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To my parents

KOBAYASHI-HITCHIN CORRESPONDENCE FOR TAME HARMONIC BUNDLES AND AN APPLICATION

Takuro Mochizuki

***Abstract.* —** We establish the correspondence between tame harmonic bundles and μ_L-polystable parabolic Higgs bundles with trivial characteristic numbers. We also show the Bogomolov-Gieseker type inequality for μ_L-stable parabolic Higgs bundles.

Then we show that any local system on a smooth quasiprojective variety can be deformed to a variation of polarized Hodge structure. As a consequence, we can conclude that some kind of discrete groups cannot be a split quotient of the fundamental group of a smooth quasiprojective variety.

***Résumé* (La correspondance de Kobayashi-Hitchin pour les fibrés harmoniques modérés et une application)**

Nous établissons la correspondance de Kobayashi-Hitchin entre les fibrés harmoniques modérés et fibrés de Higgs paraboliques μ_L-polystables dont les deux premiers nombres de Chern sont nuls. Ensuite, nous montrons que tout système local sur une variété quasi-projective lisse peut être déformé vers une variation de structure de Hodge polarisée. En conséquence, nous pouvons conclure que certains groupes discrets ne peuvent pas apparaître comme quotient scindé d'un groupe fondamental d'une variété quasi-projective lisse.

CONTENTS

CHAPTER 1

INTRODUCTION

1.1. Background

1.1.1. Kobayashi-Hitchin correspondence. — We briefly recall some aspects of the so-called Kobayashi-Hitchin correspondence. (See the introduction of [**38**] for more detail.) In 1960's, M. S. Narasimhan and C. S. Seshadri proved the correspondence between irreducible flat unitary bundles and stable vector bundles with degree 0, on a compact Riemann surface ([**47**]). Clearly, it was desired to extend their result to the higher dimensional case and the non-flat case.

In early 1980's, S. Kobayashi introduced the Hermitian-Einstein condition for holomorphic bundles on Kahler manifolds ([**30**], [**31**]). He and M. Lübke ([**37**]) proved that the existence of Hermitian-Einstein metric implies the polystability of the underlying holomorphic bundle. S. K. Donaldson pioneered the way for the inverse problem ([**12**] and [**13**]). He attributed the problem to Kobayashi and N. Hitchin. The definitive result was given by K. Uhlenbeck, S. T. Yau and Donaldson ([**64**] and [**14**]). We also remark that V. Mehta and A. Ramanathan ([**40**]) proved the correspondence in the case where the Chern class is trivial, i.e., the correspondence of flat unitary bundles and stable vector bundles with trivial Chern classes.

On the other hand, it was quite fruitful to consider the correspondences for vector bundles with some additional structures like Higgs fields, which was initiated by Hitchin ([**22**]). He studied the Higgs bundles on a compact Riemann surface and the moduli spaces. His work has influenced various fields of mathematics. It involves a lot of subjects and ideas, and one of his results is the correspondence of the stability and the existence of Hermitian-Einstein metrics for Higgs bundles on a compact Riemann surface.

1.1.2. A part of C. Simpson's work. — C. Simpson studied the Higgs bundles over higher dimensional complex manifolds, influenced by the work of Hitchin, but motivated by his own subject: Variation of Polarized Hodge Structure. He made great