des années 1820 et 1830. Tout d'abord, en reconstruisant le processus social par lequel un jeune homme peut devenir un mathématicien reconnu à cette époque, cet article

Texte reçu le 10 avril 2011, accepté le 17 juin 2011.

C. Ehrhardt, IFÉ/ENS-Lyon, Service d'histoire de l'éducation, 45, rue d'Ulm, 75230 Paris Cedex 05 (France).

Courrier électronique : caroline.ehrhardt@ens-lyon.fr

2000 Mathematics Subject Classification : 01A55, 01A60, 01A80; 11–03, 12–03.

montre que la trajectoire de Galois est loin d'être exceptionnelle et, surtout, qu'il n'a pas été traité différemment des autres aspirants-mathématiciens de sa géneration. Ensuite, cet article propose de passer de la biographie des mathématiciens à la biographie des textes mathématiques. De fait, le sens d'un texte mathématique est le produit d'un long processus social et scientifique, un processus qui, dans le cas de Galois, a pris plus de cent ans. Pendant cette longue période, le texte de Galois a été lu, interprété et reformulé, dans des contextes locaux, par un grand nombre d'acteurs qui ne s'accordaient pas nécessairement sur son sens. Ce n'est qu'au début du XX^{e} siècle, alors que la théorie de Galois était en train de devenir une matière d'enseignement, qu'elle a acquis un sens plus uniforme. Mais à ce moment là Galois, l'aspirant-mathématicien qui n'avait pas réussi à convaincre les membres de l'*Académie des sciences*, était déjà en train de devenir une légende.

1. INTRODUCTION

Évariste Galois (1811–1832) has received extensive treatment by historians of mathematics, who have written dozens of biographies and monographs about his work, along with numerous studies of the development of 19th century algebra in which he is given an important role.[1] To these works one may add texts produced since the beginning of the 20th century for larger audiences, texts which have established a legend around Galois's personality.[2] This abundance of publications reflects the importance of Galois and his writings in mathematics and in the history of mathematics. It also invites us to explore in this paper questions of method. Some of these questions are immediately relevant to the social and cultural history of mathematics, but Galois's case actually leads straight to the more general question of how mathematical knowledge is constructed historically. This article is based on the empirical study I conducted for my doctoral dissertation, which focused on Galois's afterlife during the 19th century [Ehrhardt 2007]; detailed results of my work are about to be published; see [Ehrhardt 2011a] and [Ehrhardt 2011b].

The starting point of this study is a simple question: how did *Galois theory*, which is one of the fundamental theories in modern algebra, come to be

---

[1] With no intention to cover one century of historiography, one may mention the biographies [Dupuy 1896], [Dalmas 1956], [Toti Rigatelli 1996], Taton's articles [Taton 1947], [Taton 1971], and [Taton 1983], the article [Infantozzi 1968], as well as the studies [Kiernan 1971], [Wussing 1984], [Hirano 1984], [Toti Rigatelli 1989], [Dahan-Dalmedico 1983], [Friedelmeyer 1991] and, more recently, [Galuzzi 2001].

[2] Here one may cite biographical novels like [Bell 1937], [Infeld 1948], and most recently [Auffray 2004], as well as books like [Verdier 2003].

known as such? More precisely, why do we continue to attribute to Évariste Galois a theory which not only reaches far beyond the scope of his own writings, but which is entirely based on a mathematical machinery—the structural algebra of the 1920s and 1930s—which is completely alien to Galois. Indeed, looking at Évariste Galois's life and manuscripts, one is first struck by the fact that this mathematician, when he died at age twenty, did not leave behind more than some 60 sheets of manuscripts.[3] Among those, his most accomplished work, the *Mémoire sur les conditions de résolubilité des équations par radicaux*, is a short and cryptic text from which proofs are mostly missing and which was refused in 1831 by the *Académie des sciences*.[4] And it actually does not contain what one is used to recognize as "Galois theory." Furthermore, in Galois's case, the problem of the authorship of the theory generates another question, related to how one can properly write the history of mathematics; just as Galois theory is the product of a collective historical construction, is not Galois's personality equally the product of such a construction, due in no small part to historians and philosophers of mathematics?

The label "Galois theory" thus makes us wonder about the way in which mathematical knowledge is created and distilled over time, transforming a simple *mémoire* into a fully crystallized theory, and what this means for the alleged solidity and universality of the knowledge thus created. When we use the expression "Galois theory" both with respect to 1850, and to 1900, or 1950, we ought to be speaking of the "same" thing. But Galois's name in this expression already betrays a tension between historicity and transcendence of mathematical results—a tension which is germane to both mathematics and its historiography. Evariste Galois therefore offers an especially interesting occasion to analyze the way in which a text with a specific known origin in time and place, acquires the status of certified, transhistorical knowledge.

