TOPICS IN STATISTICAL MECHANICS

Cédric Boutillier, Béatrice de Tilière,
Kilian Raschel (eds.)

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***Abstract.* —** This volume aims at giving an overview of the "États de la recherche on Statistical Mechanics," organized by the French Mathematical Society in 2018, which took place in Institut Henri Poincaré (Paris) in 2018. It was a successful event bringing together 125 mathematicians, ranging from master students to young and confirmed researchers.

There were four mini-courses by Francesco Caravenna, Hugo Duminil-Copin, Thierry Bodineau & Isabelle Gallagher & Laure Saint-Raymond, Vincent Vargas. This was complemented by thirteen research talks, altogether giving an overview of a wide number of models of statistical mechanics such as the Ising model, Potts model, percolation, perfect gases, Coulomb gases, particle systems, kinetically constrained spin models, the dimer model etc.

This volume contains an introduction with a summary of the four mini-courses and of all the talks. The heart of this publication consists of five original contributions by: Djalil Chafaï, Ewain Gwynne & Nina Holden & Xin Sun, Arnaud Le Ny, Sébastien Ott & Yvan Velenik, Rémi Rhodes & Vincent Vargas.

***Résumé.* (Quelques aspects de la mécanique statistique) —** Ce volume a pour but de donner un aperçu des « États de la recherche en Mécanique Statistique », organisés par la Société Mathématique de France en 2018, à l'Institut Henri Poincaré (Paris). La conférence a été un franc succès, rassemblant 125 mathématiciennes et mathématiciens de niveau master à celui de chercheuses et chercheurs confirmés.

Il y a eu quatre mini-cours par Francesco Caravenna, Hugo Duminil-Copin, Thierry Bodineau & Isabelle Gallagher & Laure Saint-Raymond, Vincent Vargas. Ces mini-cours ont été accompagnés de treize exposés de recherche donnant ainsi un panorama sur un grand nombre de modèles de mécanique statistique tels que le modèle d'Ising, le modèle de Potts, la percolation, les gaz parfaits, les gaz de Coulomb, les systèmes de particules, les modèles de spins cinétiquement contraints, le modèle de dimères etc.

Ce volume contient une introduction avec un résumé des quatre mini-cours et de tous les exposés. Le cœur de cette publication est constitué de cinq contributions originales de : Djalil Chafaï, Ewain Gwynne & Nina Holden & Xin Sun, Arnaud Le Ny, Sébastien Ott & Yvan Velenik, Rémi Rhodes & Vincent Vargas.

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ABSTRACTS

Coulomb gases are special probability distributions, related to potential theory, that appear at many places in pure and applied mathematics and physics. In these short expository notes, we focus on some models, ideas, and structures. We present briefly selected mathematical aspects, mostly related to exact solvability and first and second order global asymptotics. A particular attention is devoted to two-dimensional exactly solvable models of random matrix theory such as the Ginibre model. Thematically, these notes lie between probability theory, mathematical analysis, and statistical physics, and aim to be very accessible. They form a contribution to a volume of the *Panoramas et Synthèses* series around the workshop *États de la recherche en mécanique statistique*, organized by Société Mathématique de France, held at Institut Henri Poincaré, Paris, in the fall of 2018.

We survey the theory and applications of mating-of-trees bijections for random planar maps and their continuum analog: the mating-of-trees theorem of Duplantier, Miller, and Sheffield (2014). The latter theorem gives an encoding of a Liouville quantum gravity (LQG) surface decorated by a Schramm-Loewner evolution (SLE) curve in terms of a pair of correlated linear Brownian motions. We assume minimal familiarity with the theory of SLE and LQG.

Mating-of-trees theory enables one to reduce problems about SLE and LQG to problems about Brownian motion and leads to deep rigorous connections between random planar maps and LQG. Applications discussed in this article include scaling limit results for various functionals of decorated random planar maps, estimates for graph distances and random walk on (not necessarily uniform) random planar maps, computations of the Hausdorff dimensions of sets associated with SLE, scaling limit results for random planar maps conformally embedded

in the plane, and special symmetries for $\sqrt{8/3}$-LQG which allow one to prove its equivalence with the Brownian map.

This review-type paper is based on a talk given at the conference *États de la Recherche en Mécanique statistique*, which took place at IHP in Paris (December 10–14, 2018). We revisit old results from the eighties about one dimensional long-range polynomially decaying Ising models (sometimes called *Dyson models* when focusing on dimension one) and describe more recent results about interface fluctuations and interface states in dimensions one and two.

Based on a series of joint works with R. Bissacot, L. Coquille, E.O. Endo, A. van Enter, B. Kimura and W. Ruszel [**37**, **10**, **25**, **8**, **35**, **36**].

We present a brief survey of rigorous results on the asymptotic behavior of correlations between two local functions as the distance between their support diverges, concentrating on the ferromagnetic Ising model on $\mathbb{Z}^d$.

The purpose of these notes, based on a series of lectures given by the second author at IHÉS and at IHP (états de la recherche SMF: mécanique statistique), is to explain the probabilistic formulation of Liouville conformal field theory (LCFT) and of the DOZZ formula for the three point correlation functions of the theory. In Lecture 1, we review the probabilistic construction of the N point correlation functions of LCFT on the Riemann sphere for $N \geq 3$ (based on a series of works of the authors with F. David and A. Kupiainen). In Lecture 2, we explain the construction of the two point correlation functions of LCFT, also called the reflection coefficient. In Lecture 3, these probabilistic constructions are justified from the point of view of Polyakov's path integral formulation of LCFT. In Lecture 4, we give a sketch of the proof of the DOZZ formula. Finally, in Lecture 5, we present very recent work on LCFT (the conformal bootstrap approach), open problems and extensions of LCFT (the so-called Toda conformal field theory).