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CHARLES HERMITE'S PRACTICES AND THE PROBLEM OF THE UNITY OF MATHEMATICS

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Abstract. — The theme of the unity of mathematics developed during the nineteenth century as specialized articles proliferated and it has often been associated for this period with the definition of new types of mathematical objects in a structuralist setting. This article focusses on the almost opposite point of view of Charles Hermite. Although his work was praised by his contemporaries for beautifully contributing to and displaying the unity of mathematics, he himself strongly opposed the idea of free conceptual creation in mathematics and favored explicit, extensive computations with algebraic forms and classical functions. Hermite's way of testifying to the unity of mathematics must thus be reconstructed by a close reading of his papers, here based on a focus on a few keywords. The result appears proteiform; Hermite operates sometimes by constructing bridges within mathematics through formulas, sometimes by recycling and adapting well-known algebraic expressions, and even occasionally by providing alternative proofs of a theorem. The coherence of these practices with Hermite's general viewpoint on mathematics leads us to advocate for a richer history of the problem of the unity of mathematics.

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Mots clefs. — Charles Hermite, unité des mathématiques, formules mathématiques, analyse algébrique arithmétique.

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Résumé (Les pratiques de Charles Hermite et le problème de l'unité des mathématiques)

Le thème de l'unité des mathématiques s'est développé au cours du dix-neuvième siècle en même temps proliféraient les articles spécialisés et il a souvent été associé, pour cette période, à la définition de nouveaux types d'objets mathématiques dans un cadre structuraliste. Cet article se concentre sur le point de vue presque opposé de Charles Hermite. Bien que son travail ait été loué par ses contemporains pour avoir brillamment contribué à l'unité des mathématiques et même pour l'avoir mise en évidence, il s'est lui-même fermement opposé à l'idée d'une création conceptuelle libre en mathématiques et a privilégié les calculs explicites et étendus sur les formes algébriques et les fonctions classiques. La manière dont Hermite témoignait de l'unité des mathématiques doit donc être reconstituée par une lecture attentive de ses articles, ce que nous ferons ici en suivant les indications de quelques mots-clés. Le résultat apparaît protéiforme, Hermite opérant tantôt en construisant des ponts à l'intérieur des mathématiques par le biais de formules, tantôt en recyclant et en adaptant des expressions algébriques bien connues, et même occasionnellement en fournissant des preuves alternatives d'un théorème. La cohérence de ces pratiques avec le point de vue général d'Hermite sur les mathématiques nous conduit à plaider pour une histoire plus riche du problème de l'unité des mathématiques.

The theme of the unity of science became classic in the philosophy of science during the twentieth century, mathematics being from the start a key feature in this edifice. One of the most famous testimonies of this trend is, of course, the First International Congress for the Unity of Science, which took place in 1935 in Paris, in parallel with the long-term project of the *International Encyclopedia of Unified Science*. Besides Otto Neurath and Rudolf Carnap, the leading figures of the Vienna Circle at the origin of the project, the Congress gathered several representatives of the cream of the mathematical crop of the time: Elie Cartan, Jacques Hadamard, Federigo Enriques, Bertrand Russell, and Richard von Mises, among others. [1] "Recent years have witnessed a striking growth of interest in scientific enterprise and especially in the unity of science," says the front flap of the first volume of the *Encyclopedia*. Boosted in part by the successes and hopes of general relativity and its developments, the theme of the unity of science was often at the time associated with that of the unity of the world—both human and natural—on one side and, on the other, that of the unity of mathematics which was supposed to reflect and to warrant it.

In an address fittingly entitled "The Unity of Mathematics" at the American Association for the Advancement of Science in 1937, the mathematician James Byrnie Shaw, professor at the university of Illinois, claimed for

1 [Neurath et al. 1938]. For a brief history of this movement, see [Morris 1960]; see also [Kremer-Marietti 2003] and [Bourdeau et al. 2018].

instance that "there is unity in architecture, in sculpture, in painting, in poetry, in music, in drama, in dancing, in mathematics. This unity is due to the central ideas which permeate the whole work [...] Mathematics was so interwoven with life that its central ideas are also those of life" [Shaw 1937, p. 402]. Describing these ideas as those of form, identity, invariance, dependence and ideality, Shaw concluded on an epic tone:

> The primeval gods were born of chaos, but their immense power is hurrying the particles of chaos and the ripples of its ocean, its intense fields and its creative spirits, into the unity of a universe. Through the ages of human life mathematics has come to be the screen upon which we may glimpse this unity.
> [Shaw 1937, p. 411]

Besides the spiritual, and even sometimes theological, component illustrated in Shaw's quote, the theme of the unity of mathematics then operates in several ways. One is material: confronted with a potentially discouraging proliferation of results, some argued in favor of new classifications of knowledge embodied in appropriate textual tools, from reviewing journals, such as the *Jahrbuch über die Fortschritte der Mathematik* or the *Répertoire bibliographique des sciences mathématiques*, to all-encompassing encyclopedias, such as Felix Klein and Wilhelm Meyer's *Encyklopädie der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen* or, of course, general books encapsulating new principles of unity, from Giuseppe Peano's *Formulario mathematico* to Nicolas Bourbaki's *Éléments de mathématique*, in whose title the singular "mathématique" emphasizes the unity of the domain.

But the two most well-known and well-studied components of this striving for unity in mathematics are methodological and conceptual. At the beginning of the twentieth century, the unity of mathematics was usually attached to a reduction process (often including a kind of axiomatization, as in Hilbert's program). At a conceptual level, this fostered the emergence of mathematical structures that were supposed to capture the bare bones of various objects or theories at the forefront of research. For the mathematicians promoting them, structures themselves possessed an intrinsic character of unity and at the same time helped to warrant the unity of mathematics, as they can be recognized and used in various mathematical subdomains and situations [Corry 2004].

