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with notes on the birth of Number Theory*

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FACTOR TABLES 1657–1817, WITH NOTES ON THE BIRTH OF NUMBER THEORY

MAARTEN BULLYNCK

ABSTRACT. — The history of the construction, organisation and publication of factor tables from 1657 to 1817, in itself a fascinating story, also touches upon many topics of general interest for the history of mathematics. The considerable labour involved in constructing and correcting these tables has pushed mathematicians and calculators to organise themselves in networks. Around 1660 J. Pell was the first to motivate others to calculate a large factor table, for which he saw many applications, from Diophantine analysis and arithmetic to philosophy. About a century later (1770), J.H. Lambert launched a table project that was to engage many (human) computers and mathematicians in the (re)production and extension of Pell's table. Lambert also pointed out

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that a theory of numbers, of divisors and factoring methods was still lacking. Lambert's ideas were taken up by his colleagues at the Berlin Academy, and indirectly by L. Euler in St. Petersburg. Finally, the many number-theoretical essays that were written in the context of Lambert's table project contributed significantly to the birth of higher arithmetic around 1800, soon to be marked by the works of A.-M. Legendre and C.F. Gauss.

RÉSUMÉ (Tables des diviseurs 1657–1817, avec des notes sur la naissance de la théorie des nombres)

La fabrication, l'organisation et la publication des tables de diviseurs, de 1657 à 1817, constitue non seulement une histoire fascinante en soi, mais soulève en même temps des enjeux plus généraux de l'histoire des mathématiques. Le travail considérable que demande la fabrication et la correction de ces tables a poussé les mathématiciens et calculateurs à s'organiser en réseau scientifique. John Pell a été aisé le premier qui incita, autour de 1660, d'autres mathématiciens à produire une grande table de diviseurs. Il en vantait l'utilité, non seulement pour l'analyse diophantienne, mais aussi pour l'arithmétique et même pour la philosophie. Un siècle plus tard, en 1770, Jean Henri Lambert lança un vaste projet, engageant beaucoup de calculateurs et mathématiciens, pour (re)construire et étendre la table de Pell. Ce faisant Lambert insistait sur le fait qu'une théorie des nombres, des diviseurs et des méthodes de factorisation faisait toujours défaut. Ces idées étaient reprises par les collègues de Lambert à l'Académie de Berlin et indirectement par Leonhard Euler à St. Petersburg. Les nombreux textes sur la théorie des nombres écrits dans le cadre de ce projet de Lambert contribuaient de manière importante à la naissance de l'arithmétique supérieure aux alentours de 1800 dans les travaux de A.-M. Legendre et C.F. Gauss.

1. INTRODUCTION

The aim of the present paper is a historiographical appraisal of the construction of factor tables as a proper part of the scientific and social history of mathematics and as an important chapter or tradition within the history of number theory. Of course, like other scientific experts, the makers of tables which list prime numbers or factors of positive integers have, at least since the 18th–19th century, duly cultivated the memory of the work of their forerunners. In fact, in the case of factor tables, there was also a more specific reason to document and consult older tables. Since the entries in a factor table can neither be approximated nor interpolated with the help of the surrounding values, comparison with already existing tables was and is essential to test a new table. The most complete list of prime and/or factor tables compiled before the twentieth century can be found

in [Lehmer \[1909, i–vii\]](#) and [Dickson \[1919–1927, I, pp. 347–356\]](#); the most extensive commentary on and analysis of factor and prime tables was provided by [Glaisher \[1878\]](#), in a companion essay to his *Factor Table for the Fourth Million*.

A topic closely connected with the construction of factor tables is the development of primality tests and factoring algorithms. Especially since the advent of the digital computer and still more since the invention of RSA-encryption, primality tests and factoring algorithms are considered as an important research field for mathematics and its applications. Before 1945, however, the topic figured mainly in research on and construction of tables in number theory. The history of primality tests and factoring algorithms has already been well documented by [Dickson \[1919–1927, I, pp. 357–374\]](#) and more recently been reappraised by [Williams & Shallit \[1994\]](#) and [Mollin \[2002\]](#).

This paper wants to go beyond the mere chronological list of tables and factoring methods and embed them into their proper historical context, scientifically, socially and philosophically. We will show that the circumstances of production of the earliest prime and factor tables provide insight into the way in which mathematicians and calculators organised themselves in communities or networks in the 17th and 18th centuries. Further, it will be demonstrated that the use and production of factor tables brought up specific problems, questions and viewpoints. Conditioned by the peculiarities of factor and prime tables, a particular frame of reference with its own concepts and partial theories came, in time, to be articulated, and was eventually to have rather an important impact on the emergence of number theory. Indeed, it is claimed here that the theories and methods that provide the theoretical background for factor tables constitute one of the contexts that should complement the classical story told about the formation of number theory; viz. the Greek heritage of Pythagoras and Diophantus, its transformation in the hands of Bachet, Fermat and Euler, finally culminating in A.-M. Legendre’s *Essai* (1798) and C.F. Gauss’ *Disquisitiones Arithmeticae* (1801).¹

¹ On the role of the Pythagorean, Diophantine and Fermatian problems for the formation of number theory, see e.g. [\[Weil 1984, Chap. IV\]](#) and [\[Shanks 1993, Chap. 1\]](#).