Looking more closely, we see that we are asking for the historical construction of what makes mathematical objects ideal. This process in general involves a great deal of everyday practices, graphic procedures (the use of curves, tables, diagrams…), symbolic manipulations (computations, combinations of objects in space), an institutional framework (teaching, places of learning and research, etc.), and also representations of research domains (what is interesting? what does a certain concept "mean"?). Such a

3 Galois's works have recently been edited completely: [Galois 1962/1997].

4 On this point, see [Ehrhardt 2010a].

historical construction therefore takes place in social spaces of relevance for mathematical work that are themselves anything but universal.

The *Science Studies* approach however, with its focus on local studies and short term time horizons, fails to capture one of the most fundamental aspects of the constitution of mathematical knowledge: its claim to universality. Both the ways in which local mathematical practices are extended and elevated and the stakes involved in this process, remain a blind spot in the historiography. The ideotypical character that is easily granted to mathematical objects fits badly with the small scale at which they are actually produced.[5]

In order to retrace the combined establishment of the various mathematical theories that originated from Galois's work as well as the legend of the cursed mathematician which is today attached to his name, I offer the hypothesis that the historical character of mathematical knowledge actually lies in the way in which it is passed on. Far from being a neutral operation, this process of transmission is the very place where the historicity of mathematics lies, because it brings to the fore the successive, or intertwined, categories by which this knowledge is conceived, elaborated, and understood.[6] This means we need to analyze the composition and the reading of a mathematical text as resulting from the interaction between mental sets of tools, which depend on the apprenticeship of the authors and the social domains in which they move. Such an interaction takes place within a control system that is proper to the mathematical field in a given social space and time.

Galois's case further invites reflection on the multiple historical contexts into which a mathematical text is embedded and which serve to make sense of it: if Galois's *mémoire* was at first read in 1831 according to the criteria of the *Académie des sciences*, it subsequently remained a topical part of mathematical research all through the 19th century through several revivals. The passing of time did not transform this work into a "historical" mathematical text, for mathematicians renewed its intelligibility by continued usage.[7] In this sense today it still belongs to contemporary mathematics, as is revealed by the adjective use of the name Galois—or

5 Mathematics have been studied only very rarely along those lines of research. Recent works inspired by this approach—in spite of their great quality and the seminal way in which they open up new lines of research—are either locally focussed ([Warwick 2003]) or work in a short time range ([Rosental 2003]).

6 This point of view has been developed in [Cifoletti 1998].

7 The question of how the shelf live and the identity of mathematical objects are related to the their ongoing use is at the center of [Goldstein 1995].

the French adjective "galoisien", and similarly in other languages—to describe many notions and ideas. The *Mémoire sur les conditions de résolubilité des équations par radicaux* has thus accompanied the mathematical present for almost two centuries—but this mathematical present asks ever changing questions, from one generation to the next, because it takes different criteria and practices for granted, and because it takes place in cultural and social environments which evolve over time.

Finally, the posterity of Evariste Galois and his work is embedded into multiple social spaces and times. It cannot be understood unless one is prepared to frequently change scales in two different ways. Firstly, one needs to move between the level of the creation of mathematical knowledge at a certain instant and the long duration of its assimilation on the other, which requires that we think in terms of "social time" that is, of the time specific to each social group and space entering into the study, and their peculiar dynamics. Secondly, taking into account the local and global contexts in which the meaning of the texts is construed and passed on implies that we reconstruct as precisely as possible the links between mathematical proofs, the degree to which they overlap, and the way mathematicians use them. Also important is the social logic, whether institutional, local, international, personal, etc., which provide the frame of relevance for these practices.[8]

2. EVARISTE GALOIS'S INTELLECTUAL BIOGRAPHY AS A HISTORIOGRAPHICAL TOOL

Galois is anything but an obscure and unknown mathematician, but paradoxically, the existing studies pose more questions than they answer. According to his biographers, Galois failed to convince his contemporaries, in particular the *Académie des sciences,* because his mathematics, which are thought to contain a glimpse of the structural viewpoint in algebra, were too far ahead of their time. In this narrative, his revolt against the unfair treatment given him by the *Académie* led him to get involved

8 Even if the subject matters are very different, our approach turns out to be quite close to the type of sociocultural history developed by historians like Daniel Roche ([Roche 1988]), Roger Chartier ([Chartier 1989]) or Bernard Lepetit ([Lepetit 1999], in particuliar pp. 88–119 and pp. 142–168). The heuristic power of changing scales is emphasized in the well-known book [Revel 1996]. We have elaborated on this approach with a view to the history of mathematics in [Ehrhardt 2010b].