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NOTES & DÉBATS

THE IMPACT OF MODERN MATHEMATICS ON ANCIENT MATHEMATICS

Wilbur R. Knorr

Abstract. — In a hitherto unpublished lecture, delivered in Atlanta, 1975, W.R. Knorr reflects on historical method, its sensitivity to modern work, both in mathematics and in the philosophy of mathematics. Three examples taken from the work of Tannery, Hasse, Scholz and Becker and concerning the study of pre-Euclidean geometry are discussed: the mis-described discovery of irrational 'numbers', the alleged foundations crisis in the 5th century B.C. and the problem of constructibility.

Résumé. — L'impact des mathématiques modernes sur les mathématiques anciennes. — Dans une conférence prononcée en 1975 à Atlanta, et restée inédite, W.R. Knorr livre quelques réflexions sur la méthode historique, sa dépendance de travaux modernes, tant en mathématiques qu'en philosophie des mathématiques. Il s'appuie sur trois exemples tirés des travaux de Tannery, Hasse, Scholz et Becker sur la géométrie grecque pré-euclidienne: la découverte mal-nommée des 'nombres' irrationnels, la dite crise des fondements du V^e siècle avant J.C. et le problème de la constructibilité.

Edith Prentice Mendez found this lecture among Wilbur Knorr's papers after his death in March, 1997. Although Knorr probably never intended to publish it – and he surely would have attended to its occasional roughness – Ken Saito and I consider it an important methodological reflection on his just completed work on the early proportion theory,[1] but with much general interest. The three main examples he

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[1] Wilbur R. Knorr, *The Evolution of the Euclidean Elements*, Dordrecht: Reidel, 1975.

discusses, the theory of irrationals, the alleged foundations crisis in the fifth century and the problem of constructibility, remain important morality tales for contemporary researchers. Among specialists, the pendulum may have swung largely in the other direction, and for that reason, it is useful to quote a letter which warns against the opposed impediment to historical understanding. I thank Joseph Dauben for drawing it to my attention by sending me his transcription of it.

Wilbur Knorr to Joseph W. Dauben
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Whipple Museum, University of Cambridge
March 27, 1975.

"[...] Now, research in the ancient materials is something of an art, and I know that many scholars are by temperament unsuited for it, as they themselves would agree. Basically, the Greek record is fragmentary; we possess a few mathematical treatises virtually complete, others in part, others in random snippets preserved by accident in derivative works, plus a small para-mathematical literature, the logical writings of Plato and Aristotle, for instance. In this circumstance, literalism would be disastrous. For instance, most of the complete treatises which have survived expound a highly formal type of advanced geometry. Does this mean the Greeks were weak in the traditional areas of practical geometry and arithmetic? It goes against reason to believe so. But some scholars ... would have us draw such a conclusion. Rather, at every step one must make allowance for the selective survival of documents. The formal geometry survived because it was also philosophically interesting (from the axiomatic viewpoint) and because it merited study by serious practitioners of geometry. But easily duplicatable computations were hardly worth preserving via manuscript traditions. What mathematician has ever preserved his rough figures, once the final draft of his study has been completed? Occasionally, papyri containing everyday arithmetic and geometric problems survive. These are invariably schoolboys' exercises, amazing for the modesty of their mathematical content. Interestingly, computation throughout Greek antiquity – commercial arithmetic – was done by the Egyptian methods. But otherwise, we are left to surmise the nature of the whole from the upper most ten per cent. In this situation, a scholar with an imagination and a feeling for organizing incomplete evidence into rational frameworks can enjoy himself. But the end-products of such studies can never be much other than this or that degree of plausibility. I find this refreshing. But many find it appalling and seek the haven of documentary objectivity. I think that the student of mathematics from 1650 or so onward has the opposite problem of contending with more documentation than is manageable. Here, if ever one makes a general statement of fact, he must expect that in the materials he could not examine contrary patterns might emerge. But didn't Pascal develop this notion of the two types of reasoning? [...]"

