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and analysis before Viète*

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WHAT IS NEW AND WHAT IS OLD IN VIÈTE'S
ANALYSIS RESTITUTA AND *ALGEBRA NOVA*,
AND WHERE DO THEY COME FROM?
SOME REFLECTIONS ON THE RELATIONS
BETWEEN ALGEBRA AND ANALYSIS BEFORE VIÈTE

MARCO PANZA

ABSTRACT. — François Viète considered most of his mathematical treatises to be part of a body of texts he entitled *Opus restitutæ mathematicæ analyseos seu algebra nova*. Despite this title and the fact that the term “algebra” has often been used to designate what is customarily regarded as Viète’s main contribution to mathematics, such a term is not part of his vocabulary. How should we understand this term, in the context of the title of his *Opus*, where “new algebra” is identified with “restored analysis”? To answer this question, I suggest distinguishing between two kinds of problematic analysis: the former is that described by Pappus at the beginning of the 7th book of his *Mathematical Collection*, which I will call “intra-configurational”; the latter is the one Viète applied, which I will call “trans-configurational”. In order to apply the latter kind of analysis, Viète relies on his new formalism. I argue, however, that the use of this formalism is not a necessary condition for applying it. I also argue that the same kind of analysis was largely applied before Viète for solving geometrical problems, by relying on geometrical inferences of a special sort which I call “non-positional”, since they do not depend on a diagram. As an example of a similar systematic application of trans-configurational analysis, I consider al-Khayyām’s *Treatise of Algebra and Al-muqābala*. Finally, I suggest that Viète, when speaking of algebra in the title of his *Opus*, refers to the system of techniques underlying trans-configurational analysis, that is, to the art of transforming the conditions of certain purely quantitative problems, using either an appropriate formalism relative to the operations of addition, subtraction, multiplication,

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division, root extraction and solving polynomial equations applied to indeterminate numbers, or appropriate geometrical, non-positional inferences.

RÉSUMÉ (Qu'est-ce qui est neuf et qu'est-ce qui est ancien dans l'*analysis restituta* et l'*algebra nova* de Viète et quelle en est leur provenance ? Quelques réflexions sur les relations entre l'algèbre et l'analyse avant Viète)

François Viète considérait la plupart de ses traités mathématiques comme des parties d'un corpus de textes auquel il donna le titre de *Opus restitutæ mathematicæ analyseos seu algebra nova*. Malgré ce titre et le fait que le terme « algèbre » ait été souvent employé pour désigner ce qui est d'habitude considérée comme la principale contribution de Viète aux mathématiques, ce terme ne fait pas partie de son langage technique. Comment doit-on l'entendre, dans le contexte du titre de son *Opus*, où la « nouvelle algèbre » est identifiée à l'« analyse restaurée » ? Pour répondre à cette question, je propose de distinguer deux sortes d'analyse problématique : la première est celle décrite par Pappus au début du VII^e Livre de sa *Collection mathématique* et que je propose d'appeler « intra-configurationnelle » ; la seconde est celle qui est appliquée par Viète et que je propose d'appeler « trans-configurationnelle ». Pour appliquer cette seconde sorte d'analyse, Viète se réclame de son nouveau formalisme. Je maintiens, cependant, que l'usage de ce formalisme n'est pas nécessaire. Je maintiens aussi que cette sorte d'analyse était souvent appliquée avant Viète pour résoudre des problèmes géométriques, en se réclamant d'un type particulier d'inférences géométriques que j'appelle « non-positionnelles », car elles ne dépendent d'aucun diagramme. Pour donner un exemple d'une telle application systématique de l'analyse trans-configurationnelle, je me penche sur le *Traité d'algèbre et al-muqābala* d'al-Khayyām. Je suggère enfin qu'en parlant d'algèbre, dans le titre de son *Opus*, Viète se référait au système de techniques sous-jacentes à l'analyse trans-configurationnelle, c'est-à-dire à l'art de transformer les conditions de certains problèmes purement quantitatifs, en utilisant soit un formalisme approprié, concernant les opérations d'addition, soustraction, multiplication, division, extraction de racines et solution d'équations entières appliquées à des nombres indéterminés, soit des inférences géométriques non-positionnelles.

