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ORBIT CLOSURES IN FLAG VARIETIES FOR THE CENTRALIZER OF AN ORDER-TWO NILPOTENT ELEMENT: NORMALITY AND RESOLUTIONS FOR TYPES A, B, D

BY SIMON JACQUES

ABSTRACT. — Let G be a reductive algebraic group in classical types A, B, D. Let e be an element of the Lie algebra of G, with $Z \subset G$ its centralizer for the adjoint action. We assume that e identifies with a nilpotent matrix of order two, which guarantees that the number of Z-orbits in the flag variety of G is finite. For types B and D in characteristic two, we also assume that the image of e is totally isotropic. We show that the closure Y of such an orbit is normal. We also prove that Y is Cohen-Macaulay with rational singularities provided that the base field is of characteristic zero, and that Cohen–Macaulayness holds in any characteristic for type A. We exhibit a rational and birational morphism onto Y involving Schubert varieties. Our work generalizes a result by N. Perrin and E. Smirnov on the Springer fibers.

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