We have provided all footnotes and hence are responsible for any failure to capture Knorr's allusions. I have also checked the quotations and adjusted some (including a slight clarification of the status of one quotation) and did some other minor editing. As to the alluring title, fans of the novelist David Lodge will no doubt recall the hapless Persse Mc Garrigle and his "The Influence of T.S. Eliot on Shakespeare" in *Small World* (1984).

Knorr left many other important papers, which I hope to bring out in due time.

Henry Mendell

TRANSCRIPT OF A LECTURE DELIVERED AT THE ANNUAL CONVENTION OF THE HISTORY OF SCIENCE SOCIETY, ATLANTA, DECEMBER 28, 1975.

Chairman: Prof. Joseph Dauben,
Lehman College, City University of New York[2]

When Joe suggested to me the possibility of speaking at this meeting, the topic then projected for the colloquium was nineteenth-century mathematics. I told him I was better prepared to speak on ancient mathematics than on 19th-century. But on thinking it over, I hit upon the idea of discussing the impact of modern mathematics on ancient mathematics.

Now, what ancient mathematics was and what ancient mathematicians did has not been influenced by more recent achievements, of course. But what we take ancient mathematics to have been is very strongly influenced by modern work, both in mathematics and in the philosophy of mathematics. It is this sensitivity of the historical criticism that I wish to examine, by way of a few examples from the study of pre-Euclidean geometry. – Afterwards, I will propose some general observations on historical method, based on these examples.

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"Why didn't the Greeks construct the irrational numbers?" This question was the subject of an article by Heinrich Scholz in 1928.[3] Scholz was examining a polemical statement by Oswald Spengler, to the effect that the Greeks, overburdened by a concrete and plastic intellectual outlook, thereby missed the mathematical abstraction accessible to us now through our algebraic conceptions. Scholz rightly branded the observation nonsense. The Greeks were not blind to an extension of the number-concept through some accidental failure of spirit. They rejected any such

[2] The other paper in the symposium was by Winifred Wisan on "Galileo's Mathematical Method: A Reassessment." They had both just arrived at the ill-fated New School of Liberal Arts, an honors division of Brooklyn College, whose mission was immediately modified by an open admissions policy and which was to suffer under the budget crunches of New York City in the late seventies. As a result, the positions of each were terminated.

[3] Heinrich Scholz, Warum haben die Griechen die Irrationalzahlen nicht aufgebaut, in Helmut Hasse und Heinrich Scholz, *Die Grundlagenkrisis der Griechischen Mathematik*, Berlin: Pan-Verlag, 1928, pp. 35–72.

extension on scientific and philosophical grounds: the *arithmos* must be whole-number; even the rational numbers, a necessary preliminary to irrational numbers, were excluded from the classical number theory; the problem of irrationals was thus resolved by Eudoxus in a geometric manner instead.

Scholz' assessment is sound. But what we should at once notice is that such a debate could not have arisen before the successful resolution of the problem of irrational numbers by Weierstrass and Dedekind in the 19th century. Before that time, the Euclid-editors – Barrow and de Morgan, for instance – had to answer the charges of obscurity and verbosity levelled against Euclid in his definition of proportion in Book V.[4] But already in Dedekind's time a reversal was taking place: critics like Lipschitz[5] now questioned whether Dedekind had added anything to the Euclidean theory. Somewhat later, Thomas Heath (1921; 1926) judged that "the definition of equal ratios [by Eudoxus and Euclid] corresponds exactly to the modern theory of irrationals due to Dedekind.[6] ... It is word for word the same as Weierstrass' definition of equal numbers.[7] So far from agreeing in the usual view that the Greeks saw in their rational no *number* ... it is clear from Euclid V. that they possessed a notion of number in all its generality as clearly defined, nay almost identical with, Weierstrass' conception of it." This latter judgment, in which Heath follows the view of Max Simon,[8] is undoubtedly overstated. Nevertheless,

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[4] Cf. Thomas L. Heath, *The Thirteen Books of Euclid's* Elements, translation with introduction and commentary, 3 vols., 2nd ed., Cambridge: Cambridge University Press, 1926, vol. 2, pp. 121–122.