Tenuous threads link together these different components, with an emphasis depending on the author, the genre of the texts or the time. In 1894, Richard Dedekind, introducing what will be soon considered a key structure, that of a field—the (unfortunate for our purpose here) translation of

the word *Körper* in the original German, that is, "body"—already emphasized how it conceptually conveys unity:

> This name ["body"], similarly to that in the natural sciences, in geometry, and in the life of human society, is also intended here to designate a system that possesses a certain completeness, perfection, closure, whereby it appears as an organic whole, as a natural unity. [2]

That this "natural unity" at a conceptual level paves the road to a more global view of the unity of mathematics was elaborated by a number of mathematicians after Dedekind, from Hilbert to Emmy Noether to, of course, the Bourbaki group. [3] Charles Ehresmann (a co-disciple of Jean Dieudonné at the École normale supérieure and also a member of the Bourbaki group) would be very explicit a few decades later:

> This is a time of proliferation of mathematics; however, we can recognize also significant trends toward unity. [...] Considering the similarities of all theories, a unification is obtained by giving a general definition of the notion of a structure, or more precisely of a species of structures over sets. [Ehresmann 1966]

A decisive piece on the theme of the unity of mathematics and science—at least for the French scene, as witnessed for instance by a search of these terms on Gallica—is the double thesis of Albert Lautman, a philosopher close to Charles Ehresmann and other members of the Bourbaki group: the main thesis is devoted to structure and existence, the complementary one to the unity of mathematics itself, which contributes to strengthening the association between the various components of the theme. Lautman concludes:

> The unity of mathematics is essentially that of the logical patterns which govern the organization of its edifices. … The analogies of structure and adaptations of existences … have no other purpose than to help highlight the existence within mathematics of logical patterns, which are only knowable through mathematics itself, and ensure both its intellectual unity and its spiritual interest. [4]

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2 [Dedekind 1894, p. 452, footnote]: "Dieser Name soll, ähnlich wie in den Naturwissenschaften, in der Geometrie und im Leben der menschlichen Gesellschaft, auch hier ein System bezeichnen, das eine gewisse Vollständigkeit, Vollkommenheit, Abgeschlossenheit besitzt, wodurch es als ein organisches Ganzes, als eine natürliche Einheit erscheint."

3 A huge literature has now been devoted to more tightly link Dedekind's work and viewpoint to this trend that identifies Dedekind as one of its main precursors, see for instance [Ferreirós & Reck 2020; Sieg & Schlimm 2017].

4 [Lautman 1938, p. 198]: "L'unité des mathématiques est essentiellement celle des schémas logiques qui président à l'organisation de leurs édifices … Les analogies de

The same features, reorganized and illustrated in a variety of ways, were subsequently used in numerous texts, as for instance in this preface of a book aimed at a more general audience that the mathematician Georges Bouligand wrote with the engineer and teacher Jean Desbats just after World War II:

> We first notice the obstacles that mathematical activity constantly comes up against; the share of the plural and the diversified imposes a struggle at all times to reduce, simplify and encompass. [...] The unity of Mathematics requires an integral struggle. The diversity of objects subjected to reasoning, and also the plurality of hypothetico-deductive systems, come to an arrangement, and this is the essential point, with a *unitary structure* of Mathematics, whose fundamental terms are the two notions of *invariance* and *group*. This unification in the structure facilitates methodological unity. [5]

The theme of the unity of science and the key position of mathematics in it predate of course the 1930s (going back for some to Antiquity), the description of its relevant features and components varying greatly according to the period. [6] In the nineteenth century, one may think of Martin Ohm's *Versuch eines vollkommen consequenten Systems der Mathematik*, George Boole's *Laws of Thought* or of course Auguste Comte. Charles-Ange Laisant, whose 1898 book *La Mathématique: Philosophie, Enseignement* was reedited in 1907, envisioned the unification of all sciences, following Auguste Comte, through a universal method, leading from the concrete to the abstract and reciprocally, "which draws its inspiration from Mathematics only because Mathematics expresses the intrinsic traits of the

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structure et les adaptations d'existences ... n'ont pas d'autre but que de contribuer à mettre en lumière l'existence au sein des mathématiques de schémas logiques, qui ne sont connaissables qu'à travers les mathématiques elles-mêmes et en assurent à la fois l'unité intellectuelle et l'intérêt spirituel."

5 [Bouligand & Desbats 1947, p. 8]: "On constate d'abord les obstacles auquel se heurte constamment l'activité mathématique; la part du plural et du diversifié impose une lutte de tous les instants, en vue de réduire, de simplifier et d'englober. [...] L'unité de la Mathématique exige une lutte intégrale [...] La diversité des objets soumis au raisonnement, et aussi bien, la pluralité des systèmes hypothético-déductifs pactisent, c'est là le point essentiel, avec une *structure unitaire* de la Mathématique dont les termes fondamentaux sont les deux notions d'*invariance* et de *groupe*. Cette unification dans la structure facilite l'unité méthodologique."

6 See for a sample of this variety [Kremer-Marietti 2003; Krömer 2007; Maronne 2014; Stump 1997]. Still, it should be noted that there is no mention of this theme in, for instance, the eighteenth-century *Encyclopédie ou Dictionnaire raisonné des sciences, des arts et des métiers*, edited by Denis Diderot, Jean Le Rond d'Alembert and Louis de Jaucourt. For an example of a different approach which can be retroactively linked to the theme of the unity of mathematics, see [Rabouin 2009].