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CHARLES HERMITE'S STROLL THROUGH THE GALOIS FIELDS

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Abstract. — Although everything seems to oppose the two mathematicians, Charles Hermite's role was crucial in the study and diffusion of Évariste Galois's results in France during the second half of the nineteenth century. The present article examines that part of Hermite's work explicitly linked to Galois, the reduction of modular equations in particular. It shows how Hermite's mathematical convictions—concerning effectiveness or the unity of algebra, analysis and arithmetic—shaped his interpretation of Galois and of the paths of development Galois opened. Reciprocally, Hermite inserted Galois's results in a vast synthesis based on invariant theory and elliptic functions, the memory of which is in great part missing in current Galois theory. At the end of the article, we discuss some methodological issues this raises in the interpretation of Galois's works and their posterity.

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Résumé (Les promenades galoisiennes de Charles Hermite)

Bien que tout semble les opposer, Charles Hermite a joué un rôle important dans l'étude et la diffusion des travaux d'Évariste Galois en France au milieu du XIX^e siècle. Cet article étudie les travaux d'Hermite en lien direct avec ceux de Galois, en particulier sur la réduction des équations modulaires. Il montre comment les convictions mathématiques d'Hermite, sur la nécessité de calculs effectifs et sur l'unité de l'algèbre avec l'analyse et l'arithmétique, ont modelé son interprétation de Galois et des pistes ouvertes par celui-ci. Réciproquement, Hermite a inséré les résultats de Galois dans une vaste synthèse appuyée sur la théorie des invariants et les fonctions elliptiques dont la théorie de Galois usuelle a mutilé la mémoire. La fin de l'article revient sur les problèmes méthodologiques ainsi soulevés dans l'interprétation des travaux de Galois et de leur postérité.

Bringing together Galois and Hermite may seem strange at first sight. Our images of them are so contradictory: the young, cheeky rebel and the crusty professor, the precursor of structural viewpoint and the opponent of Cantorian set theory, the revolutionary, rejecting authorities and rejected by them, and the conservative supporter of the Church and the monarchy, at the centre of French academic mathematics.

FIGURE 1. Two usual representations of Évariste Galois and Charles Hermite: rotogravures of Galois's portrait at 15 (left), published in [Dupuy 1896] and of Hermite's photo at 65 (right), at the beginning of [Hermite *Œuvres*, vol. 3].

Indeed, we can well imagine the outrage that one who wrote: "It was not then without pain and indignation that places in the Restoration government were seen to become the prey of the highest bidder in terms of monarchial and religious ideas,"[1] would have felt had he been privy to the other's explanation on one of his votes:

> In the person of M. Wurtz, M. B[ertrand] has a rival in the Academy for [election to] the High Council [for Public Education] but I have yet another reason, aside from that of kinship, to vote in favor of the latter. M. Wurtz, who may be an excellent man, will represent in the Council if he ends up there, M. Ferry's Article 7, that is, the expulsion from education of the Jesuits and religious communities.[2]

This opposition extends even to their views on how to do mathematics. "I thought I noticed a tendency of my mind to avoid calculations in subjects I was dealing with and, moreover, I recognized an insurmountable difficulty for whoever might wish to carry them out generally in the questions I dealt with," wrote Galois from the Sainte-Pélagie jail.[3] And his famous: "Take a flying leap over these calculations" and "here we do the analysis of analysis" have become legend. Hermite, on the contrary, presented himself as "doing this good old analysis which above all wants to be simple and clear, following such masters as Euler, Lagrange, Gauss and Jacobi,"[4] and put computations at the core of his conceptual practice: "Nothing seems to me more opposed to the truth and reality of things," he wrote, "than when M. Poinsot says: 'Computation is an instrument which produces nothing

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1 E.G., Lettre sur l'enseignement des sciences, *Gazette des écoles*, January 2, 1831, repr. in [Galois 1962, p. 21]: *Ce n'était donc pas sans douleur et indignation que dans le gouvernement de la Restauration, on voyait les places devenir la proie des plus offrants en fait d'idées monarchiques et religieuses.*