When he published the first edition of his *In artem analyticem isagoge*, in 1591, François Viète presented it as the first of ten treatises that were to form a systematic body of mathematical texts entitled *Opus restitutæ mathematicæ analyseos seu algebra nova*.¹ Despite the identification of “restored

¹ Cf. [Viète 1591a, pp. 1r–1v]; Viète also provides a list of the treatises that were to form his *Opus*: cf. footnote 2. Some copies of the printed texts of the dedicatory letter to the Princess Catherine de Parthenay and of the *Isagoge* were bound together with a different title page mentioning the title of the *Opus* as a whole (*Francisci Vietae Fontenæensis Opus restitutæ mathematicæ analyseos seu algebra nova*, and the same publisher and date as Viète [1591a]). A copy of such a volume is now kept at the Library of the Arsenal, in Paris, under the signature “FOL-S- 1091 (2), Pièce n° 2”. Presumably Ritter was using this volume for his French translation of the dedicatory letter to the Princess Catherine de Parthenay and of the *Isagoge*: cf. [Ritter 1868], pp. 224 (footnote by B. Boncompagni) and 225.

analysis” and “new algebra” suggested by this title, Viète’s vocabulary in the *Isagoge* and other treatises of his *Opus*² includes the term “analysis”, but not the term “algebra”.

Still, Viète’s title had some consequences. In 1630, two different French translations of the *Isagoge* and the *Zeteticorum libri quinque* (the second treatise of Viète’s *Opus*), appeared, prepared by Vaulézard and Vasset, respectively.³ The former published his translations separately, and entitled the first of them *Introduction en l’art analytique, ou Nouvelle algèbre*; the latter united his translations in a unique volume, under the title *L’algèbre nouvelle de Mr Viète*. Also in 1630 Ghetaldi’s posthumous treatise on Viète’s art was published. Though, for Ghetaldi, this art pertained to analysis and synthesis (which, contrary to Viète, he called with their Latin names: “*resolutio*” and “*compositio*”), it applied a sort of “algebra”: “not the vulgar sort [...], but that of which François Viète is the author”.⁴

When van Schooten published his edition of Viète’s *Opera mathematica*, sixteen years later, in 1646, he did not follow the practice of Vaulézard, Vasset and Ghetaldi: while he included in his collection all the treatises included in Viète’s *Opus* (including Anderson’s⁵), he did not mention this *Opus* as such, and attributed to Viète no sort of algebra. However, he transformed Viète’s terminology too and called him “the first author of analysis by species [*analyseos speciosæ*]”, whereas Viète himself had used the term “logistic by species [*logistice speciosa*]”.

The comparison of the wording of Viète, Vaulézard, Vasset, Ghetaldi, and van Schooten shows that the technical vocabulary of Viète’s school was unstable. Consequently, the mutual relations that, according to Viète and his followers, should have held between analysis, algebra and logistic, in particular when they were supposed to deal with species, are quite difficult to clarify by a literal examination of their works. As a matter of fact,

² Viète’s *Opus* was to comprise ten treatises. Eight of them correspond eight of his later publications: [Viète 1631], [Viète 1591b], [Viète 1600], the first part of [Viète 1615a], [Viète 1591c], [Viète 1593a], [Viète 1593b]. The remaining two are denoted as *Ad logisticem speciosam notæ posteriores* and *Analytica angularum sectionum in tres partes tributa*. Ritter [1895, p. 396] has suggested that the former identifies with the second part of [Viète 1615a]. A. Anderson reconstructed the first part of the latter in [Viète 1615b], while there is no trace of the two other parts.

³ Cf. [Vaulézard 1630b], [Vaulézard 1630a], and [Vasset 1630].

⁴ Cf. [Ghetaldi 1630, pp. 1–2]: “*Duplex autem est resolutionis genus alterum quidem ad Theoremata pertinet [...] alterum vero ad Problemata [...]. sed omnia fere Theoremata, & Problemata, quæ sub Algebram cadunt facillime resolvuntur, ac per resolutionis vestigia componuntur: non quidem vulgaris Algebræ beneficio; quæ resolutionis vestigia omnino confundit; sed illius, cuius auctor est Franciscus Vieta [...].*”

⁵ Cf. footnote 2.