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NOTES & DÉBATS

MATHEMATICAL RECONSTRUCTIONS OUT, TEXTUAL STUDIES IN: 30 YEARS IN THE HISTORIOGRAPHY OF GREEK MATHEMATICS

Ken SAITO (*)

History of Greek Mathematics Before and After 1970

Thirty years ago, at the end of the sixties, the history of Greek mathematics was considered an almost closed subject, just like physics was at the turn of the twentieth century. People felt that they had constructed a definitive picture of the essence of Greek mathematics, even though some details remained unclear due to irrecoverable document losses. Critical editions had been established, mainly by Heiberg, while two of the great scholars of the history of Greek mathematics, Tannery and Zeuthen, had built on this material. Then, the standard book [Heath 1921] brought much of this material together. Through his discoveries in Mesopotamian mathematics, Neugebauer was led to think that he had given substance to legends about the Oriental origin of Greek mathematics. Originally published in Dutch in 1950, the book [van der Waerden 1954] reflected scholars' self-confidence in this period.

One may well compare what happened after 1970 in the historiography of Greek mathematics to the developments of physics in the first decades of this century. In some sense the change in the history of Greek mathematics was even more dramatic, because no new important material was discovered since 1906, at which time the *Method* was brought to light by Heiberg. This great interpretative change was mainly due to a shift in scholars' attitudes.

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Ken SAITO, Osaka Prefecture University, Department of Human Sciences, Sakai 599–8531 (Japan). http://wwwhs.cias.osakafu-u.ac.jp/~ksaito/.

In the following, the historiography of Greek mathematics before 1970 will be briefly contrasted, with no pretense of being exhaustive, with that which followed[1].

The Origins*: *Who Was the First Mathematician?

A tradition reaching as far back as Eudemus (late 4th century BC), *via* citations found in Proclus' *Commentary on the First Book of Euclid's Elements*, considers Thales (*fl. ca.* 585 BC) to have been the founder of the Greek mathematical sciences. However, if by the phrase "*the origins of Greek mathematics*" we mean an embryo of the rigorous deductive structure found in the *Elements*, Thales had little to do with it. Eudemus may well have constructed a story of a mathematician from fragmentary sources at his disposal, which described the practical knowledge of a wise man (see [Dicks 1959], and also [Vitrac 1996]).

Dismantling the myth of origins became the subject of hot debates centered around the figure of Pythagoras (*ca.* 572–*ca.* 494 BC), who enjoys no less enthusiastic advocates today than in the ancient world. However, considerable if not decisive, is the damage done to "*Pythagoras the mathematician*" by the blow of the epoch-making study [Burkert 1972][2]. Now we see Pythagoras as the founder of a prevalently (but not exclusively) religious community, established on doctrines of reincarnation and metempsychosis. To be a Pythagorean meant choosing a certain way of life based on these doctrines, without being necessarily involved in philosophical or scientific inquiries.

Thus we are rather concerned, now, with the role of the Pythagoreans in the development of Greek mathematics after the middle of the fifth century. The picture once prevailed that the discovery of incommensurability was a scandal for the Pythagoreans and provoked a *crisis*. This belief was

[1] Accounts given below are a personal view, even if I tried to be as impartial as possible in the bibliography. I restricted myself to studies of mathematics in the classical period, that is, mathematics before Apollonius, although developments of research in late antiquity, including the rediscovery of part of the lost books of Diophantius' *Arithmetica* in Arabic, should not be underestimated.

[2] Reasonable doubts over whether Pythagoras actually was the first mathematician and philosopher go back at least to [Vogt 1908/09], and perhaps even to [Zeller 1844–1852]. Burkert's central thesis, as well as scholarly developments after 1972, are very skilfully, and with extraordinary clarity and concision, described in [Centrone 1996]. For opinions sympathetic to the view of Pythagoras and the Pythagoreans as scientists, see [van der Waerden 1979] and [Zhmud 1997].

deduced from 1) the alleged Pythagorean monopoly on the mathematical sciences in the fifth century; 2) their central dogma "all is number"; and 3) Iamblichus' testimony. However, 1) has no good evidence to support it; 2) is very likely an Aristotelian summary deduced from Philolaus' (*ca.* 470–*ca.* 390 BC) book; and 3) is so confused that it is hardly reliable (which means that we have no authentic document to credit the Pythagoreans with the discovery of incommensurability). The scandal, or foundations-crisis thesis has thus turned out to be scarcely plausible (see [Freudenthal 1966], [Knorr 1975, p. 21–61], [Fowler 1987, p. 294–308]). More recently [Fowler 1994] has even suggested that this discovery itself may have been no more than an incidental event. After all, the above thesis may have been a retroprojection of early twentieth-century interests in the foundations of mathematics.

Therefore, the roles traditionally ascribed to Pythagoreans are also to be reconsidered and greatly modified, a point to which we shall later return. For the moment let us examine modern studies devoted to the theory of proportions.

Mathematical Reconstructions

If a foundations-crisis theory was soon dismissed, the assumption that incommensurability constituted a turning point in Greek mathematics enjoyed better support. In fact, it seems natural to us, today, to suppose that the discovery of incommensurability called for a new definition of proportions (sameness of ratios) applicable to incommensurable magnitudes. This assumption gave birth to the most influential historical approach in this century: mathematical reconstruction.