2 Letter XVII of April 5, 1880 in [Hermite & Mittag-Leffler *Corresp.*, 5 (1984), p. 70]: *M. B[ertrand] a un rival à l'Académie pour le Conseil supérieur [de l'Instruction publique] dans M. Wurtz, mais j'ai pour voter en sa faveur un autre devoir encore que celui de la parenté. M. Wurtz qui est cependant un bien excellent homme, représentera dans le Conseil, s'il y parvient, l'article 7 de M. Ferry, c'est-à-dire l'expulsion des jésuites et des communautés religieuses de l'enseignement.*

3 Préface à deux mémoires d'analyse, manuscript written at Sainte-Pélagie, December 1831, in [Galois 1962, p. 11]: *J'ai cru observer cette tendance de mon esprit à éviter les calculs dans les sujets que je traitais et qui plus est j'ai reconnu une difficulté insurmontable à qui voudrait les effectuer généralement dans les matières que j'ai traitées.* The two other quotes, *Sauter à pieds joints sur ces calculs* and *ici on fait l'analyse de l'analyse* also come from this preface, p. 9 and 11.

4 Letter of February 22, 1878 in [Hermite & Genocchi *Corresp.*, p. 9–10]: *[N]ous faisons de cette analyse de la bonne vieille roche, qui avant tout veut être simple et claire, en suivant les maîtres qui se nomment Euler, Lagrange, Gauss et Jacobi.*

by itself and which in some way only gives back the ideas entrusted to it'."[5] The reference to Poinsot is especially relevant, because Poinsot's explicit views, not only on computations but also on the need to study permutations and relations *per se*, are remarkably close to those of Galois.[6]

But this is not the whole story and Galois and Hermite have more in common than one might suspect. Both were students in Louis Richard's mathematics class at the Lycée Louis-Le-Grand and both benefitted from his encouragement to read significant authors outside curricular limits. Richard's support of Galois is well documented, see [Dupuy 1896]. As for Hermite, he explained for instance to Gösta Mittag-Leffler:

> I studied first in Nancy and then in Paris, at the Collège Henri IV and the Collège Louis-le-Grand, where I had an excellent and highly qualified man for a mathematics teacher. I have kept a most grateful memory of M. Richard: he would send me to the Sainte-Geneviève Library to read articles in Gergonne's *Annales*. I spent long hours browsing through the publications of the Academies of Science of Paris, Berlin, and Saint-Petersburg, before I even knew differential calculus, simply concentrating on those that I could understand.[7]

Both then began to publish original work while still students and both first failed their entrance to Polytechnique. Hermite linked his mediocre achievement in intellectual competitions to his early involvement with real mathematics; he confessed to Thomas Stieltjes:

> I also abhorred examinations, and I passed one year, when I was a student in *Mathématiques spéciales*, reading the works of Euler, the publications of the Academies in the Sainte-Geneviève Library, etc. instead of revising problems in

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5 Hermite to Leo Königsberger, March 2, 1876, Staatsbibliothek zu Berlin, Handschriftenabteilung: *Rien ne me semble plus contraire à la vérité et à la réalité des choses que ce que dit M. Poinsot, dans les termes suivants: 'Le calcul est un instrument qui ne produit rien par lui-même et qui ne rend en quelque sorte que les idées qu'on lui confie'.*

6 See for instance [Grattan-Guinness 1990, p. 1231–1232], [Goldstein, Schappacher & Schwermer 2007, p. 35], [Ehrhardt 2007, p. 94–100]. Poinsot's work on equations and permutations, and his view of algebra as the "science of order" are analyzed in depth in [Boucard 2011].

