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Secrétariat :
Nathalie Christiaën
Société Mathématique de France
Institut Henri Poincaré
11, rue Pierre et Marie Curie, 75231 Paris Cedex 05
Tél. : (33) 01 44 27 67 99 / Fax : (33) 01 40 46 90 96
Mél : revues@smf.ens.fr / URL : http//smf.emath.fr/

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EDITORIAL

This issue of the *Revue d'histoire des mathématiques/Journal for the History of Mathematics* opens with an analysis of a mathematical dispute. This methodological approach, one associated with precise case studies, has been used in the history of science for some thirty years. There, it has often been viewed as a strong means for reinterpretation and for overcoming the old opposition between internalist and externalist studies. In the history of mathematics, where there are a number of famous and well-known controversies, this methodology has, however, not often been used to link the study of mathematical content with other facets involved in the making of mathematical knowledge, be they social, philosophical or political. In the first paper published in this issue, Frédéric Brechenmacher offers such an analysis. The case he studies concerns a rather poorly known controversy that ensued following the encounter, to use Brechenmacher's imagery, in 1874 of two theorems that would today be considered equivalent: Jordan's theorem (1870) on the decomposition of matrices and Weierstrass's theorem (1868) on elementary divisors. Brechenmacher carefully follows the two principal interlocutors in the quarrel, Camille Jordan and Leopold Kronecker, as well as the statements, both public and private, that they were led to make. The first blow was struck in public and, according to Brechenmacher, it highlighted two opposing ways of organizing the theory of bilinear forms. This clash was followed by an epistolary exchange in which Jordan aimed to lead the quarrel back into a private sphere. Although this aim was not realized, the episode allowed Jordan not only to understand the effects of the Berlin network in which Weierstrass and Kronecker participated but also to become more familiar with that network's local practices, practices concerned with bundles ("Schaaren") of bilinear forms. In fact, Brechenmacher succeeds in showing with great subtlety that the two protagonists actually shared a common practice devised in order to solve the problem of small oscillations in a mechanical system (Lagrange 1766), but that they inserted into different theoretical frameworks and invested with different epistemological values. While Jordan laid claim to simplicity, Kronecker ridiculed it, stressing efficiency instead. It was an historical perspective, even if it had different manifestations for each of the two authors, that allowed them to recognize the algebraic nature of their methods, methods that would, in the 1930s, be encompassed in linear algebra.

The second paper, in its own way, also bears witness to the impact that different epistemological positions may have, although less on the acceptance and recognition of mathematical practices than on their transmission. Cinzia Cerroni considers the reception of David Hilbert's famous *Grundlagen der Geometrie* (1899) and the influence of his Göttingen student Max Dehn on Italian geometers. In his *Grundlagen* and in his 1902 lecture, Hilbert treated the relations between the axioms of geometry and worked out a procedure, also used by Dehn, to prove the independence of one axiom from the others, namely, exhibit a geometry that satisfies all of the axioms, except the one the independence of which is to be demonstrated. This procedure leads to the construction of non-Archimedean geometries, in which not only the axiom of Archimedes fails to hold but also there are infinitely many lines parallel to a given straight line and passing through a point not on the line. These geometries are called non-Legendrian (if the sum of the inner angles of a triangle is greater than two right angles), semi-Euclidean (if the said sum is equal to two right angles) and hyperbolic (if it is smaller than two right angles). Dehn was at the inception of a research program based on the above procedure and created a school, while Giuseppe Veronese, who had been the very first to try to construct a non-Archimedean geometry, was unable to gather Italian geometers around him. Cerroni shows that for Federigo Enriques and his student Roberto Bonola the problem of the foundations of geometry belonged to elementary mathematics, while for Hilbert and Dehn it was part of a highly valued, fundamental research program. It was these respective stances that, on the one hand, allowed Dehn to formulate such a program and, on the other, resulted in the absence of the Italians from the international debate.

In the section on "Notes & débats," Sabine Rommevaux treats the difficulties encountered by editors of medieval mathematical texts. If, in the first paper, Brechenmacher, insists on the fact that contemporary mathematical formulations can hide the multiple identities of mathematical statements, the situation is even more complex in Rommevaux's example. There, the statements lie in the remote past, making it even more difficult to reconstruct their cultural context; the editor of the 21st century can no longer observe what time has definitively erased. The example Rommevaux details concerns the critical edition published in 2005 by Hubert Busard of Campanus's version of Euclid's *Elements*. In the 13th century, Campanus revised and provided commentary on one of the versions of the *Elements* given by Robert of Chester in the 12th century. Rommevaux shows that changing one word — *simul* to *similes* — in the definition of proportionality in Book V changes considerably the sense and the importance of the

definition. Her textual choice conforms to what may be found in the most ancient manuscripts and allows her most notably to change our understanding of Campanus's commentary. According to Rommevaux, Campanus makes visible in his formulation of the definition of continuous proportionality a circularity that was already implicit in the 12^{th}-century translations. Although he recognized this circularity, Campanus did not reject the definition. It was as if mathematical coherence was of little importance to him, an impression that later earned him much criticism. Rommevaux justifies her editorial choice through textual and doctrinal coherence, while the reasons for Busard's choice, on which she speculates, seem to be linked to his search for a proximity to the Greek version of the *Elements*.

The publication of this note also provides an occasion for the *Revue d'histoire des mathématiques / Journal for the History of Mathematics* to honor the huge editorial and explanatory work done by Hubert Busard on medieval versions of the Euclidean text. Deceased on 2 December 2007, he can unfortunately neither respond to the critique formulated by Rommevaux nor justify his editorial choice.

The Editors-in-Chief