[Becker 1932/33] pointed out that a passage of Aristotle's *Topics* can be construed as evidence for the existence of a definition of proportions based on *anthyphairesis* (Euclidean algorithm), which can be dated to a period between the discovery of incommensurability (probably second half of the fifth century) and Eudoxus' time (*ca.* 390–*ca.* 337). This paper not only called attention to the technique of *anthyphairesis*, but also encouraged scholars to use mathematical reconstructions in order to venture new conjectures and hypotheses. One eminent example of this technique is [Fritz 1945], which proposed, with no direct textual evidence, that incommensurability had first been found in a study of the relation between the side and diameter of regular pentagons by the method of *anthyphairesis*. Even [Knorr 1975], the most critical and thoroughgoing study of the development of incommensurability theory to date, remained highly speculative, and in a sense, this book marked a culmination in the tradition of the reconstruction approach opened by Becker[3].

Although the significance of this kind of study cannot be denied, its danger also is obvious: one has no general criterion to judge whether the reconstructed argument ever existed in antiquity. Moreover, while most reconstructions deal with the period around and before 400 BC, sources come from later periods[4].

From Mathematical Reconstructions to Textual Studies

However, since any significant interpretation of ancient mathematics is bound to involve some kind of conscious or unconscious reconstruction, one may well ask whether it is actually possible to distinguish recent research from previous reconstructive approaches. Let me try to justify this distinction, granted that, here, my account inevitably is more personal than other parts of this paper.

Previous scholars (say, from Tannery and Zeuthen to van der Waerden) were, I believe, confident in the power of something like universal reason, and took it for granted that a careful mathematico-logical reasoning was able to restore the essence of ancient mathematics. Today scholars are more skeptical: the type of reasoning that once played an essential role tends to be regarded as a mere rationalising conjecture. They are even convinced that the modern mind will always err when it tries, without the guide of ancient texts, to think as the ancients did (I personally think

[3] I exclude from this tradition the book [Fowler 1987] whose reconstructions undoubtedly are more sophisticated: see below.

[4] Anomalies and idiosyncrasies in the logical structure of the *Elements* have been used by many scholars (including myself) in order to reconstruct the earlier phase of Greek mathematics. For example, the first four books of the *Elements* contain several demonstrations more easily proved with the theory of proportions. These demonstrations have been either located in the period when no adequate theory of proportions was available or attributed to some mathematician who compiled earlier versions of the *Elements*. [Artmann 1985] and [Artmann 1991] are a remarkable outcome of this approach. However, this approach relies on the assumption that Euclid's editorial intervention was minimal and the extent to which this assumption can be justified is unknown to us. With a bit of irony, Vitrac called this kind of approach an "*enquête archéologique*" [Vitrac 1993, p. XI]. See also [Gardies 1998], which developed very specific reconstructions based on logical analyses, and [Caveing 1994–98].

that this opinion can be attributed to an indirect influence of Thomas Kuhn).

Thus, texts are read in a different manner by recent historians of mathematicians, as well as by historians of science in general. For example, apparently redundant or roundabout passages call for more attention, because these might reveal some of the ancients' particular thoughts of which modern minds are unaware[5]. This is one of the attitudes typical of what I call "textual studies" in a broad sense (I do not restrict them to textual criticism), an attitude based on reasonable doubts as to the validity of logical conjectures.

Happily for the French-reading public, the spirit and results of this new textual approach are best embodied in the French translation of the *Elements* now in progress (see [Euclide 1990–]), but a review of other studies will also help us understand the new historiography. Renewed interest in text led to careful examinations of the extant mathematical documents and their logical structure. Since most of these documents are series of propositions, the logical interdependence of propositions, or the "deductive structure," is one of the important subjects of recent studies. Most of this work has been limited to Euclid's *Elements*: see [Beckmann 1967], [Neuenschwander 1972/73], [Neuenschwander 1974/75], and the comprehensive and influential book [Mueller 1981][6]. Lately, [Netz (forthcoming)] has proposed brand-new, insightful approaches to texts[7].

The shift from reconstructions to textual studies can also be illustrated

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[5] Here, one cannot but recall the attractive work of Árpád Szabó (I am thinking of [Szabó 1969] and less known [Szabó 1964]), who, using philosophical arguments, was the first seriously to criticise the trend of mathematico-logical reconstructions. His approach predated the present research trend. He was however concerned with finding traces of the earliest developments of Greek mathematics, and his arguments inevitably remain no less speculative than the theses he challenges.

[6] Concerning the proposition used by later authors, indices devoted to Papus, Apollonius and Archimedes are available on my web page, where one will find how propositions of the *Elements* were used (or not used) in other mathematicians' works. My paper [Saito 1994] indicates the reasons that prompted me to assemble such indices.

[7] In this book Netz examines the form and style of Greek mathematical texts, which, like Homer's epics, largely depended on "formulae" — fixed expressions regularly used to denote certain mathematical objects or relations. He illustrates the way in which mathematical deduction is constructed and how formulae work. He moreover analyses the relation between text and diagram, and elucidates their interdependence, showing the indispensable role played by diagrams.