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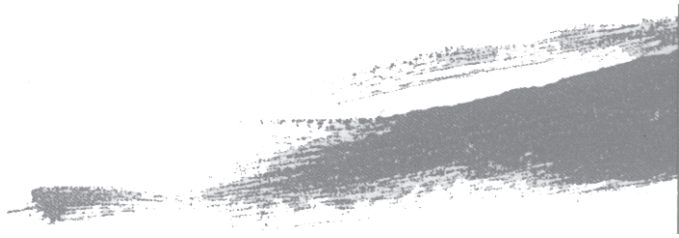

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SOCIÉTÉ MATHÉMATIQUE DE FRANCE

ÉDITORIAL

"We are all Hermite's heirs," claimed Cyparissos Stephanos (1857–1917), who studied with Hermite in Paris in the 1880s and became professor of mathematics at the University of Athens. [1]

And that would probably be even truer today. The databases MathSciNet and zbMATH contain more than 5,500 articles published between, say, 1950 and 2024, whose *title* includes the name "Hermite" and more than 6,000 articles whose title includes the derived adjective "Hermitian". They belong to an impressive variety of fields, from number theory to numerical analysis, from linear algebra to computer science, from functional analysis to probabiltity theory, from fluid mechanics to partial or ordinary differential equations and special functions. There are Hermite inequalities and a Hermite constant, Hermite polynomials, Hermite schemes, Hermitian forms, matrices or varieties.

Hermite's career, however, was not a smooth one. [2] Hermite was born in Dieuze (Lorraine) on December 24, 1822 as the son of a merchant family. He studied in Nancy, then in Paris and was admitted at the École polytechnique in 1842, without distinction (ranking 68th out of 134). He left the Polytechnique only a year later, both because of a disability that made him ineligible for the usual careers open to Polytechnique students, and because Joseph Liouville, who taught there, advised him to concentrate on his main passion, mathematics. Indeed, by this time, Hermite was already publishing on topics that would occupy him for the rest of his life: algebraic equations (with a proof of the impossibility of solving the general quintic equation by radicals) and elliptic functions and their generalization (which brought him

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1 See Phili (Christine), Sur le développement des mathématiques en Grèce durant la période 1850-1950, *Istorico-matematicheskie issledovania*, 3 (1997), pp. 80-102, quote p. 85.

2 On Hermite's life, aside the substantial introduction by his son-in-law Émile Picard to Hermite's complete works, one can mention among his numerous obituaries: Noether (Max), Charles Hermite, *Mathematische Annalen*, 55 (1902), pp. 337-385 and Darboux (Gaston), Notice historique sur Charles Hermite [1905], in *Éloges académiques et discours*, Paris: Hermann, 1912, pp. 117-172. In addition, two books have been devoted to Hermite's scientific biography: Ozhigova (Elena Petrovna), *Charles Hermite*, Leningrad: Nauka, 1982 (in Russian) and Brezinski (Claude), *Charles Hermite, père de l'analyse moderne*, Paris: Société française d'histoire des sciences et des techniques, 1990.

into epistolary contact with one of the most important mathematicians of the time, Carl Gustav Jacob Jacobi). [3]

During the 1850s, Hermite extended his research to number theory (in particular to the theory of quadratic forms), approximation, invariant theory and algebra. [4] He visited Berlin and made contact with several other mathematicians, from Gotthold Eisenstein, Carl Borchardt and Peter Gustav Lejeune-Dirichlet to James Sylvester, Arthur Cayley and Augustin Cauchy. [5]

Yet, he only secured a modest job as a *répétiteur* at the Polytechnique in 1848. [6] Nevertheless, it allowed him to marry Louise Bertrand, the sister of a co-disciple at Polytechnique, Joseph Bertrand, thus joining an influential academic family. They will have two daughters, one who will marry Georges Forestier, an engineer of the Ponts-et-Chaussées and an automobile expert

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3 On Hermite's work in these topics, see Houzel (Christian), Fonctions elliptiques et intégrales abéliennes, in Dieudonné (Jean) (dir.), *Abrégé d'histoire des mathématiques 1700-1900*, Paris: Hermann, 1978, vol. 2, pp. 1-113; Belhoste (Bruno), Autour d'un mémoire inédit: la contribution d'Hermite au développement de la théorie des fonctions elliptiques, *Revue d'histoire des mathématiques*, 2 (1996), pp. 1-66; Bottazzini (Umberto) and Gray (Jeremy), *Hidden Harmony—Geometric Fantasies: The Rise of Complex Function Theory*, New York: Springer, 2013.

