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A CASE OF MATHEMATICAL EPONYMY: THE VANDERMONDE DETERMINANT

Bernard Ycart

Abstract. — We study the historical process that led to the worldwide adoption, throughout mathematical research papers and textbooks, of the denomination "Vandermonde determinant". The mathematical object can be related to two passages in Vandermonde's writings, of which one inspired Cauchy's definition of determinants. Influential citations of Cauchy and Jacobi may have initiated the naming process. It started during the second half of the 19th century as a teaching practice in France. The spread in textbooks and research journals began during the first half of 20th century, and only reached full acceptance after the 1960's. The naming process is still ongoing, in the sense that the volume of publications using the denomination grows significantly faster than the overall volume of the field.

Résumé (Le déterminant de Vandemonde). — Nous étudions le processus historique qui a conduit à l'adoption dans le monde entier de la dénomination « déterminant de Vandermonde ». L'objet mathématique peut être relié à deux passages dans les écrits de Vandermonde, dont l'un a inspiré Cauchy pour sa définition des déterminants. Les citations de Cauchy et Jacobi ont pu déclencher le processus de dénomination. Celui-ci a démarré au cours de la seconde moitié du xixe siècle comme une pratique pédagogique. Cette pratique a précédé, plutôt que suivi, la diffusion dans les livres et les articles de recherche, qui a commencé pendant la première moitié du xxe siècle, et n'a atteint un

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réel consensus qu'après les années 1960. Le processus de dénomination est encore en cours, au sens où l'usage du nom croît significativement plus vite que le volume global de publications du domaine.

1. INTRODUCTION

The Vandermonde determinant has become a standard example of Stigler's law of eponymy: "No scientific discovery is named after its original discoverer" (see [Stigler 1999, p. 277]). The source? An authority: Henri Lebesgue (1875–1941). On October 20, 1937, he gave a conference at Utrecht University, entitled "L'œuvre mathématique de Vandermonde". The text of that conference was published in 1939, reproduced in 1956, and again in a 1958 monography [Lebesgue 1958] to which we shall refer. In order to enhance Vandermonde's [1774] main achievement on the resolution of algebraic equations, Lebesgue [1958, p. 21] downplays his three other memoirs:[1]

> Thus the Vandermonde determinant is not due to Vandermonde; his theory of determinants is not very original, his notation of factorials is unimportant; his study of situation geometry is somewhat childish, what is left?[2]

Actually, the memoir on combinatorics Vandermonde [1775] contains more than just a notation for factorials: the identity

$$\binom{n}{k} = \sum_{i=0}^{k} \binom{m}{i} \binom{n-m}{k-i}$$

is still referred to as "Vandermonde's theorem" in probability and combinatorics textbooks (*e.g.* p. 315 of Santos [2011]). Though "childish", the memoir on situation geometry Vandermonde [1776b] made him regarded as a precursor of knot theory (see Przytycki [1992]).

The life of Alexandre Théophile Vandermonde (1735–1796), his engagement during the French revolution, his interests in music, mechanics, and political economy, and his short mathematical carrier, have all been amply documented: see Lebesgue [1958], Hecht [1971], Gillispie [1976],

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[1] All translations are from the author.

[2] Ainsi le déterminant de Vandermonde n'est pas de Vandermonde ; sa théorie des déterminants n'est pas très originale, sa notation des factorielles est sans importance ; son étude de géométrie de situation est un peu enfantine, que reste-t-il ?

Faccarello [1993], and Sullivan [1997]. We shall not attempt a new biography nor a mathematical assessment of Vandermonde's contribution. Neither shall we review here the early history of determinants. T. Muir's *Theory of determinants in their historical order of development* is the indispensable basis, and we shall often refer to the first two volumes: Muir [1906] and Muir [1911]. Our focus here is exclusively on the Vandermonde determinant, and more precisely on how that particular object came to be known under that name. We call *Vandermonde Determinant*, and denote by VD hereafter, the following determinant, depending on n variables $a_1, \ldots, a_n$:

$$\begin{vmatrix} 1 & a_1 & a_1^2 & \ldots & a_1^n \\ 1 & a_2 & a_2^2 & \ldots & a_2^n \\ \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \vdots \\ 1 & a_n & a_n^2 & \ldots & a_n^n \end{vmatrix}.$$

The VD has different mathematically equivalent interpretations, as a product of differences or an alternating polynomial, that will be developed in section 2.2.

Lebesgue makes the following assertion:

> What could have been personal, is the Vandermonde determinant? Yet it is not there, nor anywhere else in Vandermonde's work.[3] [Lebesgue 1958, p. 21]

Why then was Vandermonde's name given to that determinant? Lebesgue has a conjecture.[4]

> Vandermonde considers linear equations of which the unknowns are denoted by $\xi 1, \xi 2, \xi 3, \ldots$, and the coefficient of ξi in the k-th equation by $\overset{k}{i}$. The resolution of such a system, *e.g.* of
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> $$\begin{aligned} \overset{1}{1}\xi 1 + \overset{1}{2}\xi 2 + \overset{1}{3}\xi 3 + \overset{1}{4} &= 0, \\ \overset{2}{1}\xi 1 + \overset{2}{2}\xi 2 + \overset{2}{3}\xi 3 + \overset{2}{4} &= 0, \\ \overset{3}{1}\xi 1 + \overset{3}{2}\xi 2 + \overset{3}{3}\xi 3 + \overset{3}{4} &= 0, \end{aligned}$$

[3] Ce qui aurait pu être personnel, c'est le déterminant de Vandermonde ? Or il n'est pas là, ni nulle part ailleurs dans l'œuvre de Vandermonde!

[4] Lebesgue's notations have been reproduced.