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Astérisque, tome 64 (1979), p. 1

<http://www.numdam.org/item?id=AST_1979__64__1_0>

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Astérisque, tome 64 (1979), p. 3-86
<http://www.numdam.org/item?id=AST_1979__64__3_0>

SUPERSINGULAR K3 CRYSTALS

by

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§0. INTRODUCTION.

This paper is intented as propaganda for the machinery of crystalline cohomology, and in particular for the philosophy that F-crystals are a partial analogue, in characteristic p, to Hodge structures in characteristic zero. An extremely rudimentary start along this road, for "abstract" F-crystals and Hodge structures, was made in [15] ; here we turn to crystals arising geometrically, especially from supersingular abelian varieties and K3 surfaces. As we shall see, it is reasonable to hope that the moduli of such varieties are given by the moduli of their F-crystals, which in fact form explicit "period-spaces".

Here is a plan of the paper : The first section contains some refinements of generally known facts concerning crystalline Chern classes, e.g. an integral version of Bloch's theorem relating flat and crystalline cohomology (1.7), conditions guaranteeing that $c_1 : \mathrm{Pic} \otimes \mathbb{Z}/p\mathbb{Z} \to H^2_{DR}$ is injective (1.4), and a formula for certain second order obstructions to extending invertible sheaves in a family (1.15).

The second section gives applications of these results to families of polarized K3 surfaces. In particular, we slightly refine Deligne's proof of liftability of a K3 by bounding the ramification ; this allows us to prove that if $p > 2$, the map $\mathrm{Aut}(X) \to \mathrm{Aut}\, H^2_{\mathrm{cris}}(X/W)$ is injective. We also show that the geometric generic fiber of a versal family of polarized K3's is ordinary and has base number $\rho = 1$.

The next three sections are devoted to the classification of those F-crystals which have the slopes and Hodge numbers of the crystalline cohomology of a supersingular surface with $p_g = 1$, which we call "supersingular K3 crystals". In section three we give the basic structure theorems and explicit "coarse moduli". Section four introduces a fine moduli space for such crystals, suitably rigidified. This space turns out to have a beautiful smooth compactification, with a clear "modular" interpretation. In the fifth section we discuss families of crystals, make precise the term "fine moduli", and study the period map arising from a family of K3 surfaces. As Artin showed, K3 surfaces with $\rho = 22$ fit in 9 dimensional versal families ; we show that (after suitably rigidifying) the period map to our fine moduli space is étale. This is the local Torelli theorem, and, I hope, the first step towards a global Torelli theorem for supersingular K3 surfaces.

In the sixth section, we look at supersingular abelian varieties of dimension n. We prove a Torelli theorem : If Y and Y' are supersingular abelian varieties of dimension $n \geq 2$, and if there exists an isomorphism : $H^1_{\mathrm{cris}}(Y/W) \to H^1_{\mathrm{cris}}(Y'/W)$ compatible with Frobenius and the trace map, then Y and Y' are isomorphic. It is interesting to note that this is <u>false</u> if $n = 1$, or if the trace map is forgotten, and in particular the interpretation of H^1 in terms of p-divisible groups is inadequate for such a result.

The final section is devoted to the Torelli problem for K3 surfaces in characteristic $p > 2$ with $\rho = 22$, which takes the following forms :

0.1 <u>Conjecture</u>. X <u>and</u> X' <u>are isomorphic iff there exists an isomorphism</u>