4 On Hermite's works on these topics, see Brezinski (Claude), *History of Continued Fractions and Padé Approximants*, Berlin, New York: Springer, 1991 ; Serfati (Michel), *Quadrature du cercle, fractions continues, et autres contes: Sur l'histoire des nombres irrationnels et transcendants aux XVIII^e et XIX^e siècles*, Paris: Éditions de l'APMEP, 1992; Sinaceur (Hourya), *Corps et modèles*, Paris: Vrin, 1994 ; Houzel (Christian), L'équation générale du cinquième degré, in *La Géométrie algébrique: recherches historiques*, Paris: Blanchard, 2003, pp. 73-80; Brouzet (Robert), La double origine du groupe symplectique, *Expositiones mathematicae*, 22 (2004), pp. 55-82 ; Goldstein (Catherine), The Hermitian Form of Reading the *Disquisitiones*, in Goldstein (Catherine), Schappacher (Norbert) & Schwermer (Joachim) (eds.), *The Shaping of Arithmetic after C. F. Gauss's* Disquisitiones Arithmeticae, Berlin: Springer, 2007, pp. 377-410 ; Brechenmacher (Frédéric), Autour de pratiques algébriques de Poincaré: héritages de la réduction de Jordan, 2011, hal-00630959v3 ; Goldstein (Catherine), Charles Hermite's Stroll through the Galois Fields, *Revue d'histoire des mathématiques*, 17 (2011), pp. 211-270.

5 On Hermite's role in the international community, see Archibald (Thomas), Charles Hermite and German mathematics in France, in Parshall (Karen Hunger) & Rice (Adrian), *Mathematics Unbound: The Evolution of an International Mathematical Research Community, 1800–1945*, Providence, RI: American Mathematical Society, 2002, pp. 123-137; Šegan-Radonjić (Marija) & Todorčević (Vesna), Mihailo Petrović's Education in France and its Significance in Establishing the Petrović School of Mathematics, *Revue d'histoire des mathématiques*, 30 (2024), pp 31-69; and the editions of Hermite's correspondence with various mathematicians in the Bibliography of Charles Hermite, hereafter.

6 On this position, see Vincent (Yannick), "Les répétiteurs de mathématiques à l'École polytechnique de 1798 à 1900," thèse de doctorat, Université Paris-Saclay, 2019.

and the other, the mathematician Émile Picard. [7] This was also the period when Hermite moved from an apparently indifferent attitude to religion to a rigorous and committed Catholicism, partly under the influence of Cauchy.

In 1856, Hermite was elected to the French Academy of Sciences, but he had to wait until 1862 to obtain a position at the École normale supérieure. In the years 1869-1870 Hermite finally secured stable and important professorships, at the Polytechnique (until 1876) and at the Faculté des sciences de Paris, the Sorbonne, from which he retired only in 1892. [8] The war between France and Prussia and especially the advent of the Third Republic in 1871, against his conservative and Catholic views, left him deeply worried and pessimistic. During this time, however, he extended his work to various questions of analysis, in particular differential equations, and in 1873 he provided the first proof of the transcendence of a natural constant of analysis, the number e. [9] He was an influential figure in the reception of the works of Galois and Riemann in France and also maintained an intense international correspondence, both with his pairs and with younger colleagues, providing advice and often papers for their newly founded mathematical journals, for example *Mathesis*, *Jornal de Sciencias Mathematicas e Astronomicas*, *Acta mathematica* or *Archiv der Mathematik und Physik*. He died on January 14, 1901.

Hermite published his first papers in 1842, his last ones in 1901, a few days before his death, and many of them had a long and fruitful posterity. Although somewhat delayed from an institutional point of view, as mentioned above, Hermite's recognition during his lifetime was impressive: he was a member of several prestigious academies and the subscription for his jubilee gathered donations from 810 persons from 17 different countries. However, despite his central place in 19th century mathematics and the ubiquitous trace of his mathematics until now, Hermite has not been much studied by historians of mathematics. He is often portrayed as a bit out of

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7 On the Bertrand family, see Zerner (Martin), Le règne de Joseph Bertrand (1874-1900), in Gispert (Hélène), *La France mathématique*, Paris: SMF et SFHST, 1991, pp. 298-322.

8 On his courses, see Renaud (Hervé), "La fabrication d'un enseignement de l'analyse en France au XIX^{e} siècle : acteurs, institutions, programmes et manuels," thèse de doctorat, Université de Nantes, 2017.

9 See Waldschmidt (Michel), Les débuts de la théorie des nombres transcendants, *Cahiers du séminaire d'histoire des mathématiques*, 4 (1983), pp. 93-115 ; Waldschmidt (Michel), La méthode de Charles Hermite en théorie des nombres transcendants, *Bibnum, Mathématiques*, (2009), `http://journals.openedition.org/bibnum/893`; Bottazzini (Umberto) and Gray (Jeremy), *Hidden Harmony—Geometric Fantasies*, op. cit.; Archibald (Tom), Hermite's "Concrete" Analysis: Research and Educational Themes in an Evolving Discipline, *Revue d'histoire des mathématiques* (2024), this issue.

step with the main innovations of his time, just missing out some important theorems, be it the structure of units in algebraic number fields or the transcendence of π, or underestimating the importance of some innovations, set theory, non-Euclidean geometry or teratological functions of analysis. A famous sentence in a letter to Thomas Stieltjes, in which Hermite "turns away with fright and horror from the lamentable plague of continuous functions without derivatives" is often taken out of context and used as evidence of his old-fashioned views on analysis, when in fact it was a simple joke after a miscalculation. [10]

In 2001, on the occasion of the centenary of Hermite's death, a special session of the seminar on the history of mathematics of the Institut Henri Poincaré in Paris was organized in the Hermite auditorium. [11] One of the most striking conclusions of this meeting was that little was actually known about Hermite. During the following decades, several investigations have revealed the complexity of his mathematics, both in its individual singularity and in its collective dimensions. Since 2001, Hermite's works have been discussed in the *Revue d'histoire des mathématiques* in connection with those of Évariste Galois, Henri Poincaré, Édouard Lucas, James Joseph Sylvester, Junius Masseau, Sylvestre François Lacroix, Élie Cartan, Leopold Kronecker, Charles-Ange Laisant, Giuseppe Peano, Hermann Minkowski, Camille Jordan, Louis Poinsot, Émile Picard, Gaston Darboux, Mihailo Petrović...