[5] Knorr's source may be a letter from Richard Dedekind to Rudolf Lipschitz dated 6 July 1876, in which he quotes extensively from Lipschitz' previous letter to him. Lipschitz wonders if Dedekind's account of real number is merely Euclid, *Elements* V, def. 5, which he quotes in Latin, "rationem habere inter se magnitudines dicuntur, quae possunt multiplicatae sese mutuo superare [...]" (cf. Richard Dedekind, *Gesammelte mathematische Werke*, ed. by Robert Fricke, Emmy Noether, and Öystein Ore, vol. 3, Braunschweig, 1932, pp. 469–470. Lipschitz' letters are now published, *Briefwechsel mit Cantor, Dedekind, Helmholtz, Kronecker, Weierstrass und anderen*, ed. by Winfried Scharlau, Braunschweig: Vieweg, 1986. For this letter of 6 July 1876, see pp. 70–73.

[6] Thomas L. Heath, *A History of Greek Mathematics*, Oxford: Clarendon Press, 1921, p. 327.

[7] Heath, *op. cit.*, 1926, (see note 4), vol. 2, p. 124.

[8] Maximilian Simon, *Euclid und die sechs planimetrischen Bücher*, Leipzig: Teubner, 1901, p. 110.

we meet later writers like A.E. Taylor,[9] who insist on finding traces of the modern real-number concept in obscure passages from ancient authors. When Plato is reported to have described how "the One equalizes the Great-and-Small",[10] this is read as the definition of an irrational number as the limit of an alternating rational sequence. Again, a puzzling, and likely corrupt, passage from Aristotle, that "number is also predicated of that which is not commensurable",[11] has recently been used to affirm the conception of irrational *numbers* in the early 4th century B.C. In letting such evidence over-ride the unanimous restriction to whole numbers in the pre-Diophantine literature on number theory, these writers clearly betray a distortion of critical viewpoint owing to their awareness of the modern real-number concept.

Thus, the successful "arithmetization of the continuum" in the 19th century has had perceptible effects on the interpretation of the ancients. In a positive way, it has drawn new attention to certain areas, here the Eudoxean proportion theory, until then not fully understood or appreciated. But once such a problem in mathematics has received a modern solution, this solution tends to be given an absolute status and

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[9] Alfred E. Taylor, *Plato*: *the Man and his Work*, 7th ed., London: Routledge, 1960, pp. 509–513.

[10] Cf. Taylor, *ibid.*, p. 512. Our primary source, Aristotle, *Metaphysics* M 8.1083^{b}23-32, N 3.1091^{a}23-5, attacks this view as part of Plato's position on number.

[11] The text would appear to be *Met.* D15.1021^{a}5-6. Of the three principal manuscripts (labelled E, J,A^{b}) used by W.D. Ross in *Aristotle's Metaphysics* (Oxford: Oxford University Press, 1924), E and J and Alexander of Aphrodisias have: *kata mê summetron de arithmon legetai* (or *legontai*), and so it is printed in every text before Ross, and which he translates in his first Oxford translation (1908), "but this relation may involve a 'non-commensurate number'." A^{b} has instead: *kata mê summetron de arithmos ou legetai*, which Ross emends to: *kata mê summetrou de arithmos ou legetai* (number is not said of the non-commensurate). In general, where E, J, and Alexander agree against A^{b}, Ross sides with them against A^{b} (cf. introduction to his text, p. clxi), but not always (cf. 1008^{a}25 and introduction p. clxii). All texts and most translations follow Ross (exceptions are translations by R. Hope and H. Apostle, who seem to translate unstated emendations along the lines of EJ). However, if E, J are in error, it still remains interesting that someone before Alexander (ca. 300 C.E.) wrote 'non-commensurable number', i.e. if they wrote it intentionally. It is possible that Knorr refers to Julius Stenzel, Zur Theorie des Logos bei Aristoteles, *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*, Abt. B, Bd. I, 1929, pp. 34–66, in particular pp. 57–60 (reprinted in *Kleine Schriften zur griechischen Philosophie*, Darmstadt: Wissenschaftliche Buchgesellschaft, 1956, pp. 188–219, in particular pp. 210–212). However, the reference may well be to a more recent (and less sophisticated) interpretation of the passage.