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TEXTS & DOCUMENTS

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LOUIS BACHELIER'S 1938 MONOGRAPH ON THE CALCULUS OF SPECULATION: MATHEMATICAL FINANCE AND RANDOMNESS OF ASSET PRICES IN BACHELIER'S LATER WORK

Hichem Ben-El-Mechaiekh & Robert W. Dimand

Abstract. — Louis Bachelier's 1900 dissertation on the theory of speculation is now recognized as a landmark in the history of mathematical finance and stochastic processes, but his later work receives much less attention. Over the four decades following the defense of his dissertation on March 29, 1900, Bachelier repeatedly published new formulations of his theory of speculation: a more mathematically rigorous version in 1912, a less formal and more accessible chapter in 1914, and finally, in 1938, a monograph that was more concise and readable and more mathematically elegant than his earlier statements of the theory. That long-neglected monograph, Bachelier's final statement of his theory of speculation, is translated here into English for the first time, making it accessible to a larger audience.

Résumé (TEXTES & DOCUMENTS : La monographie de Louis Bachelier (1938) sur le calcul de spéculation : les mathématiques financières et l'aléatoire des prix des actifs dans l'IJuvre tardive de Bachelier)

Alors que la thèse de doctorat de Louis Bachelier sur la théorie de la spéculation est considèrée à juste titre comme une fondation des mathématiques financières et des processus aléatoires, ses derniers travaux sont loin d'avoir reçus la même attention. Tout au long des quarante années qui suivirent la date

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Mots clefs. — Louis Bachelier, mathématiques financières, théorie de la spéculation, mouvement brownien, prix des actifs.

de la publication de la thèse le 29 mars 1900, Bachelier s'employa a publier de nouvelles versions de sa théorie: une reformulation mathématiquement plus rigoureuse en 1912, une moins formelle et plus accessible en 1914, et finalement une exposition plus concise et mathématiquement plus élégante que les versions antérieures sous la forme d'une monographie en 1938. Ce compte-rendu final de la théorie de la spéculation de Bachelier, traduit ci-dessous en langue anglaise pour la première fois, est mis à la disposition d'un lectorat élargi.

INTRODUCTION

Louis Bachelier's doctoral dissertation *Théorie de la Spéculation*, defended on 29 March 1900 [Bachelier 1900], achieved renown after half a century of neglect. At first, mathematicians paid little attention to an introductory study of the analytical valuation of the pricing of options on government bonds not only as being outside the usual range of topics studied by contemporary mathematicians, but also because of over-simplifications that may appear natural to a physicist with hands-on experience in the Paris stock exchange, it lacked the mathematical rigor, formalism and depth [Bernstein 2005] despite its remarkable originality. On the other hand, the absence of any echo to Bachelier [1900] from those actively involved in speculation in bonds in early twentieth century Paris could only have resulted from the fact that either they have not heard of it, or simply did not have the mathematical background necessary to understand it.

While mathematics and finance have been in close relationship since antiquity, Bachelier's modeling of stock prices by equations for Brownian motion is heralded today as the birth of financial mathematics [Courtault et al. 2000]. But this area of human thought has only gained recognition as a scientific field after 1940 under the formidable impetus of the MIT modern finance pioneers such as Nobel laureates Paul Samuelson, Franco Modigliani, Myron Scholes, and Robert Merton. The limited early resonance of Bachelier's seminal ideas largely finds explanation in the absence of an "organized scientific community interested in his research" [Jovanovic 2010].

As pointed out by an anonymous referee—whom we wholeheartedly thank for insightful remarks—it is important to mitigate the myth of a disenchanted Bachelier who went totally unnoticed only to be incidentally rediscovered by Benoît Mandelbrot [1967] decades later (see e.g., [Weatherall 2013, 22–24]). As a matter of fact, Bachelier himself published various new accounts of his early ideas between 1906 and 1941,

even though without novel inspirations or vitality after 1915. And his 1912 *Traité* was relatively well cited between 1913 and 1923 but enjoyed weak dissemination between 1924 and 1960 (see [Jovanovic 2012], for a detailed bibliometric analysis of the dissemination of Bachelier's work). The situation changed dramatically when A. James Boness's English translation of Bachelier [1900] was published in Paul Cootner's *The Random Character of Stock Market Prices* [Cootner 1964] and when William Feller [1966, 181], influenced by Lévy [1948], introduced the term Wiener-Bachelier process as a synonym for Brownian motion, an identification also made on the opening page of Itô & McKean Jr. [1965]. Benoît [Mandelbrot 1989, 86] holds that the subject of finance "has its Gregor Mendel in Louis Bachelier" (see also [Mandelbrot & Hudson 2004]), while Bernard Bru (in [Taqqu 2001] and in [Courtault et al. 2000, 98]) considers Bachelier [1900] as "the Newton of the Bourse" and Jules Regnault [1863] as "its Kepler" (see also [Dimand 1993], [Jovanovic & Le Gall 2001], and [Jovanovic 2006]).

Mark Davis and Alison Etheridge note that Bachelier "defined Brownian motion—predating Einstein by five years—and the Markov property, derived the Chapman-Kolmogorov equation and established the connection between Brownian motion and the heat equation [Davis & Etheridge 2006, xiii]. The purpose of all this was to give a theory for the valuation of financial options. Bachelier came up with a formula which, given his mathematical model of asset prices, is correct" (although, because of limited liability, which prevents asset prices from going below zero, later writers since Paul Samuelson and M. F. M. Osborne have assumed geometric Brownian motion of the logarithm of asset price, rather than of the level of asset prices). Peter Bernstein [1992], Paul Samuelson (in his foreword to [Davis & Etheridge 2006]), [Jovanovic 2000], [Dimand & Ben-El-Mechaiekh 2006], and [Read 2013] tell the remarkable story of the rediscovery of Bachelier [1900]. Bachelier [1900] is justly celebrated, with [Courtault & Kabanov 2002] publishing the proceedings of a symposium held on 29 March 2000 to mark the centenary of Bachelier's thesis defense.

The difficulties of Bachelier's academic career after his return from the First World War are well covered in the existing literature: a series of limited-term sabbatical replacement positions (as *chargé de cours*) in the provinces until he finally obtained a chair in mathematics in Besançon in 1927 at the age of fifty-seven, a mere decade before retirement (see details in [Taqqu 2001]). However, it seems to be much less well known that Bachelier offered new mathematical statements of his theory of spec-