

# Bulletin

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**ORBIT CLOSURES IN FLAG VARIETIES FOR THE  
CENTRALIZER OF AN ORDER-TWO NILPOTENT ELEMENT:  
NORMALITY AND RESOLUTIONS FOR TYPES A, B, D**

BY SIMON JACQUES

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ABSTRACT. — Let  $G$  be a reductive algebraic group in classical types A, B, D. Let  $e$  be an element of the Lie algebra of  $G$ , with  $Z \subset G$  its centralizer for the adjoint action. We assume that  $e$  identifies with a nilpotent matrix of order two, which guarantees that the number of  $Z$ -orbits in the flag variety of  $G$  is finite. For types B and D in characteristic two, we also assume that the image of  $e$  is totally isotropic. We show that the closure  $Y$  of such an orbit is normal. We also prove that  $Y$  is Cohen-Macaulay with rational singularities provided that the base field is of characteristic zero, and that Cohen-Macaulayness holds in any characteristic for type A. We exhibit a rational and birational morphism onto  $Y$  involving Schubert varieties. Our work generalizes a result by N. Perrin and E. Smirnov on the Springer fibers.

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