7 Letter LXXX, c. September 15, 1882, in [Hermite & Mittag-Leffler *Corresp.*, 5 (1984), p. 168–169]: *J'ai été d'abord au collège de Nancy, puis à Paris au collège Henri IV et au collège Louis-Le-Grand, où j'ai eu un homme excellent et d'un mérite supérieur pour professeur de mathématiques. M. Richard me laisse le plus reconnaissant souvenir; il m'envoyait à la bibliothèque Sainte Geneviève lire des articles des Annales de Gergonne. J'y passais de longues heures à feuilleter les mémoires des Académies des sciences de Paris, de Berlin, de Saint-Pétersbourg, que j'ai parcourus avant de connaître le calcul différentiel, en m'attachant à ceux que je pouvais comprendre.* On Louis Richard, see his obituary by Olry Terquem, [Terquem 1849], as well as [Brasseur 2010], for which I am indebted to Norbert Verdier.

> geometry, statics, etc. M. X… had taken an aversion to me and I paid for my whims of an *écolier savant* by a humiliating failure.[8]

And if Hermite, in contrast to Galois, finally succeeded—a more docile nature? fewer family and money problems? examiners in a better mood?—he remained at Polytechnique only one year before leaving and devoting himself completely to mathematics, a path Galois probably would have liked to follow. It was to Hermite that Richard chose to pass on Galois's surviving schoolwork, see [Ehrhardt 2008]. And it was Galois's ghost that a visitor to the Bertrand family home felt he saw when meeting there Hermite for the first time—an anecdote which fittingly closes Joseph Bertrand's review of Paul Dupuy's biography of Galois:

> One of my father's brothers, Dr. Stanislas Bertrand, who had never studied mathematics, lived in close relation with Galois. He had met him in 1830, both in the offices of the *La Tribune* newspaper, and in the secret meetings of the Society *Aide-toi, le ciel t'aidera* ('Help Yourself and Heaven Will Help You'); practices which found them sitting together on police station benches. Fifteen years later, my Uncle, coming to visit me, found me speaking with a young man whom he seemed to observe with particular attention and to listen to with astonishment. He told me the next day: 'I have undergone a great shock; for a quarter of an hour I thought I was seeing and hearing Évariste Galois!' He had seen and heard Charles Hermite.[9]

These intimations of a resonance between Galois and Hermite took on some deeper substance in 1847, when the 24-old Hermite, out of Polytechnique and still jobless, wrote to Carl Gustav Jacob Jacobi about number theory:

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8 Letter 59 of July 2, 1884, in [Hermite & Stieltjes *Corresp.*, vol. I, p. 129]: *J'ai eu aussi les examens en horreur, et j'ai passé une année, étant élève de mathématiques spéciales, à lire à la bibliothèque Sainte-Geneviève les mémoires des collections académiques, les ouvrages d'Euler, etc. au lieu de me mettre en mesure de répondre sur les questions de géometrie, de statique, etc. M. X. m'avait pris en aversion et j'ai expié par un humiliant échec mes fantaisies d'écolier savant.* M. X. is Camille Gerono, the co-founder of the *Nouvelles Annales.*

9 [Bertrand 1899, p. 400]: *Un des frères de mon père, le Dr Stanislas Bertrand, qui jamais n'étudia les Mathématiques, a vécu dans l'intimité de Galois. Il le rencontrait en 1830, tantôt dans les bureaux du journal* La Tribune*, tantôt dans les réunions secrètes de la Société* Aide-toi, le ciel t'aidera*; ce qui les conduisit à s'asseoir ensemble sur les bancs de la police correctionnelle. Quinze ans après, mon oncle, venant me voir, me trouva causant avec un jeune homme, qu'il semblait regarder avec attention et écouter avec étonnement. Il me dit le lendemain: j'ai éprouvé une grande émotion, j'ai cru pendant un quart d'heure voir et entendre Évariste Galois. Il avait vu et entendu Charles Hermite.* The mathematician Joseph Bertrand was Hermite's brother-in-law. The Society 'Aide-toi, le ciel t'aidera' was a liberal society created in 1827 in order to lobby against Charles X for legislative elections. *La Tribune des départements* was a radical newspaper, opposed to monarchy, and published between 1829 and 1835.