However, Charles Hermite was still the elephant in the room. The bicentenary of Hermite's birth in 2022 was thus an occasion to retrace his footsteps, to look back at some of his less studied contributions, at his institutional role in the Paris mathematical scene, [12] at his international relations

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10 On Hermite's general views on mathematics, see Brezinski (Claude), Charles Hermite: père de l'analyse mathématique moderne, op. cit. ; Goldstein (Catherine), Un arithméticien contre l'arithmétisation: les principes de Charles Hermite, in Flament (Dominique) & Nabonnand (Philippe), *Justifier en mathématiques*, Paris, Éditions de la Maison des sciences de l'Homme, 2011, pp. 129-165.

11 Several aspects of Hermite's mathematical works had been discussed on this occasion, with contributions by Catherine Goldstein on number theory, Tom Archibald on theta functions and differential equations, Jean Mawhin on Hermite's patronage of Poincaré, Hélène Gispert and Philippe Nabonnand on his role in the French mathematical community. See the program on the website of the Institut Henri Poincaré, accessed December 18, 2024, `https://www.ihp.fr/fr/1995-2008`.

12 On the institutional roles played by Hermite in the 1880s at the Paris Academy of Science as well as on his patronage on the mathematical journals edited by Gaston Darboux and Camille Jordan, see Croizat (Barnabé), *Gaston Darboux: naissance d'un mathématicien, genèse d'un professeur, chronique d'un rédacteur*, Thèse de doctorat, Université Lille 1, 2016, and Frédéric Brechenmacher, Un journal de rang « élevé »: le *Journal de mathématiques pures et appliquées* sous la direction de Camille Jordan, 2021, hal-04252496v1.

with Italy, Russia, and Serbia, and at his resonances in today's mathematics. A conference, once again in the Hermite auditorium of the Institut Henri Poincaré, took place in December 2022 in Paris. [13] The speakers at this conference included, besides the two editors of this special issue, Karen Hunger Parshall, Yannick Vincent, Natalia Ingtem, Livia Giacardi, Rossana Tazzioli, Jean-Benoît Bost, Anne Quéguiner-Mathieu, Alin Bostan, Tom Archibald, Marija Šegan-Radonjić, Vesna Todorcevic, François Lê, Barnabé Croizat, and Hélène Gispert. [14]

The present special issue of the *Revue d'histoire des mathématiques* brings together specialists from different branches of the history of mathematics. It aims to highlight, in the light of recent and ongoing research, lesser-known aspects of Hermite's work, such as invariant theory and numerical equations, and further explores Hermite's conceptions of mathematics through an analysis of his writing style, the key role played by a concrete approach to analysis in both his academic works and his teaching, and the way he constructs a unity of mathematics. Hermite's preference for effective formulas and transfers by analogy from arithmetic to algebra to analysis contrasted with the contemporary development of foundational programs, abstract objects and general frameworks. This situation may have been one of the main factors troubling his overall image over the course of the 20th century. Taking seriously the way in which Hermite and many of his interlocutors conceived of mathematics, its disciplinary division, the introduction of new subjects of study, as well as his convictions on matters of proof and results, prompts us to rewrite the development of mathematics in the 19th century in a less linear way.

The scattering of Hermite's results among notes, lectures and letters, is another major obstacle to access his mathematical thought. His works, edited in four volumes by Picard in the early 20th century, do not contain a complete bibliography and there are sometimes substantial differences between the original version and the version given in the Œuvres. Moreover, several parts of Hermite's extensive correspondence with various mathematicians have been published in various monographs and journals from the 19th to the 21st century. We have therefore supplemented the papers in

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13 The conference was supported by the LinX Laboratory of the École polytechnique, the Institut de mathématiques de Jussieu-Paris Rive Gauche, the UFR of mathematics of Sorbonne Université, and the GDR 3398 of the History of Mathematics of the CNRS. We particularly thank Sylvie Benzoni, Émilie Faure, Sébastien Gauthier, Sitti Mchinda and Frédéric Zantonio for their help and support.

14 See the program on the website of CNRS, accessed 18 December 2024, `https://indico.math.cnrs.fr/event/8758/`.

this special issue with both a list of Hermite's original publications and of his published correspondence to date.

Frédéric Brechenmacher (LinX, École polytechnique)
Catherine Goldstein (IMJ-PRG, CNRS, Sorbonne Université, Université Paris Cité)