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This volume contains the contributions to the congress Complex Analytic Methods in Dynamical Systems held at Instituto de Matemática Pura e Aplicada, IMPA, in January 1992.

We hope this book will provide an extensive overview on the subject.

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Rio de Janeiro, September 10, 1993

C. Camacho
A. Lins Neto
R. Moussu
P. Sad,

The Editors

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ABSTRACTS

C. BONATTI and X. GOMEZ-MONT. *The index of holomorphic vector fields on singular varieties I*

Given an analytic space V with an isolated singularity p, a Poincaré-Hopf type of index, $\mathrm{Ind}(X, V, p)$ is associated to every holomorphic vector field X tangent to V for which p is an isolated zero.

In this paper this topological index is related to the algebraic multiplicity $\mu_V(X, p)$. In particular, it is shown that the set of indices $\mathrm{Ind}(X, V, p)$, where X is tangent to V with an isolated zero at p, admits a minimum which is reached for X in the open dense subset of vector fields of smallest V-multiplicity.

M. BRUNELLA. *Vanishing holonomy and monodromy of certain centres and foci*

In this paper we study a particular class of germs of analytic differential equations on the real plane, which present a singularity of the type centre - focus. For these singularities it is defined a monodromy map, which is a germ of analytic diffeomorphism on the real line. A complexification of these germs allows to introduce (following R. Moussu and D. Cerveau) their vanishing holonomy. We study the relation existing between monodromy and vanishing holonomy; corollaries about normal forms are obtained.

D. CERVEAU. *Théorèmes de type fuchs pour les tissus feuilletés*

Après avoir rappelé des résultats pour certains bien anciens - et souvent méconnus - concernant les d-tissus sur un ouvert de $\mathbf{C}^n$ on s'intéresse à la dynamique des 3-tissus feuilletés hexagonaux globaux. Bien souvent - c'est le cas sur les espaces projectifs - un tel objet va présenter des singularités. On se propose, moyennant des hypothèses de type Fuchs, de donner une description des feuilles comme niveaux de fonctions multivaluées de type Liouville ($\Sigma\lambda_i \log f_i + H$, f_i et H holomorphes). Ce travail est motivé par la description de la variété des feuilletages algébriques de codimension un sur des espaces projectifs $\mathbf{CP}(n)$.

D. CERVEAU and A. LINS NETO. *Codimension one foliations in $\mathbf{C}P^n$, $n \geq 3$, with Kupka components*

We consider holomorphic foliations of codimension one in $\mathbf{C}P(n)$, $n \geq 3$, with a Kupka component. We prove that if the Kupka component is a complete intersection, then the foliation has a first integral of the type f^p/g^q, where p, q are positive integers with $(p, q) = 1$, f and g are homogeneous polynomials in $\mathbf{C}^{n+1}$ such that p degree $(f) = q$ degree (g) and the Kupka component is $\{f = g = 0\}$ in homogeneous coordinates.

J. ECALLE. *Compensation of small denominators and ramified linearisation of local objects*

We show, on typical examples, how local objects (i.e. germs of analytic vector fields or diffeomorphisms of $\mathbf{C}^\nu$) which, due to resonance or small denominators, fail to possess an analytic linearisation, may still be reduced to their linear part by means of ramified changes of coordinates. The latter are not merely formal, but canonically resummable in spiral-like neighbourhoods of the ramified origin $\underset{\bullet}{0}$ of $\underset{\bullet}{\mathbf{C}}^\nu$. Apart from its obvious bearing on local dynamics, ramified linearisation leads to an extension of the concept of holonomy.

J.E. FORNAESS and N. SIBONY. *Complex dynamics in higher dimension I*

We study global questions of iteration for holomorphic self maps of $\mathbf{P}^k$. After discussing some basic properties of holomophic and meromorphic maps in $\mathbf{P}^k$, we describe the maps f in $\mathbf{P}^2$ for which there exists a variety V satisfying $f^{-1}(V) = V$. We show that for a Zariski dense set of holomorphic maps in $\mathbf{P}^2$ the complement of the critical orbit is Kobayashi hyperbolic. We then study expansive properties of the maps in the interior of the complement of the critical orbit, under suitable hyperbolicity assumptions. We finally classify maps in $\mathbf{P}^2$ such that the orbit of the critical set is a variety.

Y. ILYASHENKO. *Normal forms for local families and nonlocal bifurcations*

The study of nonlocal bifurcations from the topological point of view requires not only topological, but smooth normal forms of the families of differential equations near singular points. In the first part of the paper a survey of these normal forms is presented. In the second part these normal forms are applied to the study of the bifurcations of planar vector fields. A complete list of polycycles appearing in generic two or three parameter families (Zoo of Kotova) is presented.

The proof of the finite cyclicity of elementary polycycles occuring in typical finite parameter families of planar vector fields is outlined.

V.P. KOSTOV. *Regular linear systems on* $\mathbf{CP}^1$ *and their monodromy groups*

In this paper we prove that the $p+1$ Jordan normal forms of the monodromy operators of a regular linear system on $\mathbf{C}P^1$ with $p+1$ poles and the possible reducibility of the monodromy group define an analytic stratification of $(GL(n, \mathbf{C}))^p$ - the space of monodromy groups of such systems.