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HIGHER DIMENSIONAL COMPLEX GEOMETRY

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A Summer Seminar at the University of Utah,
Salt Lake City, 1987

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Introduction

These notes originated at a seminar that was held during July and August of 1987 at Salt Lake City. The original aim of the seminar was to get an overview of the following three topics:

1. Recent advances in the classification program of three (and higher) dimensional algebraic varieties.
2. Existence of rational curves and other special subvarieties.
3. Existence and nature of special metrics on varieties.

We also hoped to then go further and study the relationships between these three approaches. Time however proved to be insufficient to complete even the limited program.

The first part of the program was considered in detail. In that part, the central theme is the investigation of varieties on which the canonical class is not numerically effective. For smooth threefolds this was done in [M1] and later extended considerably. The original approach of [M1] is geometrically very clear, therefore it is given in detail. Subsequent generalizations were also considered at length.

Considerable attention was paid also to the study of special curves on hypersurfaces and some related examples. There seems to be a lot of experimental evidence to indicate that there is a very close relationship between the Kodaira dimension of a threefold (a property of a threefold from classification theory) and the existence of rational curves. These problems are very interesting but they also seem quite hard. Our contribution in this direction is mostly limited to presenting some examples and conjectures.

In the second direction, one of the questions we were interested in was that of understanding rational curves on quintic hypersurfaces in $\mathbf{P}^4$. Later this was scaled down to understand *lines* on quintic hypersurfaces in $\mathbf{P}^4$, but even this seems a hard problem. We began to understand it more completely only after the seminar had ended (see [J]).

Very little time was left to consider the third direction. We were fortunate to have a series of lectures, but we could not pursue this interesting and important direction in any detail.

The style of the seminars was very informal. We tried to keep them discussion-and-problem oriented. Notes were taken by H. Clemens who typed them up by the next day. These notes constituted the first version of the present text. During the seminar and afterwards, these notes were considerably revised, cut, expanded and edited. During this process we tried to keep the original informality of the talks alive.

The regular participants of the seminar were J. Jimenez, T. Luo, K. Matsuki and the three of us. Several other people joined us for various length of time. A hopefully complete list is:

J. Carlson, L. Ein, M. He, Y. Ma, D. Ortland, S. Pantazis, P. Roberts, D. Toledo, S. Turner, and Stephen Yau. We are very grateful for their contribution to the success of the seminar.

We are especially thankful to those people who gave talks. The following is a list of the lectures of a mathematician other than one of the three of us.

J. Carlson:	Maximal variations of Hodge structures;
L. Ein:	Submanifolds of generic complete intersections in Grassmanians;
L. Ein:	A theorem of Gruson-Lazarsfeld-Peskine and a lemma of Lazarsfeld;
K. Matsuki:	Cone Theorem;
K. Matsuki:	Non-vanishing Theorem;
D. Toledo:	Kähler structures on locally symmetric spaces;
D. Toledo:	Proof of Sampson's theorem;
D. Toledo:	Abelian subalgebras of Lie algebras.

At the final editing of these notes some talks were left out. This was the fate of the following talks:

H. Clemens:	Abel-Jacobi maps;
S. Turner:	Elliptic surfaces in characteristic p;
S. Yau:	Euler characteristic of Chow varieties.

These talks were about topics that we had no time to pursue further, and therefore they did not fit neatly into the final version of the notes.

Our aim was to keep the notes advanced enough to be of interest even to the specialists, but understandable enough so that a person with a good general background in algebraic geometry would be able to understand and enjoy them. Especially at the beginning, the lectures are rather informal and concentrate on the geometric picture rather than on a proof that is correct in every technical detail. We hope that this informal introduction to [M1] will be helpful. These matters occupy the first two lectures.

The classification theory of surfaces is reviewed from the point of view of threefold theory in Lecture 3. This leads naturally to the next lecture which is an introduction to the study of cones of curves. Lecture 5 discusses the aims of Mori's program in more detail, concentrating mainly on flips, the presence of which is perhaps the most important difference between algebraic geometry in two and in three dimensions. At the end of this lecture, a table compares the basic results in the birational geometry of surfaces and threefolds. Even though the list was selected with bias, the similarities are striking.

Lecture 6 is a little more technical. It discusses the singularities that arise naturally in the study of *smooth* threefolds. These are the three dimensional analogues of the rational double points of surfaces. Their structure is however more complicated and not completely known.